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TO THE UNIVERSITY
BY
ROBERT FINCH, M. A.
OF BALLIOL COLLEGE.

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THE

DIARIAN MISCELLANY:

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All the Uſeful and Entertaining Parts, both Mathematical and Poetical, extracted from the

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THE

MATHEMATICAL PARTS

OF THE

LADIES' DIARIES.

1732.

Of the Eclipſes in 1732.

TO the inhabitants of our terraqueous globe there will happen five eclipſes: Three times will the moon, in her wandering courſe, interpoſe and hide the ſplendid rays of the ſun from our view; and twice will the earth, in its courſe, ſo fall in the line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Moon eclipſed May 28, at 2 in the afternoon, inviſible.

2. Sun eclipſed June 11, about noon; but by reaſon of the moon's ſouth latitude and parallax, is inviſible here.

3. Sun eclipſed November the 6th, at 4 in the afternoon; but ſo ſmall, as not viſible to the naked eye.

4. Moon eclipſed on Monday the 20th day of November, at three quarters after nine at night, total and viſible.*

Computed by	Begin.	Begin	End	Midd	End	Digits
	h. m.	to. da.	to. da.			
Aſtronom.Carol.Coventry	viii 2	ix 0	x 36	ix 48	xi 33	21 17
Mr. Chattock, London	8 1	9 0	10 42	9 51	11 41	21 27
Mr. Leadbetter, London	8 13	9 11	10 46	9 58	11 44	20 48
Mr. Bulman, Lewiſham	8 9	9 6	10 43	9 54	11 40	21 14
Mr. Bulman, Carliſle	7 58	8 55	10 32	9 43	11 29	21 14
Mr. Turner, Hull	8 37	9 35	11 13	10 24	12 11	21 0
Mr. Oats, Givenna	7 41	8 39	10 17	9 28	11 15	
Mr. Williams, Middleton	4 2	- - -	- - -	- - -	7 38	17 2
Mr. Brown, Bridgenorth	7 58	8 55	10 32	9 44	11 29	21 14
Mr. Paternoſter, Hitchin	7 50	8 49	10 32	9 41	11 31	21 35

5. Sun eclipſed December the 6th, at 15 minutes after nine in the morning; viſible, but ſo very ſmall, that not above one three hundred and ſixtieth part of his diameter will be obſcured, and ſo not perceptible to a naked eye.

New

---

* This eclipſe was obſerved thus:

At	By	Begin.	Immerſ.	Emerſ.	End
		h. m. s.	h. m. s.	h. m.	h. m. s.
London	Geo. Graham	8 1 30	8 59 30	10 38	11 37 0 a.t.
London	Mr. Hodgſon	8 1 30			11 36 30 a.t.
York	John Turner	8 1 0	8 59 0	10 40	11 41 0 a.t.
Rome	Di. Revillas, Jo. Bottarius, & Euſt. Manfredi	8 51 49	9 48 24		12 26 55 t.t.

New Queſtions.

I. QUESTION 163, *by Mr.* Sam. Aſhby.

In verdant fields one ſummer's morning fair,
In walking forth to take the pleaſant air,
Pleas'd with the harmony o'th' warbling notes
Of larks and nightingales' extended throats.
And pleaſing zephyrs, with a gentle breeze,
Spread o'er the plains, did waft the verdant trees;
And Sol's refulgent rays join to complete
This lovely ſcene: Where I by chance did meet
A Geodecian, in a park, by th' way,
Was thither come, the ſame for to ſurvey:
Whoſe form he a right-angled triangle found,
In which was made a walk exactly round,
And touch'd all ſides of the ſaid triangle;
In which round walk four other walks quadrangle
Were made; denoted by *A*, *B*, *C*, *D*,
Meeting in the round walk's periphery.
The area * of the whole triangle's known,
And each ſide † of the quadrilateral's ſhown;
By which the following he was to produce,
Baſe, perpendicular, and hypothenuſe?
But finds his ſkill will not reſolve this doubt,
So begs you'll lend your aid to help him out.

* *The area* = 55296 *ſquare chains.*

† $\begin{cases} AB = 152{\cdot}99 \\ BC = 191{\cdot}11 \\ CD = 93{\cdot}03 \\ DA = 41{\cdot}56 \end{cases}$

II. QUESTION 164, *by Mr.* John Turner.

Two men, *A* and *B*, buy a piece of ground in an unknown northern latitude: but it was obſerved that on a certain day in the year, alſo unknown, the ſun's altitude upon the ſouth part of the meridian, at the ſaid place, was 42° 30′; and upon the north part of the meridian, his altitude above the horizon was 4° 30′: The limits of the ground were to be marked out by the ſhadow of the vertex of a tree 20 yards high, on that ſame day when the altitude of the ſun on each part of the meadow was obſerved as above-mentioned. It is required hence to find the latitude of the place, and the ſun's declination; and alſo the ſhare of the ground belonging to each man; *A* being to have for his part the greateſt triangle that can be cut out of the ſaid conic ſection deſcribed by the ſhadow of the tree's top, and *B* to have the remainder.

III. QUESTION 165, *by Mr.* Tho. Grant.

Pray, gentlemen gaugers, be pleaſed to lend
Your aſſiſtance and aid to a brother and friend;
Who lately has met with a caſk in his round,
The content of which by him cannot be found
From any problem or theorem taught,
By thoſe who have on ſtereometry wrote.
A ſpheroidical fruſtum it ſeemeth to be,
Whoſe dimenſions are ſuch as hereunder * you ſee;
Hence you are deſir'd to ſhew its content
By a general rule, and how much of the length
Is on each ſide the greateſt bulge of the caſk;
Which done will reſolve him in all he does aſk.

* *Gr. head* 32·0. *Leſ. head* 27·0. *Bung* 36·0. *Length* 45·0.

IV. QUESTION 166, *by Mr.* Chr. Maſon.

A canon give, that will exhibit fair
All perfect numbers; and alſo declare
What thoſe from unit to ten millions are?

V. QUESTION 167, *by Mr.* Turner.

Let there be a triangle whoſe 3 ſides are given, viz. 415, 353, and 488: And upon the three angular points, as centers, let there be deſcribed three circles whoſe radii are 130, 80, and 70: Let a fourth circle be drawn, which ſhall touch theſe three circles. It is required to find its diameter?

VI. QUESTION 168, *by Mr.* Chr. Maſon.

I once ſupinely trifling time away,
With two old quondams who at dice would play.
Each ſtak'd his guinea, fifteen up the game;
And I by chance had juſt got ten o'th' ſame.
The other two had not ſuch luck to thrive;
The one being eight, the other only five:

When they propos'd no farther to advance,
But part the ſtakes, according to each chance.
And I well weighing gameſters fickle caſe,
With feign'd denial, did their choice embrace.
I now deſire ſome artiſt to unfold,
How much each gameſter is to have o'th' gold.

The PRIZE QUESTION, *by Mr.* Rob. Fearnſide.

A whimſical merchant of late did import,
Than buſineſs more for diverſion and ſport,
Cylindric and conical poles not a few,
Whoſe dimenſions * in part, you have here in full view.
Now it happen'd, as with him one ev'ning I ſat,
By degrees did begin mathematical chat;
Till by ſome how or other this bargain at laſt,
Gave riſe to this queſtion, he ſtarted in haſt.
' The greateſt of thoſe ſort of poles I wou'd know,
' That's poſſible up this ſame chimney to go,
' Whoſe width, I remember, when meaſur'd, to be
' Juſt forty-eight inches at the mantle tree;
' And likewiſe between the ſaid tree and the floor,
' The diſtance was found to be twice as much more:
' The man who the eaſieſt method can ſhew,
' On demand twelve diaries may claim as his due.'
Half aſſur'd of ſucceſs, I reſolv'd to begin,
His queſtion to ſolve, and the diaries to win;
But I found after all, to my grief and vexation,
The *X*'s quite vaniſh'd out of my equation.
Therefore, ladies, the manner to ſolve it pray ſhew,
And when reading the diaries I'll think upon you.

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* *The baſes of the cylindric poles were 12 inches, and the ſides of the recti-conical cones, were to their baſes as 4 to 1.*

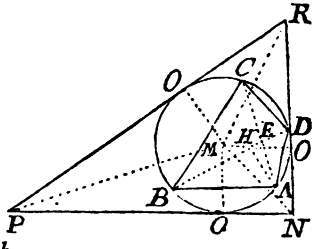
Queſtions

1733.

Queſtions anſwered.

I. QUESTION 163 *anſwered by Mr.* W. Grimmett.

DRAW the diagonals BD and CA; and from the center of the inſcribed circle draw PM, RM, and NM: Let AH be perpendicular to BD, as alſo MO's to PN, PR, and RN.



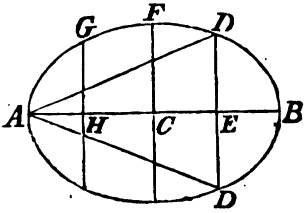
Put $s = BC = 191{\cdot}11$, $c = AD = 41{\cdot}56$, $b = CD = 95{\cdot}03$, $d = BA = 152{\cdot}99$, and $a = BE$. Then from the ſimilarity of triangles we have as, $d : a :: b : \frac{ab}{d} = CE$; and after the ſame way of reaſoning DE will be had $= \frac{bca}{sd}$; and $AE = \frac{ca}{s}$. But $a + \frac{bca}{sd} = BD$, and $\frac{ab}{d} + \frac{ca}{s} = AC$; and then we ſhall have $\frac{baa}{d} + \frac{caa}{s} + \frac{bbcaa}{sdd} + \frac{bccaa}{ssd} = cs + db$; which reduced will be $a = \sqrt{\frac{ssscdd + d^3bss}{bdss + ddcs + bbcs + ccdb}} = 167{\cdot}65 = BE$; and then BD will be found $= 189{\cdot}81$.

Again, from the circle and its inſcribed triangle BAD, in which the perpendicular AH is let fall, it will be as $AH : DA :: BA$: the diameter of the circle $\doteq 191{\cdot}6832$; and conſequently MO the radius is $95{\cdot}8416$.

Put $b =$ area of the triangle PNR; $n = MO$; $x = RN$. Then $PN = \frac{2b}{x}$, and $\sqrt{xx + \frac{4bb}{xx}} = PR$; whence $nx + \frac{2bn}{x} + n\sqrt{xx + \frac{4bb}{xx}} = 2b$: Per reduction there comes forth $xx - \frac{bx}{n} - nx + 2b = 0$; which ſolved, x will be found $= 285{\cdot}7084 = NR$: Then $PN = 387{\cdot}079$, and $PR = 481{\cdot}1031$ chains. *Q.E.I.**

II. QUESTION 164 *anſwered by Mr.* Rob. Fearnſide.

'Tis obvious that the declination of the ſun is equal to half the ſum of the meridian altitudes, which conſequently is 23° 30′; and the latitude 71°.



Now the tree being ſuppoſed to be placed in *H*, 'tis evident, as the ſun does not ſet, that its ſummit will deſcribe the ellipſis *AGFDBDA*; therefore, by plain trigonometry, AH will be found $= 21{\cdot}826$, and $BH = 254{\cdot}124$; and (finding the altitude of the ſun when due eaſt or weſt) $GH = 43$, and conſequently FC the ſemi-conjugate diameter $= 79{\cdot}5$.

Then put $AB = 2a$, $FC = b$, $CE = x$, and $DE = y$. Then, per conics, $aa - xx : yy :: aa : bb$; therefore $y = \frac{b}{a}\sqrt{aa - xx}$. Now $\overline{a + x} \times \frac{b}{a}\sqrt{aa - xx}$, = area of the triangle *DAB*, muſt be a maximum; which put into fluxions and ordered, x will be $= \frac{1}{2}a$, and the area of the greateſt triangle will be $= 7124{\cdot}37 = 2$ a. 3 r. 32 p. = *A*'s ſhare,
and $27335{\cdot}88 = 4$ 0 29 = *B*'s ſhare.

Mr. *Grimmett* having diſcovered a new property of the ellipſis, after a ſolution to this queſtion, concludes with this other following method.

Suppoſing a circle inſcribed in the ellipſis, then it will be as the radius of the inſcribed circle, is to the perpendicular height of the equilateral triangle inſcribed therein; ſo is the ſemi-tranſverſe of the ellipſis, to the perpendicular of the

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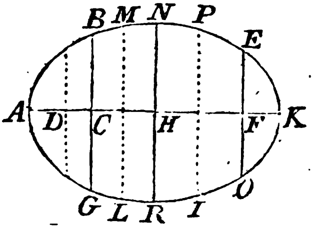
* I. QUESTION 163.

To inſcribe a quadrilateral, whoſe ſides are given, in a circle, may be ſeen in VIETA's *Opera Mathematica* p. 277, and in SIMPSON's *Select Exerciſes pr.* 35.

greateſt triangle inſcribed in the ellipſis = 206·9652. The anſwer will be 14277·7 = 2 a. 3 r. 32 p. = *A*'s } ſhare.
20251·5 = 4 0 29 = *B*'s }

Mr. *Charles Forreſt* has calculated the ſolution to this queſtion trigonometrically, with the ſun's altitude, azimuth, and length of the ſhadow at every hour of the day, and from thence according to the doctrine of conics has given the true ſolution as above.*

III. QUESTION 165 *anſwered by Mr.* Ri. Lovatt.



Given $\begin{cases} CF = 45 = d, \\ NH = 18 = b, \\ BC = 16 = n, \\ EF = 13·5 = m. \end{cases}$

Put $a = AH$, $e = CH$, and $h = \frac{bb}{dd}$. Then, per conics, $aa : bb :: aa - ee : nn$; hence $\frac{aabb - aann}{bb} = ee$. Subſtitute $pp = \frac{bb - nn}{bb}$: Then $ap = e$; and hence $d - ap = HF$.

Again,

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* II. QUESTION 164.

Having found the declination and latitude as in the firſt ſolution above, viz. by taking half the ſum of the greateſt and leaſt altitudes for the declination, and then by taking the complement of the difference between this declination and the greateſt altitude for the latitude of the place, which are general rules; next compute the altitude when due eaſt or weſt, and then ſay as radius : the height of the tree : : the cotangent of each of theſe three altitudes : each of the three lines *HA*, *HB*, *HG*.——All the reſt of the firſt ſolution above is very clear.

The truth of Mr. Grimmett's theorem above may appear thus: From Mr. Fearnſide's ſolution we find that the altitude of the greateſt triangle is 3-4ths of the tranſverſe axe; and by geometry we know that the altitude of an equilateral triangle is alſo 3-4ths of the diameter of its circumſcribed circle; wherefore as the diameter of any circle is to the altitude of its inſcribed equilateral triangle, ſo is the tranſverſe axe of an ellipſe to the altitude of its greateſt *inſcribed triangle.*——We may hence remark alſo that the *equilateral is the greateſt* triangle that can be inſcribed in a circle.

Again, $aa:bb::aa-dd+2dpa-ppaa:mm$; and hence $aamm=aabb-ddbb+2dpbba-bbppaa$. Subſtitute $c=bb-mm-ppbb$; and $k=2dpbb$; then $a=\sqrt{\frac{ddbb}{c}+\frac{\frac{1}{4}kk}{cc}}-\frac{\frac{1}{2}k}{c}=40$. Hence $CH=18{\cdot}311$, and $HF=26{\cdot}68$.

To find the content; put $m=AH=40$, and $n=CH=18{\cdot}311$; find 18311000000000000000000 ordinates rightly applied betwixt C and H, which will give the equal cylinder; thus put $d=1831100$, &c. then $\frac{{}^{*}3mmd-\frac{3}{2}nn\frac{1}{2}n-nnd}{3d}\times\frac{3}{4}b=$ the ſquare of the equal cylinder; and $\frac{{}^{*}\sqrt{3mmd\frac{3}{2}nn-\frac{1}{2}nn-\frac{1}{2}n-nnd}}{3d}\times 4b=34{\cdot}7198814$ $=ML=$ the diameter of the cylinder $=BNRG$; and by the ſame method the cylinder $=$ to $NEOR$ is $=33{\cdot}2216007$ $=PI$; hence the content is $\left\{\begin{matrix}BNRG=61{\cdot}470027\\NEOR=82{\cdot}041786\end{matrix}\right\}143{\cdot}5118$ ale gallons.

The ſame anſwered by Mr. J. Turner.

Put $\left\{\begin{matrix}\text{bung diam.} & 36=m\\ \text{greater head} & 32=n\\ \text{leſſer head} & 27=s\\ \text{length} & 45=2t\end{matrix}\right\}$ and AC, the part wanting, $=2x$. Then is $HF=t+x$, and $CH=t-x$.

As $tt+2tx+xx:mm-ss::tt-2tx+xx:mm-nn$, by the property of the ellipſis, (as per Ward's Introduct. p. 448) therefore $ttmm+2tmmx+mmxx-ttnn-2tnnx-nnxx=ttmm-2tmmx+mmxx-ttss+2tssx-ssxx$; by reduction and tranſpoſition, $ssxx-nnxx+4mmtx-2tnnx-2tssx=ttnn-ttss$.

Put $ss-nn=-b$, and $4mmt-2tnn-2tss=c$, and $ttnn-ttss=d$; then $-bxx+cx=d$; and, extracting the root, $x=\frac{c}{2b}-\sqrt{\frac{cc}{4bb}-\frac{d}{b}}=4{\cdot}10$ inches; then $2x=8{\cdot}20$ inches $=$ the part wanting DC: from whence we find HF the diſtance from the greateſt bulk of the caſk to the leſſer head to be $=26{\cdot}6$ inches, and CH to the greater head $18{\cdot}4$ inches. And the content is $143{\cdot}9$ ale gallons.

Mr. *Grimmett*, after two different ſolutions to this queſtion, delivers this theorem:

Theorem. { The ſquare root of each difference between the ſquare of half the bung diameter and the ſquare of half the diameter of each head, put into one ſum: It will be as the ſum is to either of thoſe roots, ſo is the length of the caſk to the diſtance of the reſpective head from the bung.*

IV. QUESTION 166 *anſwered by Mr.* Grimmett.

If from unity be taken how many numbers ſoever in double proportion continually, until the whole added together be a prime number; and if this whole be multiplied by the laſt term of the ſeries which conſtitutes the prime, the product will be a perfect number. 36 Euclid 9.

From ſuch a ſeries it may be obſerved, that any term made leſs by unity, will be = the ſum of all the preceding terms. Put therefore $a = 2$; and $x =$ its variable exponent (for in the firſt operation it will repreſent 1, in the next 2, and then 3, &c. till it be raiſed to a^{x+1} and being leſſened by unity may be a prime number. Thus

x being

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* III. QUESTION 165.

By the nature of the ellipſe, $\sqrt{HN^2 - CB^2} : \sqrt{HN^2 - FE^2} :: CH : HF$; and, by compoſition, &c. $\sqrt{HN^2 - CB^2} + \sqrt{HN^2 - FE^2} : CF :: \left\{ \begin{matrix} \sqrt{HN^2 - CB^2} : CH \\ \sqrt{HN^2 - FE^2} : HF \end{matrix} \right\}$, which therefore both become known; and this is Mr. Grimmett's theorem, mentioned above.

Again, by the nature of the ellipſe, $\sqrt{HN^2 - CB^2} : CH :: HN : HA$ the ſemi-tranſverſe; which being thus found, the contents of the two parts $BNRG$, $NEOR$, of the caſk being computed ſeparately by the common rules, their ſum will be the whole content.—— Or indeed their contents are eaſily computed without the tranſverſe axe by rule 1 page 278 of my Menſuration.

N. B. The two expreſſions marked * in Mr. Lovatt's ſolution are wrong printed; but they are here given as they ſtand in the *original, as it is not* eaſy to diſtinguiſh what are the true expreſſions meant.

$$x \text{ being} = \left\{\begin{array}{c}1\\2\\3\\4\\5\\6\\\&c.\end{array}\right\}\overline{a^{x+1}-1}\times a^{x} = a^{2x+1}-a^{x} = \left\{\begin{array}{ll}\left.\begin{array}{l}6\\28\end{array}\right\} & \text{perf. num.}\\ — & \text{abounding}\\ 496 & \text{perf. num.}\\ — & \text{abounding}\\ — & \text{dimumitive}\\ & \quad\&\text{c.}\end{array}\right.$$

Whence the perfect numbers are 6, 28, 120, 496, 2016, 8128, 32640, 130816, 523776, 2096128, and 8386560; all the perfect numbers required per queſtion.

Mr. C. Maſon *the propoſer gives this rule:*

$\overline{1+2}\times 2 = 6$; $\overline{1+2+4}\times 4 = 28$; $\overline{1+2+4+8}\times 8 = 120$, &c.

Mr.

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* IV. QUESTION 166.

By *Eucl.* IX. 36, : $1+2+2^{2}+2^{3}+2^{4}+$ &c. to $2^{n} : \times 2^{n}$ is a perfect number when the ſum of the ſeries is a prime number; but the ſum of the geometrical ſeries is $2^{n+1}-1$, therefore $\overline{2^{n+1}-1}\times 2^{n}$ is a perfect number when $2^{n+1}-1$ is a prime number. Taking $n=0$; then $2^{n+1}-1$ is $=1$ a prime, and $1\times 2^{0} = 1\times 1 = 1$ the firſt perfect number: If $n=1$; then $2^{n+1}-1 = 3$ a prime, and $3\times 2^{1} = 6$ the next perfect number: If $n=2$: then $2^{n+1}-1 = 7$ a prime, and $7\times 2^{2} = 28$ the 3d perfect number: If $n=3$; then $2^{n+1}-1 = 15$ which is not a prime, and therefore $15\times 2^{3} = 120$ is not a perfect number: In like manner it will appear that no other greater odd number can be put for n ſo as to make the expreſſion $\overline{2^{n+1}-1}\times 2^{n}$ a perfect number; n muſt therefore be always an even number for finding the other perfect numbers; but it cannot be any even number, as ſome have falſely aſſerted. Dr. Harris ſays that there are only ten perfect numbers between 1 and 1,000,000,000,000.

This rule of Euclid's only demonſtrates that a number found by it will be a perfect number; but neither it nor any other that I know of, ſhew that there may not be other perfect numbers beſides thoſe found by this rule.

Mr. Sam. Aſhby *anſwers thus:*

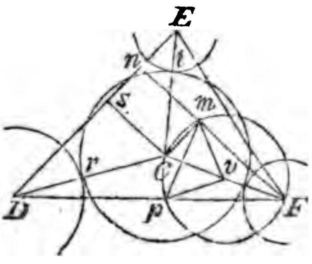
The canon. { If from any power of 2 be ſubtracted unity, and that remainder be a prime number, multiply it by half the ſaid power, and that product will be a perfect number.

Mr. Robert Fearnſide's *anſwer.*

Let $y^n x$ be the number ſought; its aliquot parts will be $1 + y + y^2 + y^3$, &c. till the exponent becomes n; and $x + yx + y^2 x + y^3 x$, &c. till the exponent be likewiſe n; then, from the nature of a perfect number, $1 + y + y^2 + y^3 +$ &c. $+ x + yx + y^2 x + y^3 x +$ &c. $= y^n x$; and conſequently $x = \dfrac{1 + y + y^2 + y^3 \text{ \&c.}}{y^n - 1 - y - y^2 - y^3}$: Now, that x may be a whole number, 'tis requiſite that $y^n - 1 - y -$ &c. be $= 1$, which only happens when y is $= 2$; whence the canon required becomes $2^n x$. If $n = 1$, x will be $= 1 + 2$, and the firſt perfect number $= 6$. If $n = 2$, x will be $= 1 + 2 + 4$, and the ſecond perfect number will be $= 28$. If $n = 4$, the third perfect number $= 496$. If $n = 6$, the fourth perfect number is 8123. If $n = 8$, the fifth number is 130816. If $n = 10$, the ſixth number will be 2196128, which are all the perfect numbers from unity to ten millions.

Woolfius in his *Elem. Math.* ſuppoſes n to be ſucceſſively the numbers 1, 2, 3, 4, 5, &c. which will not hold when n is ſuppoſed = the odd numbers 3, 5, 7, 9, &c. and Mr. *Cunn's* rule for finding a perfect number will not find all the above numbers; ſee p. 12 of his Decimals.

V. QUESTION 167 *anſwered by Mr.* Turner *the propoſer.*



This problem is taken from lemma 6 book 1 of Sir Iſaac Newton's Principia, where the geometrical conſtruction may be ſeen.

Having the three ſides of the triangle, the perpendicular Fn, and ſegments Dn and En, may be found.

Draw Cs perpendicular to DE, and Cm perpendicular to Fn; and put

$Cr = Ct = Cv = a$, $Fn = 345·8 = b$,
$Cm = sn = e$, $En = 70·7 = c$,
$mn = Cs = u$, $Dn = 344·3 = d$. Then is
$Dr = 130 = r$, $Ds = d - e$, and
$Fv = 70 = s$, $Es = c + e$; alſo
$Et = 80 = n$, $Fm = b - u$: Then

$$\left.\begin{matrix}(1)\ rr+2ra+aa=dd-2de+ee+uu\\(2)\ nn+2na+aa=cc+2ce+ee+uu\\(3)\ ss+2sa+aa=bb-2bn+uu+ee\end{matrix}\right\}\text{per 47 Euc. 1.}$$

Subtract the third ſtep from the firſt, and tranſpoſe the terms, and dividing by $2b$, we ſhall have

$$(4)\ u=\frac{rr-ss+2ra-2sa+bb-dd+2de}{2b}.$$

Put $rr-ss+bb-dd=f$; and $2r-2s=g$; then $u=\frac{f+ga+2de}{2b}$; and $uu=\frac{ff+ggaa+2fga+4fde+4dgae+4ddee}{4bb}$

Subtract the ſecond ſtep from the firſt, and tranſpoſing makes $2de+2ce=dd-cc-rr+nn+2na-2ra$.

Put $2d+2c=h$; $dd-cc-rr+nn=k$; and $2n-2r=-l$;
(5) Then $e=\frac{k-la}{h}$; and $ee=\frac{kk-2kla+llaa}{hh}$.

In the firſt ſtep ſubſtitute the value of uu found, then $rr+2ra+aa=dd-2de+ee+$ $\frac{ff+2fga+ggaa+4fde+4dgae+4ddee}{4bb}$, or $4bbrr+8bbra+4bbaa-ggaa-2fga-ff-4bbdd=4bbee+4ddee+4fde-8bbde+4dgae$: Put $4bb+4dd=m$; $4fd-8bbd=-n$; and $4dg=p$; then $4bbrr+8bbra+4bbaa-ggaa-2fga-ff-4bbdd=mee-ne+pae$; for e and ee put the values above found and bring it out of the fractions, will give this quadratic equation:

$$\left.\begin{matrix}+4bbhh\\+plh\\-gghh\\-mll\end{matrix}\right\}aa\left.\begin{matrix}+8bbhhr\\+2mkl\\-2fghh\\-nlh\\-pkh\end{matrix}\right\}a=\left\{\begin{matrix}4bbddhh\\+ffhh\\+mkk\\-4bbrrhh\\-nkh.\end{matrix}\right.$$

In numbers $aa+195·085a=52778·24$ when divided by the coefficients of the higheſt power; and, extracting the root, $a=152·048$ the radius of the circle touching the three circles given in poſition and magnitude; conſequently the diameter is $=304·096$ the anſwer.

Mr. *George Brown*, after giving the true anſwer in a very conciſe way, ſays, ſince it is not limited how the three circles ſhall be placed on the angular points, it will admit of ſo many anſwers as the circles are to be varied.

Mr. Rich. Lovatt's *anſwer to the ſame.*

Put $b = \frac{488^2 - 130^2 - 70^2}{488 \times 2}$; $d = \frac{130 - 70}{488}$; $n = \frac{353^2 - \overline{80 - 70}^2}{353 \times 2}$; $m = \frac{80 - 70}{353}$; $p = \frac{DFE}{2}$; and $a = CF$. Then, per axiom 4, $b - ad = pE$; and $n - am = mF$; and CF will be the diameter of a circle, whoſe periphery will paſs through mCF; and (20 Eucl. 3) the angle at the center, v, is double to the angle, F, at the periphery; alſo (5 Eucl. 1) the angle $vpm = vmp$. Put $h = \text{s.} \angle vmp = 33^\circ\, 34'$; $k = \text{s.} \angle mvp = 112^\circ\, 56'$; then $h : \frac{a}{2} :: k : \frac{ak}{2h} = mp$. And by that noble theorem which I mentioned in the Diary 1731, queſt. 143, we have this analogy: $\overline{b - ad} \times \overline{n - am} : \frac{b - ad - n + am}{2} + \frac{ak}{2h} \times \frac{-b + ad + n - am}{2} + \frac{ak}{2h} :: 1 : pp$; and $pp \times \overline{b - ad} \times \overline{n - am}$

$$= \frac{b - ad - n + am}{2} + \frac{ak}{2h} \times \frac{-b + ad + n - am}{2} + \frac{ak}{2h}.$$

In numbers, $1682aa + 9{\cdot}071a = 9473{\cdot}58 + 1023{\cdot}9$, and $a = \sqrt{63140{\cdot}3} - 27 = 224{\cdot}273$; hence $224{\cdot}773 - 70 = 154{\cdot}273 = Cr$, which doubled is the diameter $308{\cdot}56$ of the circle required.

VI. QUES-

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* V. QUESTION 167.

This is one of the problems of APOLLONIUS on *Tangencies*, and is conſtructed by his reſtorers VIETA and our countryman the Rev. Mr. JOHN LAWSON, who has lately publiſhed an engliſh reſtoration of this piece of APOLLONIUS's works, where it appears that the problem hath ſeveral caſes according as the fourth circle is to touch the other three either all internal or all externally, or elſe ſome internally and the reſt externally.

This problem has alſo been attended to by ſeveral other reſpectable perſons, it being conſtructed by Sir ISSAC NEWTON in *Lemma* 16 *lib.* 1 of the *Principia*, and in his *Univerſal Arith.* prob 47; by the MARQUIS DE L'HOSPITAL in his *Sections Conique* liv. 10 *ex.* 4 *cor.* 1; and by Mr. THO. SIMPSON at the end of his *Geometry.*

Con-

* VI. QUESTION 168 *anſwered by* Mr. Rob. Fearnſide.

Let A, B, C, repreſent the three players; A wants 5 of being up; B 7, and C 10. Now it is plain the game will be ended in 20 throws at moſt; then $A+B+C$ muſt be raiſed to the 20th power; and as the players here are ſuppoſed equal, the coefficients of every term where the 5th power of A and upwards including the 20th is found, are to be added together, as alſo the coefficients where the 7th power of B and upwards is found, and the coefficients of the 10th power of C and upwards are to be added alſo together; theſe three totals will be in proportion to one another as the reſpective ſhares they are to have of the guineas.

He

Concerning this problem I ſhall alſo inſert the following extract from the *Hiſtoire des Mathematiques* par. M. MONTUCLA *Tom.* I. p. 263. . . .

VIETA, in a diſpute which he had with ADRIANUS ROMANUS, propoſed to him this queſtion. The ſolution which ROMANUS gave to it, though obvious, was very indifferent, viz. by determining the center of the circle ſought in the point of interſection of two hyperbolas; for as the problem is a plane one, it may be ſolved by plane geometry; by this VIETA ſolved it, and very elegantly: his ſolution is the ſame as that given in NEWTON's *Univerſal Arithmetic*. Another ſolution may alſo be ſeen in the 1ſt book of the *Principia* (this queſtion being there neceſſary for ſome determinations in *Phyſical Aſtronomy*) wherein NEWTON, by a remarkable dexterity, reduces the two ſolid loci of ROMANUS to the interſection of two right lines.——Moreover, DESCARTES attempted to ſolve this problem by the help of the Algebraical Analyſis, but without ſucceſs; for of the two ſolutions which he derived from thence, he himſelf acknowledges (ſee Lett. Tom. III. let. 80, 81) that one furniſhed him with ſo complicated an expreſſion, that he would not undertake to conſtruct it in a month; whilſt the other, though ſomewhat leſs complicated, was not ſo very ſimple, as to encourage him to ſet about a conſtruction of it.——Laſtly, the Princeſs ELIZABETH of *Bohemia*, who, it is well known, honoured DESCARTES with her correſpondence, deigned to communicate a ſolution to this Philoſopher; but as it is deduced from the algebraical calculus, it labours under the ſame inconveniences as that of DESCARTES.

* VI. QUESTION 168.

The method of *ſolving* queſtions of this kind, may be ſeen at *page 43 or 192 of* DE MOIVRE, or in ſome other books on Chances.

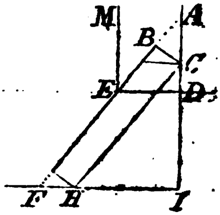
	l.	*s.*	*d.*
He who got 10 muſt have	2	3	10 +
He who got 8 muſt have	0	16	$1\frac{3}{4}$ +
He who got 5 muſt have	0	3	0 +

Mr. *John Ommanney*'s numbers are the ſame.

Mr. *James Hemmingway* and Mr. *Chr. Hale* alſo anſwered this queſtion.

The PRIZE QUESTION *anſwered.*

Let FI repreſent the floor, and $ADEM$ the chimney; then put $BC = d$, $ED = b$, $DI = c$; $LB = x$. Now, per ſimilar triangles, $x : d :: d : \frac{dd}{x} = BA$; and $d : \frac{dd}{x} :: b : \frac{bd}{x} = AD$. After the ſame manner of reaſoning we ſhall find $AC = \frac{d}{x}\sqrt{dd + xx}$; $CI = \frac{bd + cx}{x} - \frac{d}{x}\sqrt{dd + xx}$; and $CH = \frac{bd + cx}{dx}\sqrt{dd + xx} - \frac{dd}{x} - x$; which muſt be a maximum; conſequently put into fluxions, &c. the following equation will come out; *i.e.*

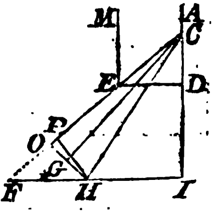


$$\left.\begin{matrix}cc\\-dd\end{matrix}\right\}x^6 + d^4x^4 - 2bcd^3x^3 + d^6x^2 + \left.\begin{matrix}bbd^6\\-d^6\end{matrix}\right\} = 0.$$

Brought into numbers and reduced, $x = 9{\cdot}74$ inches; and conſequently $CH = 175{\cdot}2$ inches $=$ 14 feet 7 inches 2 tenths, the length of the cylindric pole.

For the Conical Pole.

Let $ED = b$; $DI = c$; and $r = 4$; $CD = x$; $HO = 2y$: then $1 : r :: 2y : HC = 2ry$; $GC = y\sqrt{4rr - 1} = ny$ (putting $\sqrt{4rr - 1} = n$); as $2ry : ny :: 2y : PH = \frac{ny}{r}$; $PO = \frac{y}{r}$. Again, $x : \sqrt{bb + xx} :: c + x : FC = \frac{c + x}{x}\sqrt{bb + xx}$; and $x : b :: \frac{ny}{r} : PF = \frac{bny}{rx}$. Conſequently,



bny

$\frac{bny}{rx} - \frac{y}{r} = \frac{c+x}{x}\sqrt{bb+xx} - 2yr$. Ergo (putting $2rr - 1 = m$) $y = \frac{cr+rx}{bn+mx}\sqrt{bb+xx}$. Now it is evident that $GC = \frac{cnr+nrx}{bn+mx}\sqrt{bb+xx}$ muſt be a maximum; which in fluxions, &c. the following equation will come out, viz. $x^3 + \frac{2bn}{m}x^2 + \frac{bcn}{m}x = b^2c - \frac{b^3n}{m}$; brought into numbers and reduced, gives $x = 44{\cdot}88$ inches, and $GC = 168{\cdot}48$ inches $=$ 14 feet 0 inches, 48, the length of the conical pole required.*

The lot of 10 diaries fell to *F. R. S.*

Of

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* *The* PRIZE QUESTION.

It is true the proceſs above will bring out the longeſt pole which can be put quite into or up the chimney, but ſome of the expreſſions uſed in it are very improper: thus the expreſſion for *CH* in the former caſe, and *CG* in the latter, is not a maximum, but a minimum; for it has no maximum but infinity; and the thing to be found, though it be the longeſt pole that can be put quite up the chimney, is the minimum of *CH* or *CG*, that is the ſhorteſt pole which can reſt with one end on *FI*, the other on *AI*, and its ſide touch the point *E*: for it is evident that whether way this line be moved from this narroweſt or ſhorteſt poſition, its ſide will fall below the point *E*, and ſo it may be put up the chimney; but a longer cannot be put into the ſaid ſhorteſt poſition, and therefore not up the chimney.

The former part of the proceſs, for determining the length of the cylinder, may be brought out by a ſimple cubic equation thus:

Put $AD = z$; alſo $DE = b$, and $DI = c$, as above. Then $AI = c+z$, and $z : b :: c+z : \frac{c+z}{z} \times b = IF$; hence $\overline{c+z}|^2 + \frac{\overline{c+z}|^2}{z^2} \times b^2 = AF^2$, which muſt be a minimum. This in fluxions, &c. we get $z^4 + cz^3 - b^2cz - b^2c^2 = 0$; the root of which is evidently $z = \sqrt[3]{b^2c} = AD$, from which the poſition is determined, and the length of the cylinder can be eaſily expreſſed in terms of *b*, *c*, and *d* its diameter; thus the length *CH* is $= \frac{c+bq}{\sqrt{1+q^2}} \times q - dq$, putting $q = \sqrt[3]{\frac{c}{b}}$.

Of the Eclipſes in 1733.

To the inhabitants of our terraqueous globe, there will happen four eclipſes: Twice will the moon in her wandering courſe interpoſe and hide the ſplendid rays of the ſun from our view; and twice will the earth in its courſe ſo fall in the line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Sun eclipſed on wedneſday the 2d of May, at 6 in the evening, which will be a great and viſible eclipſe; and three-fourths of the ſun's diameter will be darkened.*

Computed by		Begin.		Mid.		End		Dur.		Digits	
		h.	m.	h.	m	h.	m.	h.	m.	d.	m
Aſtronom. Carolina,	Coventry	v	41	vi	35	vii	26	i	45	ix	45
Mr. Chattock,	London	5	44	6	38	7	29	1	46	9	6
Mr. Leadbetter,	London	5	42	6	36	7	27	1	45	9	20
Mr. Tho. Sparrow,	Notting	5	40	6	37	7	31	1	50	9	49
	Royſton	5	49	6	45	7	40	1	50	9	51
Mr. Chriſt. Hale,	Derby	5	38	6	35	7	23	1	44	9	35
	London	5	43	6	38	7	27	1	43	9	19
Mr. Samuel Travis,	Utoxeter	5	39	6	34	7	28	1	49	9	20
Mr. Will. Brown,	Bridgenor.	5	37	6	31	7	22	1	45	9	46
	Glouceſter	5	38	6	33	7	23	1	45	9	39
	Rome	6	53	7	39	8	24	1	21	8	12
	Edinburgh	5	30	6	24	7	17	1	47	10	31
	Paris	6	4	6	59	7	43	1	39	9	11
Mr. Will. Lovatt,	Mansfield	5	40	6	35	7	29	1	49	9	15
Mr. John Browne,	London	5	43	6	38	7	29	1	46	9	53
	Benhall	5	48	6	43	7	34	1	46	10	8
Mr. Tho. Wright,	Sunderlan	5	29	6	25	7	17	1	47	9	47
Mr. John Bulman,	Lewiſh.	5	36	6	41	7	20	1	44	9	43
	Carliſle	5	24	6	30	7	10	1	46	10	1
Mr. Nicholas Oats,	Falmouth	5	3	6	56	6	52	1	49	8	4
Mr. John Turner,	Hull	5	18	6	13	7	6	1	48	10	12
Mr. Tho. Williams,	Middleto	5	41	6	36	7	27	1	45	9	35
Mr. Rich. Lovatt,	Derby	5	38	6	35	7	23	1	44	9	35

2. Moon

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* This eclipſe was obſerved thus:

At *Gottenburg* in *Sweden* by *D. Birgerus Voſſenius.*

h.	m.	s.	
6	26	40	Before this was the beginning.
7	14	46	Total immerſion.
7	16	54	Emerſion.
8	5	50	End.

2. At

2. Moon eclipſed on thurſday the 17th of May, at 6 in the evening.

Computed by	Begin.		Mid.		End		Dur.		Dig.	
	h.	m.	h.	m.	h.	m.	h.	m.	h.	m.
Aſtronom. Carolina, Coventry	5	51	7	21	8	50	2	58	8	22
Mr. Sparrow, Nottingham	5	47	7	17	8	46	2	59	8	27
Mr. Hale, Derby	5	26	6	57	8	28	3	2	8	3
Mr. Browne, Bridgenorth	5	47	7	16	8	46			8	23
Mr. Bulman, Lewiſham	5	57	7	26	8	56	2	59	8	22

3. Sun eclipſed the 26th of October, at 5 in the evening, inviſible.

4. Moon eclipſed the 10th of November, at 1 in the afternoon, inviſible.

New

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2. At *Wittemberg* in *Saxony* by *John Frid. Weidler*.

h.	m.	s.	
6	36	5	Beginning.
7	29	20	Eleven digits.
7	46	5	Sun ſet eclipſed.

At	By	Begin.	Middle	End	Dig.
		m. s.	h. m. s	m. s.	
3. London	G. Graham	44 45	6 37 3	28 23 ap.t.	9$\frac{4}{5}$
4. Norton Court near Feverſham in Kent	Ste. Gray	49 15	6 40 0	7 32 30 ap.t.	9$\frac{7}{8}$
5. Otterden Place near Lenham in Kent	Granv Wheler Eſq.	49 0		7 31 40 ap.t.	
6. Yeovil *in Somerſetſhire*	*J. Milnet*	5 34 0		7 14 30	

New Queſtions.

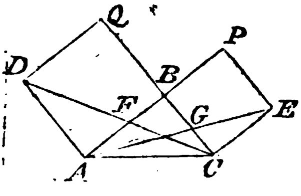
I. QUESTION 169, *by Mr.* Will. Grimmett.

In a certain dictionary, under the word *Conoid*, it is ſaid, the ſolidity of an hyperbolic conoid is to its circumſcribing cylinder as 3 to 10; and in an appendix of fluxions the ſame is alſo aſſerted; which is certainly falſe. Be pleaſed therefore to inveſtigate the expreſſion that does expound their ratio; and when you are in this way of thinking, ſuppoſe the generating hyperbola to become the plain of a weſt-declining dial, in the latitude of 50° north, and the focus to be the center of the ſame, in which, if you erect a wire perpendicular to the plain, the ſun on its firſt ſhining on the plain, the 12th of May, will caſt the ſhadow of the wire, ſo erected, exactly on the hour line of 8. Quere the declination of the plain.

II. QUESTION 170, *by Mr.* Sam. Aſhby.

You that would learn the art and myſtery
Of mathematics, learn geometry.
The firſt ſix books of Euclid are the beſt;
Which being known, you'll eaſily learn the reſt.
And then, to put in practice what you know,
Obſerve a propoſition here below;
On which, if you'll be pleas'd ſome time to ſpend,
You'll much oblige your mathematic friend.

If upon each leg *AB* and *BC*, including the right angle, be drawn a ſquare *BD* and *BE*; and the lines *DC* and *EA*, which cut the ſaid legs at *F* and *G*. I ſay, *BF* and *BG* are equal, and are each a mean proportional between the ſegments *AF* and *CG*; that is, as *AF* : *FB* : : *FB* : *GC*, &c. Quere the demonſtration geometrically.



III. QUESTION 171, *by Mr.* Geo. Brown.

One morning fair bright Phœbus did diſplay
His glorious rays over the northern ſea.
There from a port in * fifty-one degrees,
Three ſhips ſet ſail, their † courſe as here you ſee;
Then each ſhip chang'd her courſe, and did another get;
And when an equal diſtance run, they all did meet.
Now each ſhip's ſecond courſe and diſtance run,
Likewiſe the ſame from whence they firſt did come,
Unto this place where now they lie,
With its latitude, is what you're to deſcry?

* *Of latitude.* † *S. E.* 33·8. *S. S. E.* 49. *S. S. W.* 35·5 *leagues.*

IV. QUESTION 172, *by Mr.* Chr. Hale.

Suppoſe the product of two lines *Inches.*
Be as the margin here defines; 2332800
The third line then I fain would know,
That will the greateſt area ſhew;
For that exactly will deſcry
The height of All Saints at Derby.

V. QUESTION 173, *by Mr.* John Turner.

Two ſhips ſail from a certain port to ſea,
Unto two ports whoſe latitudes agree.
The firſt ſhe ſails between the ſouth and eaſt,
The other makes her way 'twixt ſouth and weſt.
If both their courſes you together join,
'Twill make degrees 112, and minutes twenty-nine;
The ſhip's departure which to the eaſtward went,
Is miles two hundred twenty-nine, and ſeven-tenths:
Their diſtances muſt this * proportion bear
Unto each other. Whence I pray declare
Each ſhip's true courſe, departure, diſtance run,
And latitude of the port where they begun?

* *As* 12 *to* 5, *weſtermoſt ſhip's diſtance run, being the greater.*
N. B. *They arrived in the latitude of* 35° 34′ *north.*

VI. QUESTION 174, *by Mr.* Ri. Lovatt.

When mighty Newton the foundation laid
Of his myſterious art; none cou'd invade
Nor take from him the honour which was due;
Great Britain's ſons will long his works purſue.
By curious theorems he the moon cou'd trace,
And her true motion give in every place;
The greateſt areas he with eaſe cou'd ſhew,
It is from him alone the art we know; 3000
And to confirm the ſame, let us ſuppoſe 2000
The greateſt area that we can incloſe 1500
In four right lines, ſuch as the margin ſhews. *a*
He that a theorem gives, ſhall have his name
Recorded in the ladies' book of fame.
Quere *a*, and the greateſt area.

The PRIZE QUESTION, *by Mr.* Fearnſide.

A young lady for ſome time a meadow has own'd,
In form of a right-angled triangle found,
Whoſe baſe I could meaſure, and found it to be
Chains ninety and five, links twenty and three:
But the other two ſides were with water o'erflown,
So their lengths, tho' attempted, then could not be known.
Now a ditch iſſuing out from the * angle at th' baſe,
Made with the hypothenuſe juſt fifteen degrees;
And in the cathetus a tree I eſpy'd,
Which in two equal parts did exactly divide
That part of the ſaid perpendicular, or ſpace
Included 'twixt where the ditch croſs'd it, and baſe;
And if to the oppoſite corner you draw,
Or ſuppoſe a line drawn, from the tree, you muſt know,
It equally cuts the whole angle in two.
Now by will it was order'd, that who of this meadow
Could find the content, might marry the lady.
An admirer I've been, and there's nothing remains
But this to compenſate my care and my pains.
Therefore if by theſe hints you aught can advance,
At the wedding, fair ladies, I invite you to dance.

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* *i. e.* of the triangular meadow.

1734.

Queſtions anſwered.

I. QUESTION 169 *anſwered.*

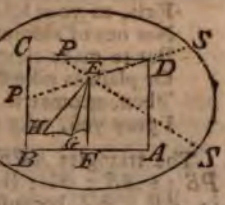
LET the abſciſſa of the hyperbola be $=x$, the ordinate $=y$, the parameter $=b$, and the tranſverſe $=a$; then the nature of the curve will be expreſſed by $yy=bx+\frac{bxx}{a}$; then by the doctrine of fluxions $\frac{pbxx}{4r}+\frac{pbxxx}{6ra}$ will expreſs the ſolidity of a conoid whoſe altitude is $=x$, and the radius of its baſe $=y$: And the ſolidity of its circumſcribing cylinder will be expreſſed by $\frac{pbxx}{2r}+\frac{pbxxx}{2ra}$: Conſequently their ratio will be expreſſed by $\frac{3}{2}a+x$ to $3a+3x$, which is agreeable to that deduced from the method of indiviſibles.

If we conſider ſuch a one whoſe altitude is equal to the tranſverſe axis of the generating hyperbola; the ratio will be expounded by that of 5 to 12.

When the ſun is in the plane of any dial, the ſhadow of the ſtyle, and that of a wire erected perpendicular to the plane in the center of that dial, will be coincident.

Demonſtration.

Let *ABCD* repreſent the plane of a dial coincident with the plane of the paper; then the point *E* will be the perpendicular wire, *EF* the hour line of 12, *EH* the ſtyle, *EG* the ſubſtyle; *SS* a parallel the ſun deſcribes, which will here become an ellipſis. Now the ſun in ſome point of the orbit will be in the plane of the dial, ſuppoſe at *S* or *S*; then its plane by the line drawn from the ſun through the center of the dial will



give

give the ſhadow of the ſtyle on the plane, as SP or SP; for a line drawn from the ſun through any other point of the ſtyle more remote from the plane will not fall on the plane. Again the ſhadow of a wire erected perpendicular on a plane, when the ſun is in that plane will be coincident with a line drawn from the ſun through the interſection of the wire with that plane (which is here ſaid to be the center of that dial) conſequently when the ſun is in the plane, the ſhadow of a ſtyle (however inclined) and the ſhadow of the wire erected perpendicular in the center will be coincident. *Q. E. D.*

Therefore from the latitude of 50 N. the declination on the 12th of May and hour of 8, the azimuth will be found 99° 48′ from the north, which is the declination of the plane from the point.

Mr. *Fearnſide* has given the ſame anſwer. Mr. *Turner*, Mr. *Smedley*, Mr. *Ommanney*, Mr. *Dunthorne*, Mr. *Quant*, Mr. *Colbourn*, and ſome others, make the declination 9° 9′, or 9° 14′, and the azimuth 80° 46′.

II. QUESTION 170 *anſwered by Mr.* Rob. Fearnſide.

'Tis plain, by ſimilar triangles, that $CB+AB : AB :: BC : FB$. Again $CB + AB : BC :: AB : GB$. Permutando $CB + AB : AB :: BC : GB$. Ergo $FB = GB$.

See the fig. to the queſt.

For the other part of the demonſtration; by ſimilar triangles $AB\ (= AD) : AF :: BC : BF$; and $AB : BG :: BC\ (= CE) : CG$. Therefore, *ex equo* $AF : BG :: BF : GC$. *Q. E. D.*

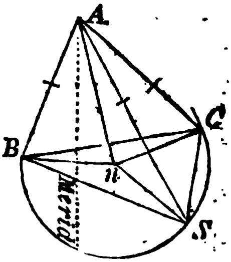
A Grubean Lady's anſwer to the ſame.

To oblige you, mathematic friend, Sam. Aſhby,
I'll prove your wond'rous prop. but not too raſhly.
Firſt, to your ſcheme let's add the *Q* and *P*,
Not out of abſolute neceſſity;
But to make clear the proof, as noſe on face,
Or plain as pike ſtaff; that is all the caſe.
Then, geometric friend, without more fuſs,
I pray you, take the demonſtration, thus.

The triangles APE and ABG, being ſimilar, it is $AP : PE :: AB : BG$; (by Euclid 4. 6) that is $AB + BC : BC :: AB : BG$, becauſe all the ſides of a ſquare are equal. In like manner the triangles CQD and CBF, being ſimilar, it is $QC : CB :: DQ : FB$; *i.e.* $AB + BC : BC :: AB : FB$. Therefore FB is $= BG$. Again, the triangles DAF and FBC, and the triangles ABG and ECG are ſimilar; therefore (DA or) $AB : BC :: AF : FB$; and $AB : BC$ ($= CE$) $:: BG : GC$. Whence $AF : FB :: BG$ (or BF) $: GC$ (by Eucl. II. 5). *Q.E.D.*

III. QUESTION 171 *anſwered by Mr.* J. Turner.

This is altogether ſolved by trigonometry. For, firſt, in the triangles BAS, CAS, there is given two ſides and a contained angle, to find BS, and the $\angle BSA$, and CS and the $\angle CSA$; add theſe two angles together, and then you have in the $\triangle BSC$ two ſides and a contained angle, to find BC: Now in the $\triangle BnC$, the angle at n is double the $\angle BSC$, and being iſoſceles, it is eaſy to find $Bn = Cn = Sn = 19{\cdot}44$ leagues, each ſhip's 2d diſtance run, from B, S, C, to meet at n. Again, in the $\triangle ABC$, find the $\angle$s ABC and ACB, to which add the $\angle$s CBn, BCn, we find that the 2d courſe of the firſt ſhip ſteered



S. S. W. from B to n is E. by N. 5° 26′ eaſterly,
2 ſhip S. E. C to n is W. S. W. 1 37 weſterly,
3 ſhip S. S. E. S to n is N. W. 3 12 northerly.

Laſtly, in the $\triangle BAn$, there is given BA, Bn, and the angle contained; to find $An = 31{\cdot}4$ leagues, the diſtance from the port A to where they all meet: And by the $\angle BAn$ found, the courſe from A to n is S. by E. 34′ ſoutherly. The difference of latitude of the ſhips when at n, $= 30{\cdot}7$ leagues.

The ſame is anſwered in this manner by Mr. *Fearnſide*, Mr. *Skews*, Mr. *P. Sharp*, Mr. *Quant*; Mr. *Wooler*, Mr. *Colbourn*, Mr. *Hemmingway*, and Mr. *Oats*; which agree ſo near to the propoſer's anſwer and one another, that I preſume I need not to exhibit any of them at large.

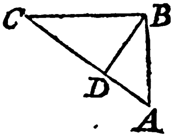
IV. QUESTION 172 *anſwered.*

I have received a great many anſwers to this queſtion, but they generally agree that it is unlimited, or elſe they do not rightly underſtand the propoſer's meaning; but if that product be broke into two equal lines, making the legs of a right-angled triangle, the hypothenuſe will be 180 feet. I ſhall here give you the propoſer's own ſolution.

Put $dd = 2332800$, and $a = AC$. $a : d + d :: d - d : o = AD - DC$: And $\frac{a}{2} = AD$.

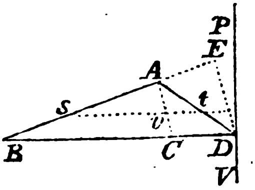
$\frac{4ddaa - aaaa}{16}$ = greateſt area ſquared.

In fluxions, $8dda\dot{a} - 4a^3\dot{a} = 0$: Reduced gives $a = 2160$ inches. The exact height of All-ſaints tower 60 yards.*



V. QUESTION 173 *anſwered by Mr.* Fearnſide.

Put $b = CD$, $d = \frac{12}{5}$, ſine of the ſum of the courſes $112° 29' = c$, coſine $= a$, $x = AE$. Then $a : x :: 1 : AD = \frac{x}{a}$, and $DE = \frac{cx}{a}$, $BA = \frac{dx}{a}$, $CA = \sqrt{\frac{xx}{aa} - bb}$. Then $BD^2 = xx + \frac{2dxx}{a} + \frac{ddxx}{aa} + \frac{ccxx}{aa} =$ (putting $p^2 = 1 + \frac{2d}{a} + \frac{dd}{aa} + \frac{cc}{aa}$) $ppxx$, therefore $px = BD$, and $px - b = BC$: Per ſimilar triangles, $px : x + \frac{dx}{a} :: \frac{dx}{a} : px - b$; hence x will be found $= \frac{pbaa}{ppaa - ad - dd} = 134{\cdot}28$ miles. Conſequently $BA = 842{\cdot}6$ miles, $AD = 351{\cdot}15$ miles, $BC = 799{\cdot}79$ miles, $\angle ABC = 18° 12' 14''$, $BAC = 71° 37'$, $CAD = 40° 51'$, and latitude $31° 45'$.



The

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* IV. QUESTION 172.

Since the product of any two ſides of a triangle drawn into the ſine of their included angle produces double the area; therefore when the product of the ſides is given, the area will be as the ſine of the angle; but the right angle has the greateſt ſine, therefore the triangle is right-angled when a maximum. And then into whatever two parts the given product is broken, the area will be ſtill the ſame; but then the hypothenuſe will vary, and is ſhorteſt when the triangle is iſoſceles, in which caſe it is $= \sqrt{2 \times 2332800} = 2160$ inches $= 60$ yards.

The ſame anſwered by Mr. Turner, *the propoſer.*

Though this queſtion may be ſolved by a ſimple quadratic equation, yet I always prefer conciſeneſs in mathematics, and ſend you the geometric conſtruction, and trigonometric calculation, which are vaſtly eaſier than the algebraic operation.

Conſtruction.

Aſſume any 2 lines as As, At, in the ratio of 12 to 5, and let them contain the given angle sAt, produce the lines indefinitely, and let fall Av perpendicular to st, and at the the diſtance of 230 draw a parallel to Av, as VP, and from where it cuts At produced as at D, draw DC parallel to ts, and where it meets As produced will form the triangle required.

Solution.

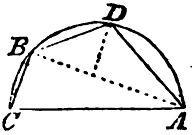
As, At, being aſſumed = any numbers in the ratio of 12 to 5, and the angle contained given: Find st, and thence vt and vA. And then it will be as $vt : vA :: CD : CA$, from whence all the reſt are eaſily known. *Miles.*

The courſe of one ſhip is S. 71° 38′ weſterly, diſt. run 843·7
the other S. 40 52 eaſterly, —— 351·5

Latitude of the port ſailed from 40° north. Difference of latitude 266 miles.

VI. QUESTION 174 *anſwered by the propoſer.*

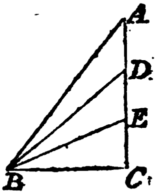
Given $AD = 3000 = m$, $DB = 2000 = b$, $BC = 1500 = 2p$. Put $\overline{AD+DB} \times \overline{AD-DB} = d$, and $AB = x$. Then per 4 and 47 Eucl. I, $\sqrt{\dfrac{4m^2x^2 - x^4 - 2dx^2 - d^2}{4}}$ = the area ABD; and px = area ABC, which is right-angled when the trapezium becomes a maximum. The



ſum in fluxions is $\dfrac{2m^2x\dot{x} - x^3\dot{x} - dx\dot{x}}{2\sqrt{4m^2x^2 - x^4 - 2dx^2 - d^2}} + p\dot{x} = 0$: Subſtitute $n = 2m^2 - d$, then $x^4 - 2nx^2 + 4p^2x^2 = 16m^2p^2 - 8dp^2 - \dfrac{4p^2d^2}{x^2} - n^2$. This adfected equation will be reduced to a quadratic $x^2 - 11875000 = 5217800x$ $= 4534·34$; then $AC = 4398·1$; and the greateſt area $5921925·2375$.

Or, when the area of the trapezium becomes a maximum, it will be incloſed in a ſemicircle, and the variable line AC will be the diameter. Therefore making $a = AC$, we have $a = 4398·1$.*

The PRIZE QUESTION *anſwered.*



Given $BC = 95·23$ chains $= b$, ſine $\angle ABD = 15^\circ = s$, $\angle ABE = EBC$, and $DE = EC$. Put $x =$ s. $\angle EBC$; then $\sqrt{1-xx} : b :: x$ $CE = \frac{bx}{\sqrt{1-xx}}$; hence $\frac{2bx}{\sqrt{1-xx}} = DC$. Then, per Euc. 47. I, $BD = b\sqrt{\frac{3x^2+1}{1-x^2}}$. Again the coſine of double the $\angle EBC$ $=$ ſine of the $\angle A$ will be found to be $= 1 - 2x^2$; then $1 - 2x^2 : s :: BD :$ $DA = \frac{bs}{1-2x^2}\sqrt{\frac{3x^2+1}{1-x^2}}$. Then by 3d prop. 6 Euclid; $BC : BA :: EC : AE$; which analytically expreſſed, and the equation ordered, becomes $4x^6 - 3s^2x^2 - ss = 0$. Reduced, the ſine of the $\angle EBC$ will be found to be that of $34^\circ\ 27'\ 47''$, and $GA = 247·13$; conſequently the area of the meadow will be $= 1176·709$ acres.

Of

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* VI. QUESTION 174.

Since the trapezium (or any other figure) will be a maximum when it is inſcribed in a ſemicircle, the unknown ſide being the diameter, as is now generally known; there is no occaſion for the uſe of fluxions in the operation, it being only to apply three given *chords ſo as to* fill up the ſemicircumference, as *Ward* has done in *his 16th problem.*

Of the Eclipſes in 1734.

There will be but two eclipſes this year, the one viſible and the other inviſible.

The firſt of the ſun the 22d of April, at 10 in the morning.*

The other of the ſun the 15th of October, at 7 at night.

New Queſtions.

I. Question 175, *by Mr.* Rob. Fearnſide.

A lady of wit, youth, and beauty beſide,
Remote from all cares, but of being a bride,
Surpriz'd her fond lover one morning in May,
And diſpatch'd him for parſon and licence away.
But how great her confuſion, when Strephon brought news
That the parſon a licence to grant did refuſe!
Her age, which the lady nor lover cou'd tell,
Was the cauſe that this fatal diſaſter befel.
Therefore, ladies, their humble requeſt is, you'll ſhow
The way how to do't from the * data below.
And this by a general method explain,
So that loveɪs may never be non-plus'd again.

* *Cycle of* ☉ 18, *golden number* 8, *roman indiction* 10, *that year the lady was born.*

II. Ques-

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* This eclipſe was obſerved at *Rome* by the Abbot *Didacus de Revillas* and *Andreas Celſius.*

h.	m.	s.	True time, p. mer.
22	22	35	The beginning a little over.
23	5	0	Middle, 2 digits.
23	52	2	End.

II. QUESTION 176, *by Mr.* John Gundy.

Whilſt I was ſurveying for his Grace the Duke of Buccleugh, it was my chance to meet with a piece of land in the form of a rectangle, or long ſquare; and the proportions were ſuch, that if it had been two perches broader, and three longer, it would have been ſixty-four perches larger than before. But, on the contrary, if it had been three perches broader, and two longer, it would then be ſixty-eight perches larger than it was by my ſurvey. Quere, What was the area of the ſaid piece of land?

III. QUESTION 177, *by Mr.* John Turner.

The dimenſions of a conical fruſtum are known
To be ſuch as hereunder[*]; from whence I'd have ſhown
The greateſt cylinder that can be inſcrib'd therein,
Its baſe's diameter, and height, I do mean.
And if, for variety's ſake, we again
Suppoſe the ſaid fruſtum to be cut by a plane
Thro' the leſſer diameter's extremity,
And parallel to the cone's axe; be ſo kind,
The ſolid content of each part for to find?

[*] *Given the greater diameter of the fruſtum* CD = 44 *inches*
leſſer diameter — — AB = 20
perpendicular height — MN = 30

IV. QUESTION 178, *by Mr.* Chr. Maſon.

I being lately in a timber yard,
Where blocks, and beams, and ſcantlings lay prepar'd;
There a young artiſt did my aid implore,
A piece to meaſure he ne'er met before.
He'd read moſt authors which on ſolids treat;
Yet this quaint ſolid did his ſkill defeat.
Four regular equal faces it did bear,
They bounded by iſoſceles triangles were.
Three feet each baſe or ſhorteſt ſide did count,
The legs or longeſt did to five amount.
I've given what is requiſite to know,
The true content and properties pray ſhew?

V. QUESTION 179, *by Mr.* J. Bulman.

At a certain place in northern latitude, the ſun was obſerved to riſe exactly at 3 h. 58 m. and at 6 o'clock his altitude was taken the ſame morning, and found to be 15 deg. 20 min. his declination being then north. Required the latitude of the place where, and day of the year when, thoſe obſervations were made?

VI. QUESTION 180, *by Mr.* John Grundy.

A gentleman, who was a great lover of the mathematics, had a large eſtate, which lay in four ſeveral entire manors, of which he had drawn four ſeveral maps. Now he, by his often looking over the contents of theſe maps, found the quantities of acres belonging to each of theſe, would make four numbers in continued proportion; whereof the ſum of the two middlemoſt numbers is 1152, and that of the two extremes 1728. He lying on his death-bed, called his four ſons to him, ſaying, My dear children, my glaſs being almoſt run, and the eſtate I have to leave amongſt you was for the moſt part gained by my own induſtry, and ſo entirely at my own diſpoſal, I ſhall leave it amongſt you, with this proviſo, that he that anſwers the aboveſaid queſtion firſt, ſhall have the largeſt manor for his ſhare; and ſo the reſt of you the others, according to your birthright. It is demanded the quantity of acres that each manor does contain?

The PRIZE QUESTION, *by Mr.* Maſon.

A worthy wight, who does in wealth abound,
And, to a wiſh, with earthly bleſſings crown'd;
Free from oppreſſion, and perfidious gain,
True Briton-like, doth liberty maintain.
On a fair ſite a ſtately fabric rais'd,
Whoſe view a Wren or Vanburgh wou'd have prais'd.
High on the north fine rural groves ariſe,
Shelt'ring from blaſts and fierce inclement ſkies.
There glades, and grots, and groteſque works appear,
And bounteous green ſtill flouriſh round the year.
The dropping rocks their trickling tears repeat,
Weeping for joy they've ſuch a ſafe retreat.
The drops unite, and into ſtreams do grow;
In peaceful *murm'rings* thro' their channels flow.

Until one ſource does all the reſt invade,
And then in haſte doth form a large caſcade,
Which rolls itſelf into a reſervoir;
Then art's requir'd where nature work'd before.
Near on the eaſt the new made ſoils produce,
The choiceſt ligums that's for kitchen uſe;
The melonry calls for the gard'ner's care,
For heat, for moiſture, and ſometimes for air.
As ſeaſons change, with changes he is ſtor'd,
For to ſupply his bounteous maſter's board.
Next, on the weſt, Pomona claims her ſeat,
Moſt free from blights, tho' not the moſt from heat.
The vernal ſcenes raviſh th' beholder's eye,
And fragrant bloſſoms, figur'd variouſly,
Diffuſe their ſweets; their vying wafts contend,
Till gentle Zeph'rus does the conteſt end.
The dancing foliage with immantlings play,
And laden branches their choice fruits diſplay.
Yet wanteth ſtill, the edifice to grace,
A large parterre the ſouthern front to face:
Then ſoon an artiſt of undoubted ſkill,
Was there produc'd the ſame for to fulfil:
His order was, ten acres he ſhou'd take,
And on three aſpects a canal to make.
The earth dug thence muſt on the ſurface lie,
To mix the ſoil, and raiſe it one foot high.
Yet no dimenſions, but proportion, giv'n,
The depth to breadth, as one is unto ſeven.
And in the midſt an oval fountain place,
Where groups of Tritons muſt the center grace.
The depth three feet, the area in roods a ſcore,
And the plus earth converted as before.
The diameters ſhou'd ſuch proportion bear,
As the garden's length and breadth (or very near.)
The ſhorter ſpace 'tween the canal and fount,
To feet, as in the margin will amount. 238
Now the dimenſions you're deſir'd to ſhow,
Both of canal, the fount, and garden too?

1735.

Queſtions anſwered.

* I. QUESTION 175 *anſwered by Mr.* Rich. Dunthorne.

THE ſolution to this queſtion is in Keil's Aſtronomy, lect. 29 p. 379 and 380, by finding three numbers $285x$, $420y$, and $532z$; ſo that the firſt divided by 28 leaves the cycle of the ſun a remainder, the ſecond by 19 leaves the golden number, the third by 15 leaves the indiction. Then if the ſum of theſe numbers be divided by 7980, the remainder will be the year of the Julian period required. Or,

$$\left.\begin{array}{llll} \text{The cycle } \odot & 18 & \text{mult. by} & 4845 = 87210 \\ \text{Golden numb.} & 8 & \text{———} & 4200 = 33600 \\ \text{Indiction} & 10 & \text{———} & 6916 = 69160 \end{array}\right\} \begin{array}{l} \text{The ſum of} \\ \text{the products} \\ \text{is } 189970. \end{array}$$

Which divided by 7980, there remains 6430, the Julian period; from which ſubtract 4713, the Julian period at our Saviour's birth, remains 1717, the year required in anſwer to the queſtion.

Anſwered

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* I. QUESTION 175.

The ſolar cycle is a period of 28 years, the lunar of 19, and the indiction a period of 15. The year before the chriſtian æra was the 9th of the ſolar cycle, the 1ſt of the lunar, and the 3d of the indiction cycle. Wherefore 9, 1, and 3 being ſeverally added to any year x of Chriſt, and the ſums divided reſpectively by 28, 19, and 15, the remainders will ſhew the ſeveral years of the cycles for that year. But, in the preſent caſe propoſed, the remainders are 18, 8, and 10; hence then $\frac{x+9-18}{28}$, $\frac{x+1-8}{19}$, and $\frac{x+3-10}{15}$ muſt be integers; or $\frac{x-9}{28}$, $\frac{x-7}{19}$, and $\frac{x-7}{15}$ integers.

Anſwered by Mr. John Ommanney.

Dr. Keil in his Aſtron. Lectures, p. 380, ſays, if 4845 be multiplied by the cycle of the ſun, and 4200 by the golden number, and 6916 by the Roman indiction, and the ſum of their products divided by 7980, the remainder (neglecting the quotient) will be the year of the Julian period; from which ſubtract 4713, there will remain the year of the chriſtian æra. And in this queſtion the anſwer is 1717, as above.

II. QUES-

Put the firſt of theſe equal to the integer m, that is $\frac{x-9}{28} = m$; then $x = 28m + 9$ a value of x anſwering the firſt condition. Write this in the 2d, then $\frac{28m+9-7}{19}$ or $\frac{28m+2}{19} = m + \frac{9m+2}{19}$ = an integer; therefore $\frac{9m+2}{19} = \frac{18m+4}{19} = m - \frac{m-4}{19}$ = an integer; therefore $\frac{m-4}{19} = n$ an integer; hence $m = 19n + 4$; which ſubſtituted in the value of x, we have $x = 28 \times \overline{19n+4} + 9 = 532n + 121$ a value of x anſwer the two firſt conditions. This value of x being written in the 3d original integer, we have $\frac{532n+121-7}{15}$ or $\frac{532n+114}{15} = 35n + 7 + \frac{7n+9}{15}$ = an integer; hence $\frac{7n+9}{15} = \frac{14n+18}{15} = n + 1 - \frac{n-3}{15}$ = an integer; therefore $\frac{n-3}{15} = p$ an integer; conſequently $n = 15p + 3$: this written in the laſt value of x, it becomes $x = 532 \times \overline{15p+3} + 121 = 7980p + 1717$ a general expreſſion for the year anſwering all the three conditions, in which p may be either nothing or any whole number. In the preſent caſe it is evident that the value of p muſt be nothing, and then $x = 1717$, the year of the lady's birth.

It is evident that ſuch a combination cannot happen again till *the year* 9697 = 7980 + 1717, when the value of p is 1; and *that the ſucceſſive years of its happening are found by the continual addition of 7980.*

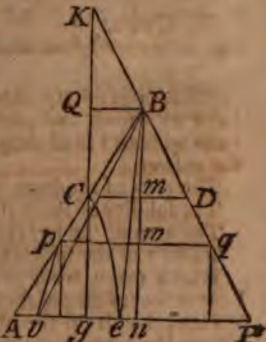
II. QUESTION 176 *anſwer'd by* Eumenes Pamphilus.

Let $x =$ length, $z =$ breadth. Then, per queſt.
$xz + 3z + 2x + 6 = xz + 64$, and
$xz + 2x + 3z + 6 = xz + 68$. Their difference is
$x - z = 4$, hence $x - 4 = z$; which being ſubſtituted for z,
there comes out, by the 1ſt equation,
$xx + x - 6 = xx - 4x + 64$, or $5x = 70$. Hence
$x = \frac{70}{5} = 14$ perches the length, and
$z = x - 4 = 10$ perches the breadth, alſo
$xz = 140$ perches, or 3 roods 20 perches the content.

Mr. *Turner*'s anſwer is in ſubſtance the ſame.

III. QUESTION 177 *anſwered by the propoſer.*

In order to find the height of the whole cone Bm. Put $Bm = x$, $Cm = b$, $mn = c$, $An = d$. Then, by ſimilar triangles, as $x : b :: x + c : d$. Hence $bx + bc = dx$, and $x = \frac{bc}{d - b} = 25$; conſequently $Bn = 55$ inches. Again, put $p = Bn$, $b = AP$, $d = \cdot 7854$, $x = nw$ the height of the greateſt cylinder; as $p : b :: p - x : \frac{bp - bx}{p} = pq$. But the ſolidity of the cylinder $= \frac{bbxx - 2bbpx + bbpp}{pp} \times dx$; in fluxions, it is $3dbbxx\dot{x} - 4dbbpx\dot{x} + dbbpp\dot{x} = 0$. Divide all by $dbb\dot{x}$, and $3xx - 4px + pp = 0$; Ergo $x = \frac{1}{3}p = 18 \cdot 3333$ inches, the cylinder's height; and the diameter of its baſe $29 \cdot 3333$. Again, from $Ag = 12$, and $Pg = 32$, being given, $gv = ge$ is found $= (\sqrt{12 \times 32})$ $19 \cdot 6$ inches, and $ve = 39 \cdot 2$: the area of the ſegment Aev (by page 404 of Ward, or much eaſier by a table of ſegments) is found to be $335 \cdot 88$ ſquare inches. Which multiplied by $\frac{1}{3}Bn = 18 \cdot 333$, gives the ſolidity of the pyramid $ABevA = 6157 \cdot 8$ cubic inches.



Now the baſe of the pyramid $CveBC$ is an hyperbola, whoſe area is thus found: $ng = 10$ is the hyperbola's conjugate ſemi-diameter $= b$, $ve = 39·2 = g$ its bounding ordinate, $Cg = 30$ the abſciſſa, CQ the tranſverſe ſemi-diameter, $= 25$, $Qg = p = 55$, $AP = d = 44$. The hyperbolic logarithm of $\frac{d+g}{2b}$ or 4·16 is $1·4255142 = S$. I ſay the area of the hyperbola is $= \frac{pg}{2} - \frac{2pbbS}{d} = 721·62$ ſquare inch. which is a contraction of Dr. Wallis's quadrature of the hyperbola, p. 328 of his Algebra. This multiplied by $\frac{1}{3}ng$, the perpendicular altitude of the pyramid, or 3·333, gives 2405·40 its ſolidity. Subtract this pyramid $CveBC$ from the pyramid $ABevA$, the remainder is the content of the ungula or cuneus $AveCA = 3752·4$ cubic inches, which was required. And laſtly, if you ſubtract this from the content of the conic fruſtum $ACDP$, which is eaſily found $=$ 25258·46, the remainder is the ſolidity of the other part or ungula $PevCP = 21506·06$ inches.

Anſwered by Mr. Geo. Brown,

Who projects the ſcheme, and all its parts exactly the ſame as above, and carries the proceſs through the whole, from whence I ſhall only collect ſuch parts as are eſſential to the anſwer.

The area of the ſegment $Aev = 335·915$ ſquare inches,
The ſolidity of the inclined pyr. $ABg = 6158·45$ cubic inch.
The area of the hyperbola $= 721·45$,
Solidity of the inclined pyramid $CBg = 2404·95$,
Which taken from ACg,
Leaves the ſolidity of the hoof $ACve = 3753·49 = 2·1721$ feet.
Which taken from the given fruſtum of the cone 14·6171,
Leaves the content of the part $PDCveP = 12·445$.

$$\left.\begin{array}{ll}\text{The height of the greateſt cylinder } wn = & 18·333 \\ \text{And its baſe's diameter} \quad — \quad — & 29·333\end{array}\right\} \text{inches.}$$

IV. QUESTION 178 *anſwered by* Juvenis Mathematicus.

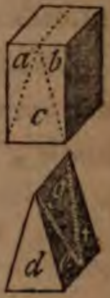
This body ſeems to agree with Euclid's definition of a pyramid. Per 47 Eucl. 1, $cc - \frac{bb}{4} = \frac{4cc - bb}{4} =$ to the perpendicular of one of the iſoſceles triangles; and if the body *be conceived* to be divided into two equal parts by a triangular plane, two ſides of which are the perpendiculars above, and the 3d $= b = 3$. Then by the 14th prop. of Keil's Plain Trigonometry, $\sqrt{\frac{4cc-bb}{4}} : \sqrt{\frac{4cc-bb}{4}} + b :: \sqrt{\frac{4cc-bb}{4}} - b$: the difference of the ſegments of the baſe. The rectangle of the means divided by the firſt extream $= \sqrt{cc - \frac{bb}{4}} - \frac{bb}{\sqrt{\frac{4cc-bb}{4}}} =$ difference of the ſegments. Hence then $\sqrt{\frac{4cc-bb}{4}} - \frac{bb}{2\sqrt{\frac{4cc-bb}{4}}} =$ the greater ſegment, and $\frac{bb}{2\sqrt{\frac{4cc-bb}{4}}} =$ the leſs ſegment; alſo $bb - \frac{b^4}{4cc-bb} =$ the ſquare of the perpendicular height of this triangle. But $\frac{1}{2}b\sqrt{\frac{4cc-bb}{4}} =$ area triangle or baſe of the ſolid body. Wherefore $\frac{1}{3}\sqrt{bb - \frac{b^4}{4cc-bb}} \times \frac{1}{2}b\sqrt{\frac{4cc-bb}{4}} = 6{\cdot}791536746428$ cubic feet, the ſolid content required.

The ſame anſwered by Mr. Ed. Golding.

A general Rule to find the Solidity. From the ſquare of one of the legs, abate the ſquare of half the baſe; and multiply the ſquare root of the remainder by the ſquare of the baſe: The laſt product divide by 6, and the quotient ſhall be the content of the propoſed ſolid.

For the perpendicular Height. The ſquare of the ſide or leg $5 = 25$; ſubtract twice the ſquare of $\frac{1}{2}$ the baſe $1{\cdot}5 = 4{\cdot}5$; there remains $20{\cdot}51$ whoſe ſquare root $4{\cdot}5277$ is the perpendicular height.

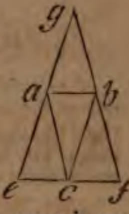
For the Solidity. Multiply the ſquare of the baſe $3 = 9$, by the perpendicular $4{\cdot}5777$, and the product $40{\cdot}7493$ is the ſolidity of the circumſcribing priſm. As $6 : 1 :: 40{\cdot}7493 : 6{\cdot}79155$ the ſolid feet in the alabrum, by which name I call this ſolid till a fitter name be impoſed. And to prove it but $\frac{1}{6}$ of the priſm. Suppoſe the firſt figure, being a ſquare priſm, be 3 feet ſquare at the ends, and 6 feet high, the content will be 54. Cut off from the priſm (1.) the 2 priſmatic wedges *a* and *b*; which both together contain 9 upon 3 = 27 feet; leaving the wedge *c*, which ſuppoſe to be the ſecond figure; from it cut off the two pyramids *e* and *f*, which both together contain 9 upon 2 = 18. Leaving the the alabrum only behind; The two wedges = 27, and two pyramids = 18, make 45, which taken from 54, remains the alabrum 9 = $\frac{1}{6}$ of the priſm. *Q. E. D.*



The ſame anſwered by Mr. J. Bulman.

From the ſquare of the given ſide of the iſoſceles triangle = 25, ſubtract the ſquare of half the given baſe = 2·25, the remainder 22·75 is the ſquare of the perpendicular to the triangle: from which ſubtract again the ſquare of half the baſe, the ſquare root of the remainder 4·527692 = the length of the circumſcribing ſquare priſm: whoſe ſolidity is 40·749228 feet, $\frac{1}{6}$ of which is 6·791538 feet, the ſolidity of the body required.

This ſolid may eaſily be cut from a priſm having ſquare baſes. Or if a piece of ſtiff paper or paſteboard be cut according to the following figure, and folded in the lines *a b*, *b c*, and *c a*, ſo as *c f* and *c e* coincide or make but one line, and *g* touches the joined points *e* and *f*, it will perfectly repreſent the body propoſed in the queſtion.



Several other true anſwers I might exhibit, were it not that I have in ſome meaſure exceeded my limits; and that the rather, becauſe ſeveral have pronounced this queſtion unintelligible and unlimited; ſuch I hope will be convinced of their error by theſe anſwers; and not haſtily judge and condemn others, that at firſt don't appear obvious to them.

V. QUESTION 179 *anſwered by Mr.* N. Oats.

Put $a =$ ſine of the altitude at 6; $d =$ ſine of the aſcen. diff. $s =$ ſine of the latitude; $\sqrt{1 - ss} =$ its coſine. Then, per ſpherics, $s : 1 :: a : \frac{a}{s} =$ ſine decl. and its coſine $= \sqrt{1 - \frac{aa}{ss}} = \frac{\sqrt{ss - aa}}{s}$. Again $\frac{\sqrt{ss - aa}}{s} : \frac{a}{s} :: 1 : \frac{as}{\sqrt{ss - aa}} =$ tang. declin. $\sqrt{1 - ss} : s :: 1 : \frac{s}{\sqrt{1 - ss}} =$ tang. lat. $1 : \frac{s}{\sqrt{1 - ss}} :: \frac{as}{\sqrt{ss - aa}} : d$; ergo $\frac{ass}{\sqrt{ss - aa}} = d\sqrt{1 - ss}$. Reduced, it is $s^4 - aass + aa - ss + \frac{aass}{dd} = 0$; hence $s = {}^{\cdot}835728 =$ the ſine lat. $= 56^\circ\ 41'\ 32''$. And the declination $= 18^\circ\ 26'\ 45''$, anſwering to May 2, or Aug. 4.

Mr. *John Turner* has given a curious ſolution to this queſtion, both trigonometrically and analytically; where he makes the ſun's declination north 18° 28′, and the latitude of the place north 56° 40′.

VI. QUESTION 180 *anſwered by Mr.* Beacham.

This queſtion is compoſed out of Sir Iſaac Newton's Arith. or Ronayne's Algebra. The firſt ſhare is 1536, the ſecond 768, the third 384, and the fourth 192 acres.

Mr. *John Corbett* has given a true algebraic ſolution to this queſtion.

The PRIZE QUESTION *anſwered by Mr.* J. Doubt.

Given 10 acres = 435600 ſquare feet $= k$; 20 roods = 5445 ſquare feet $= g$; the diſtance of fount to the canal = 476 $= b$: The area of a circle whoſe diameter is 1 = ·7854 $= m$: Put the tranſverſe diameter of the fount $= x$, and the conjugate $= y$: then $\frac{g}{xm} = y$; $y : x :: b : \frac{bxxm}{g} = a$: and alſo $\overline{a+x} \times \overline{b+y} = \frac{bbxxmm + 2bxgm + gg}{gm} = k$: and ſo $bxm + g = \sqrt{kgm}$. Hence will be found $x =$ 100·8844, $y =$ 68·7199. The length of the garden 799·677 feet; breadth 544·719 feet; and then 435600 − 5445 − 16335 = 413820 the ſolid content of the canal, which let be $= s$, and 799·6771 × 2 + 544·719 = 2144·074 (the extent of the canal) $= l$. Put breadth $= a$, the depth $= e$; then per queſtion, $7 : 1 :: a : \frac{a}{7} = e$: and $\frac{laa}{7} = s$. Therefore $a = \sqrt{\frac{7s}{l}} =$ 36·7566 feet; and $e =$ 5·2509 feet.

Some have taken this queſtion to be unlimited; indeed the words [three aſpects] do not abſolutely determine whether two ſides and one end, or two ends and one ſide; but one would rather take it in the former. But that it ſhould be an oblong is plain enough from thoſe words, that the *oval ſhould be in proportion to the garden's length and breadth.* However the author's meaning has been well underſtood by ſeveral perſons, as by the near agreement of ſome of their anſwers here below do appear.

	Length	*Bread.*	*Tranſv.*	*Conju.*	*Bred. Can.*	*Depth*
The *propoſer*	799·9	544·4	100·9	68·6	32·7	4·68
Mr. *John Turner*	799·4	544·8	101·1	68·8	38·3	5·47
F. R. S.	799·0	544·6	100·9	68·6	32·6	5·01
Mr. *John Chorley*	794·7	548·1	96·1	72·1	38·3	5·48

The prize of 10 diaries was won by Mr. *John Turner* of Hull.

Of the Eclipſes in 1735.

There will happen four eclipſes this year: twice will the moon's dark body be interpoſed between the ſun and the earth, and hinder the ſun's rays from falling on the terraqueous globe; and twice will the earth come in the direct line between the ſun and moon, and prevent the moon from receiving her borrowed light from the ſun.

1. Moon eclipſed March 27, at 11 forenoon, inviſible.
2. Sun eclipſed April 11, at 11 at night, inviſible.
3. Moon eclipſed September 21, after 1 morning, viſible.
4. Sun eclipſed October 5, at 2 morning, inviſible.

That of the moon, September 21, only viſible, as follows.

Computed by	Begin.	Mid.	End	Dur.	Dig.
	h. m.	h. m	h. m	h. m.	h. m.
By Aſtronomia for Coventry	12 23	1 39	2 56	2 33	6 2
Mr. Chattock for London	11 52	1 21	2 50	2 57	7 12
Mr. Leadbetter for London	0 16	1 33	2 50	2 34	5 37
Mr. Dunthorne for Ramſey	0 29	1 46	3 3	2 33	6 3
Mr. Hampſon, Leigh, Lancaſh.	11 39	1 0	2 22	2 43	6 29
Mr. Sparrow, { by Flamſtead's	11 52	1 14	2 35	2 43	6 18
Nottingham, { Aſtron. Carol.	12 20	1 36	2 52	2 32	5 59
Mr. J. Wooler for Whitby	11 47	1 8	2 29	2 42	6 4
Mr. J. Thomas, Penzance	11 49	1 6	2 19	2 30	5 35
Mr. Williams, Oxfordſhire	0 12	1 30	2 46	2 34	5 40
Mr. C. Forreſt, Newcaſtle	11 55	1 21	2 48	2 53	6 38
Mr. J. Hilton for London	0 16	1 33	2 50	2 34	6 37
Mr. Brown for Bridgnorth	0 18	1 35	2 52	2 33	6 2
Mr. J. Bulman	0 29	1 46	3 3	2 34	6 3

New Queſtions.

I. QUESTION 181, *by Mr.* John Turner.

In the triangle *ABC* there is given the ſide $AB = 65$ feet, and the ſide $BC = 74$; and in this triangle there is an equilateral triangle inſcribed as *SWH*, whoſe ſide *SH* is parallel to the baſe *AC*: And laſtly, in the equilateral triangle there is a circle inſcribed, whoſe area is known to be $= 307{\cdot}9$ ſquare feet. It is required to find the ſide of the equilateral, and the baſe of the external triangle?

See the fig. to the ſolution.

II. QUES-

II. Question 182, *by Mr.* Chr. Maſon.

On a rich globe where bounteous nature ſmil'd
And fragrant ſweets the ſenſe of man beguil'd;
Near a fair grove where warbling birds did ſing,
With joyous notes, to the beloved ſpring;
I thither went with orders to ſurvey
A field, with four unequal ſides there lay,
And a foot-path lay 'croſs the longeſt way.
Leſt we ſhou'd treſpaſs on the ſmiling plain,
With our rude feet, and dragging of the chain,
I took my ſtation ſtrait along the way,
And in the ſame completed my ſurvey.
One hundred rods it ſtretch'd from end to end;
Th' obtuſe angles it alſo did ſubtend;
Which ſaid angles they are plac'd * below,
And their two perpendiculars alſo;
That is, if I more plain to you muſt ſay,
They from thoſe angles fall upon the way.
Ye douty brethren, that do drag the chain,
And you who with your poles do 'thwart the plain,
And thoſe that ſtalk the Laions by Spaniſh ſtrides,
With each your art unfold the unknown ſides.

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* *The obtuſe angles are* 108° *and* 118°, *and perpendiculars* 9 *and* 7 *chains.*

III. Question 183, *by Mr.* Tho. Fearn.

A general diſpoſing his army into a ſquare battle, finds he has 284 ſoldiers over-and-above; but increaſing each ſide with one man, he wanted 25 to fill up the ſquare. Quere the number of ſoldiers.

IV. Question 184, *by* Eumenes Pamphilus.

Poets, like me, will often ſtrain and ſoar,
To make that dubious which was plain before;
And merely for to make our fuſtian chime,
We'll metamorphoze reaſon into rhime:
And mathematics too, you often hear,
In rhimes are flat and nauſeous to the ear,
And ſometimes dubious: Therefore might I chuſe,
We'd often have 'em writ in plainer proſe.

There are three numbers in continued proportion; the product of the three multiplied into one another is 512, and the ſum of the extremes 34. Three-fourths of the greater extreme being called years, and the other fourth weeks, will exactly ſhew my age; the mean, the month, and the leſs extreme, the day of the month I was born: Whence you are deſired to ſhew the year, month, and day on which I was brought to light? Nov. 27, 1733.

V. Question 185, *by Mr.* Rob. Fearnſide.

A parabolic curve, * whoſe length, in feet,
Is five times ten more ten times five complete.
The greateſt area that can be incloſ'd
By this curve, and an ordinate ſuppoſ'd
Unto its greateſt axe rightly apply'd,
Vouchſafe, ye fair, with candor to decide?

* *Of the 2d kind, whoſe equation is* $ax^2 = y^3$.

VI. Question 186, *by Mr.* Ed. Hauxley.

The uſe of a meridian line in aſtronomy, geography, dialling, &c. is very great, and on its exactneſs all depends; whence infinite pains have been taken by divers aſtronomers to have it to the laſt preciſion. M. Caſſini has diſtinguiſhed himſelf by a meridian line drawn on a copper-plate on the pavement of the church of St Petronia at Bologna in Italy, the largeſt and moſt accurate in the world. In the arched roof of the church, at a great height above the pavement, is a little hole, through which the ſun's image, when in the meridian, falling upon the line, marks his progreſs all the year. When finiſhed, Mr Caſſini, in a public writing, informed the mathematicians of Europe of a new oracle of Apollo, or the ſun, eſtabliſhed in a temple, which might be conſulted with entire confidence as to all difficulties in aſtronomy.

Bologna, by ſome tables, lies in the latitude of 44° 8′ north; and Dr. Burnet, late Biſhop of Saliſbury, in his Letters many years ago, has given us an account of this meridian line, from whence the height of the hole in a perpendicular above the pavement, and the length of that copper meridian from that perpendicular, under the ſaid hole, to its utmoſt extent, where the ſun's image marks the tropic of Capricorn, in one ſum makes 285 feet 10 inches. From hence the height of the hole, or nodus above the floor, and the length of the meridian line, and the diſtances of the tropics from the equinoxes may be ſeverally known! and are here required?

The PRIZE QUESTION, *communicated by Mr.* Samuel Aſhby, *in a Letter to* Tho. Grubbian.

Ladies, ſince you your leiſure time employ
In matters of geometry,
And have ſo curiouſly diſclos'd
The queſtions that have been propos'd;
Farther to try your ſkill in lines,
Your mathematic friend conſigns.
Deſires next year, that you'll ſend back,
His quere ſolv'd i'th' almanac.

If in any plain triangle, as ABC, you draw lines from each angle through any point E within the triangle, till they cut the oppoſite ſides, as Ca, Ab, and Bc. The rectangles of the alternate ſegments at thoſe ſides will be equal.

See the fig. to the ſolution.

viz. $Ab \times Ca \times Bc = bC \times aB \times cA$.

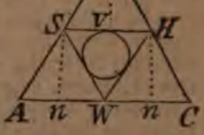
The demonſtration of this curious propoſition is required.

1736.

Queſtions anſwered.

I. QUESTION 181 *anſwered by Mr.* Chriſt. Maſon.

FIRST, per Euclid, circles are in proportion as the ſquares of their diameters, therefore the diameter of the circle will be found 19·79976. And, per plain trigonometry, as s. 30° : diameter :: s. 60° : 34·28 the ſide of the circumſcribing trigon $= SH = HW = WS$. The ſquare root of $\frac{3}{4}$ of the ſquare of the ſide is the perpendicular $VW = 29{\cdot}69$.



Let $b = AB = 65$; $c = BC = 74$; $d = SH = 34{\cdot}28$; $p = VW = 29{\cdot}69$; $a = BV$ ſought. Then 4 Eucl. 6, as $a : d :: a + p : \frac{dd + dp}{a} = AC$;

$= AC$: And per 47 Eucl. 1, $\sqrt{bb-aa+2pa+pp} = AW$; alſo $\sqrt{cc-aa+2pa+pp} = WC$; and $AW+WC = \frac{da+dp}{a}$: which, properly reduced, gives $a = 31{\cdot}42$; and $a+p = BW = 60{\cdot}735$; $AW = 23{\cdot}15$; $WC = 42{\cdot}28$; therefore $AC = 65{\cdot}43$.

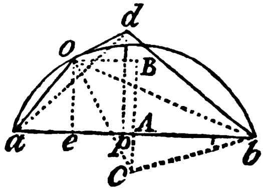
The ſame anſwered by Eumenes Pamphilus.

As 3·1415926 &c. : 1 :: 307·9 : 98·00763 = ſq. radius. Whoſe root 9·89988 = radius; and 19·79976 = diameter. Then, by Ward's Proportion, p. 336, as ·28867513 : 2 :: 9·899 : 34·29419 the ſide of the equilateral triangle required.

Now let $BA = a = 65$; $BC = b = 74$; $SH = c = 34{\cdot}29419$; $x = Ac$; and let Sn and Hn be perpendiculars let fall from the points S and H upon the baſe AC. Then will $Sn = Hn =$ three times the radius of the inſcribed circle $= 29{\cdot}69964 = d$; then, per 2 Eucl. 6, $x : c :: a : \frac{ac}{x} = BS$; and $x : c :: b : \frac{bc}{x} = BH$; $a - \frac{ac}{x} = SA$; and $b - \frac{bc}{x} = HC$; then per 47 E. 1, $\sqrt{aa - \frac{2aac}{x} + \frac{aacc}{xx} - dd} = An$; and $\sqrt{bb - \frac{2bbc}{x} + \frac{bbcc}{xx} - dd} = nC$; conſequently $x = c + \sqrt{aa - \frac{2aac}{x} + \frac{aacc}{xx} - dd} + \sqrt{bb - \frac{2bbc}{x} + \frac{bbcc}{xx} - dd}$; and by reduction $x = 67{\cdot}743589 = AC$ the baſe required.

II. QUESTION 182 *anſwered by the Propoſer Mr.* Maſon.

The obtuſe angles given are $adb = 108°$ and $aob = 118°$; the perpendiculars $pd = 9$ chains, and $eo = 7$ chains; and the baſe $ab = 25$ chains. Required bd, do, and oa; the other three ſides of the trapezium? Which will be found trigonometrically, viz. $ad = 16{\cdot}61$; $db = 14{\cdot}23$; $bo = 17{\cdot}71$; $ao = 11{\cdot}15$; $do = 5{\cdot}76$.*



III. QUES-

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* II. QUESTION 182.

In this queſtion are concerned two triangles aob, adb, on the ſame given baſe, and whoſe perpendiculars and vertical angles are alſo given; to find their ſides and the diſtances of their vertexes.

Conſtruction.

On the given baſe deſcribe two ſegments of circles capable of containing the given vertical angles; then at diſtances equal to the two perpendiculars draw two parallels to the baſe, and they will cut the reſpective circles in the required vertexes o and d of the two triangles. As is too evident to need a demonſtration.

Calculation.

Let C be the center of the circle paſſing through one of the vertexes; and CAB perpendicular and oB parallel to ab, and the other lines as per figure. Then $ACb = 62° =$ the ſupplement of the given $\angle aob$, and ACo is $=$ the difference of the $\angle$s (oab, oba) at the baſe. But s. $\angle ACb$: coſ. $\angle ACb$ :: Ab ($\frac{1}{2}ab$) : AC; hence $CB = AC + oe$; then (ſince Ab and CB are the ſines of the $\angle$s ACb and Cob to the ſame radius Cb or Co) $Ab : CB$ :: s. $\angle ACb$: s. $\angle CoB =$ the complement of the difference of the $\angle$s oab, oba, at the baſe. Hence their ſum and difference being known, the angles themſelves become known; and thence the ſides ao, bo.——In like manner are found the ſides ad, bd. And thence od.

III. QUESTION 183 *anſwered by* Ruſſellus.

Put $a =$ ſide of the firſt ſquare; $b =$ the overplus (284); and $c = 25$ the number wanting to fill up the ſecond ſquare. Then $aa + b = aa + 2a + 1 - c =$ the number of ſoldiers per queſt. $b = 2a + 1 - c$; $b + c - 1 = 2a$; and $\frac{b + c - 1}{2} = a = 154$; $aa + b = 24000 =$ the number of ſoldiers.

Nearly in the ſame manner it is anſwered by *Eumenes Pamphilus*.

Mr. *Fearne* the propoſer obſerves that in any queſtion of this kind, the two numbers given muſt be the one even, the other odd.

IV. QUESTION 184 *anſwered by* Ed. Golding.

The three numbers in continual proportion are 2, 8, 32: for $2 \times 8 \times 32 = 512$, and $2 + 32 = 34$; which exactly agrees with the propoſal: three-fourths of the greater extreme is 24 for years; and one-fourth is 8 weeks; ſo he was 24 years 8 weeks old; and he was born October 2, 1709.

The ſame anſwered by Mr. John Turner.

Put a, e, and u for the three numbers ſought; $b = 512$; $c = 34$. Then, by the queſtion, $aeu = b$; $a + u = c$; and $au = ee$: by the firſt $au = \frac{b}{e}$ (in the 3d ſtep) $= ee$; hence $eee = b$; and $e = \sqrt[3]{b} = 8$ the mean. And from hence u is found $= 32$, and $a = 2$; ſo he was 24 years 8 months and 8 weeks old.

Anſwered by Mr. Maſon.

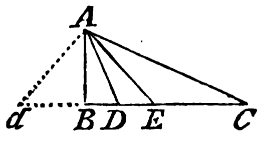
Let a, e, and u be the three numbers ſought, and $aeu = z = 512$; $a + u = s = 34$ per queſt. Then $au = ee$ per property, hence $\frac{z}{e} = ee$, therefore $z = eee$; and $\sqrt[3]{z} = 8 = e$; conſequently $a = 2$; $u = 32$; then 24 years and 8 weeks muſt be his age, the mean muſt repreſent October, and the leſſer extreme the ſecond day thereof.

V. QUEST. 185 *anſwered by Mr.* Fearnſide *the Propoſer.*

Putting the equation $ax^2 = y^3$ of the curve into fluxions, we have $2ax\dot{x} = 3y^2\dot{y}$; hence $\dot{x} = \frac{3y^2\dot{y}}{2ax} = \frac{3\dot{y}}{2}\sqrt{\frac{y}{a}}$. Then the fluxion of the area $= y\dot{x}$ is $\frac{3y\dot{y}}{2}\sqrt{\frac{y}{a}}$, whoſe fluent $\frac{3yy}{5}\sqrt{\frac{y}{a}}$ is the area a maximum. Again, the fluxion of the curve $\sqrt{\dot{x}^2 + \dot{y}^2}$ will become $\dot{y}\sqrt{\frac{9y + 4a}{4a}}$, whoſe correct fluent $\frac{\left(\frac{9y+4a}{a}\right)^{\frac{3}{2}} - 8}{27} \times a$ is $= 50 = c$. Then the value of either a or y being found from this equation, and written for it in the above area or maximum, the maximum will then contain only one of the letters a or y, and whoſe fluxion being made equal to 0, that letter will be determined, and thence all the reſt.

VI. QUESTION 186 *anſwered by Mr.* Ed. Golding.

There is given $AB + BC = 285$ feet 10 inches; required AB, BC, BE, and BD. I here uſe 23° 30′ for the greateſt declination: then $90° - 44° 8' = 45° 52'$ the elevation of the equinoctial; and the tropics and equinoctial will have the ſame angles in reſpect to each other. Of all which BD, BE, BC are tangents to the radius AB. Produce the line CB till $Bd = BA$; then, per queſtion, $Cd = 285$ feet 10 inches $= 3430$ inches. Then the angle dAB = angle $d = 45°$, and $(CAB =)\ 68° 3' + 45° = 113° 31' = CAd$.



As s. CAd : Cd : : s. d : CA = 2649·24 inches.
Rad. : AC : : s. CAB : CB = 2453·95 = 204 feet 5 inch. 95 par.
Rad. : AC : : s. ACB : AB = 976·05 = 81 feet 4 inch. 05 par.

Sum 3430 = 285 10 0

And proceeding in the ſame manner ſhall find

	Inches		Feet	Inch.	Parts
BD = — — —	380·7423	=	31	8	7423
♋ ♎ *DE* — — —	588·8556	=	49	0	8556
♈ ♑ *EB* — — —	1484·3521	=	123	8	3521
The ſum = *BC* = —	2453·95	=	204	5	95
To which add *AB* =	976·05	=	81	4	05
Total — — —	3430·	=	285	10	0

	AB	*CE*	*DE*	*CD*	*BD*	*CB*
By *F. R. S.*	83·33	121·19	49·54	171·19	31·31	202·50
Mr. *C. Maſon*	82·83	122·16	49·33	171·58	31·42	203·00
Mr. *J. Turner*	83·58	121·35	49·42	170·77	31·47	202·25
Mr. *E. Hauxley*	83·58	121·35	49·42	170·77	31·47	202·[illegible]
Mr. *J. Hampſon*		122·75	46·25	172·00		203·[illegible]

N. B. No notice is taken of the diſtance of Bologna's longitude in time from the ☉'s entering the firſt ſcruple of the equator or tropics, nor of the ☉'s refraction, in the anſwers to this queſtion, as being too curious.*

The

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* VI. QUESTION 186.

The above calculation of this queſtion is not very accurate with regard to the angles; however any one may eaſily make it as accurate as he pleaſes by the ſame method, or rather by this following.

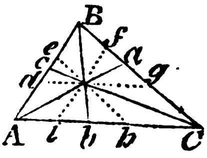
In the above figure, we have given all the angles formed at the point *A*, and the ſum of the legs *AB*, *BC*; to find the reſt. For *A* repreſents the hole, *BC* the meridian, *AB* the perpendicular on it, *AE* a ray from the ſun in the equinoctial, and *AD*, *AC* rays from him when in the two tropics: therefore the $\angle BAE =$ the lat. $= 44^\circ\ 8'$; to and from which adding and ſubtracting the declination $(23^\circ\ 29') = \angle EAC = \angle EAD$; the ſum $(67^\circ\ 37')$ will be the $\angle BAC$, and the difference $(20^\circ\ 39')$ the $\angle BAD$.

But, by plane trigonometry, 1 (rad.) : t (tang. $\angle BAC$) : : AB : BC; then, by compoſition, &c. $1 + t : AB + BC :: \begin{cases} 1 : AB, \\ t : BC. \end{cases}$

Which being known, then $1 : AB :: \begin{cases} \text{tang. } \angle BAE : BE, \\ \text{tang. } \angle BAD : BD. \end{cases}$

The PRIZE QUESTION *anſwered by Mr.* C. Maſon.

Per 31 Eucl. 1, draw lines parallel to the given ſides thro' O the given point, as dg, eh, fi. Then, per 2 Eucl. 6, will be the following proportions, viz. $dO : Og :: Ab : bC$; and $iO : Of :: Ac : cB$; alſo $eO : Oh :: Ba : aC$; from which will proceed the following compound proportions, viz. $dO \times iO \times eO : Og \times Of \times Oh :: Ab \times Ac \times Ba : bC \times cB \times aC$; and, per 4 Eucl. 6, $hO : eO :: iO : de$; and $Og : Od :: Of : de$; Ergo $\frac{eO \times iO}{hO} = \frac{dO \times Of}{Og}$; which, reduced, gives $dO \times fO \times hO = eO \times iO \times gO$; pert. alterna. & comp. ergo $Ac \times Ba \times bC = cB \times aC \times bA$. *Q.E.D.*

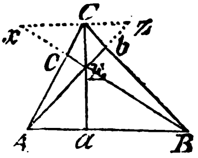


Mr. *J. Turner*'s anſwer is omitted, as being nearly the ſame.

Errata. The letter O is omitted at the point of interſection.

Another Demonſtration by Mr. Ed. Enotts.

Draw xCz parallel to AB, and produce Ab and Bc till they meet with the line xz. The $\triangle\, AEa$ is ſimilar to CEz; and xCE to aEB; and bzC to AbB; alſo xCc to AcB: therefore $xC : CE :: Ba : aE$; and $CE : Cz :: aE : aA$; alſo $xC : Cz :: Ba : Aa$; but $xC : Cz$ is compounded of $xC : AB$ and of $AB : Cz$; but $xC : AB :: Cc : cA$, and $AB : Cz :: Bb : bC$; therefore $xC : Cz$ $(:: Ba : Aa) :: Cc : cA$ and $Bb : bC$; that is $Ba : Aa :: Cc \times Bb : cA \times bC$; therefore $Ba \times Ac \times Cb = Aa \times Cc \times Bb$. *Q.E.D.*



The prize of 10 diaries was won by Mr. *Natt. Percivall.*

Of the Eclipſes in 1736.

Within the ſphere of the earth's orbit will happen ſix eclipſes this year. Four times will the moon, in her wandering courſe, interpoſe, and hide the ſplendor of the ſun from falling on the earth or its atmoſphere. And twice will the earth in its courſe ſo fall in the line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Sun eclipſed March 1, at 2h. 36m. afternoon, but the moon's latitude prevents its being viſible to us.

2. Moon eclipſed viſible and total on monday March 15, near midnight. Several calculations of which are as follow.

Computed by		Beg.	Mid.	End	Dur.	Dig.
		h. m.				d. m.
By Aſtron. Car. at Coventry		10 10	11 55	1 4	3 29	21 56
Mr. Chattock, London		10 11	12 0	1 48	3 36	22 14
Mr. Leadbetter, London		10 6	11 51	1 36	3 29	21 45
Mr. Hauxley, Kirkleatham		10 2	11 58	1 33		21 56
		9 48	11 33	1 19	3 31	21 55
Mr. R. Dunthorne, Ramſey		10 17	12 2	1 47	3 20	21 57
Mr. J. Hampſon, Leigh		10 4	11 50	1 35	3 31	22 2
Mr. S. Prichard, London		9 11	11 55	2 42	5 31	21 54
Mr. J. Bulman,	London	10 16	12 1	1 16	3 29	21 57
	Edinburgh	10 4	11 49	1 34		
	Dublin	9 48	11 33	1 6		
	Carliſle	10 5	11 50	1 55		
	Deptford	10 17	12 1	1 46		
Mr. G. Maſon,	Newcaſtle	9 42	11 28	1 14	3 32	21 55
	Edinburgh	9 48	11 34	1 20	3 32	21 55
Mr. J. Wilſon, Morpeth		10 1	11 46	1 31	3 29	21 46
Mr. T. Williams, Oxfordſhire		10 0	11 47	1 31	3 31	21 47
Mr. W. Brown, Bridgnorth		10 6	11 55	1 36	3 29	21 57
Mr. J. Thomas		9 44	11 30	1 17	3 32	21 38
Mr. W. Baylis, Oxfordſhire		10 1	11 47	1 31	3 30	22 0
Mr. T. Sparrow, Nottingham		10 3	11 48	1 33	3 20	21 45
Mr. C. Forreſt, Newcaſtle		9 41	11 30	1 18	3 37	22 19
Mr. J. Hilton, London		10 12	11 57	1 41		21 42
Mr. G. Forſter, Branſpath		9 58	11 43	1 30	3 32	22 1
Mr. T. Cowper, Wellingbrough		10 20	0 6	1 51	3 31	21 34

3. Sun eclipſed March 31, at 7 morning, inviſible.

4. Sun eclipſed Auguſt 25, at 9 morning, inviſible.

5. Moon eclipſed, total and viſible, Sept. 9, at 3 morning.

Computed by	Begin. h. m.	Mid.	End	Dur.	Dig.
Aſtronomia Car. Coventry	1 5	2 59	4 53	3 48	21 2
J. Chattock, London	12 44	2 50	4 56	4 12	22 1
C. Leadbetter, London	1 7	3 2	4 57	3 49	20 32
E. Hauxley, Yorkſhire	12 25	2 20	4 14		20 30
Streets J.	1 15		5 4	3 49	21 2
R. Dunthorne, Ramſey	1 7	3 6	5 4	3 57	21 3
J. Hampſon, Lancaſhire	1 14	3 14	5 14	3 59	21 12
S. Prichard, London	1 22	3 17	5 12	3 50	21 13
J. Bulman	3 22	5 15	7 8	3 46	18 38
G. Maſon, Newcaſtle	1 9	3 4	4 58	3 49	21 2
Edinburgh	1 15	3 9	5 4	3 49	21 2
T. Williams, Oxfordſhire	1 4	2 58	4 53	3 48	20 33
W. Brown, Bridgnorth	1 2	2 56	4 50	3 48	21 2
J. Thomas, Penzance	0 50	2 43	4 36	3 46	20 39
W. Baylis, Mixbury	1 2	2 58	4 52	3 49	20 47
Mr. T. Sparrow, Nottingham, from Flamſtead's Tables	0 53	2 51	4 49	3 56	21 10
C. Forreſt, Newcaſtle	1 10	3 8	5 5	3 54	21 0
J. Hilton, London	1 7	2 53	4 39		19 54
C. Forſter, Branſpath	1 4	2 59	4 58	3 53	20 46
T. Cowper, Wellingborough	0 59	2 56	4 54	3 55	21 14

6. Sun

The 2d Eclipſe was obſerved thus:

At	By	Beginning	Tot. Im.	Emerſion	End
		h. m. s.	h. m. s.	h. m. s.	h. m. s.
Fleet Str London	G. Graham	10 13 0	11 11 0	12 49 0	13 47 0
Ibid.	Mr. Celſius		11 10 0		13 46 0
Cov. Gar. London	Dr. Bevis	10 11 40	11 10 0	12 47 56	13 46 25 t.t.
Greenwich	Dr. Halley	10 13 37	11 9 42		
Yeovil, *Somerſet*	*John Milner*	10 6 0	11 4 30	12 43 30	13 39 15 t.t.

6. Sun eclipſed September 23, at 5 evening, part viſible.*

Computed by	Beg.	Mid.	End	Dur.	Dig.
Aſtron. Carolina, Coventry	4 56	5 28	6 9	1 22	3 41
J. Chattock, London	4 57	5 37	6 15	1 18	3 10
J. Chattock, Coventry	4 48	5 29	6 8	1 20	3 18
Mr. Ra. Hulſe, Sandbank	4 44	5 36	6 4	1 27	3 40
R. Dunthorne, Ramſey	4 53	5 34	6 31	1 20	3 36
J. Hampſon, Leigh	5 5	5 49			4 0
T. Sparrow, Nottingham	4 50	5 32	6 16	1 26	4 0
Anonymous, Leiceſter	4 39	5 23	6 3	1 24	3 44
J. Dorking, Suffolk	4 55	5 37	6 18	1 22	3 45
G. Maſon, Newcaſtle	4 45	5 27	6 8	1 22	3 40

Mr. *J. Dunthorne* gives the following account of the Tranſit of Mercury over the Sun, October 31, 1736, for Ramſey, calculated by Aſtronomia Carolina.†

Equal

The 5th Eclipſe was obſerved thus:

At	By	Beginning	Tot. Im.	Emerſ.	End
		h. m. s.	h. m. s.	h. m.	h. m.
Fleet Str. London	G. Graham & Ja. Short	12 58 0	14 3 45		a.t.
Cov. Gar. London	Dr. Bevis	12 56 50	14 2 25		a.t.
Wittemberg	J. F. Weidler	13 50 0	14 53 0	16 44	
Hudſon's Bay	C. C. Middleton		8 43 0	10 29	11 37 t. t.

* The beginning of this 6th Eclipſe Dr. Bevis obſerved, in London, at 4 h. 45 m. 31 s. app. time. The other obſervations could not be made for clouds.

† This Tranſit was obſerved by ſeveral perſons at different places; but, by reaſon of clouds, the obſervations were very imperfect, ſo that all that are worth recording here are as follows.

At	By	Beginning	Tot. Im.	Emerſ.	End
		h. m. s.	h. m. s.	m. s.	m. s.
Bologna	E. Manfredi	22 7 56	22 11 12	50 50	54 56 app. t.
Greenwich	Dr. Bevis & *Dr. Halley*				8 33

	d.	h.	m.	s.
Equal time of the true conjunction in the ecliptic	31	0	8	53
True longitude of the ſun ♏ — — —		19	23	14
of Mercury in the ecliptic ♏		19	23	14
Inclination of Mercury north — — —			31	1
Hourly motion of Mercury from the ſun —			12	49
Equal time of the neareſt approach — —		23	47	52
Equat. ad 3.58 the apparent time — —		23	51	50
Neareſt diſtance of centers 14′ 9″ ſemidiam. ſun			16	6
The beginning or ingreſs of Mercury's center		22	44	33
End or Emerſion of Mercury's center — —			59	7

New Queſtions.

I. QUESTION 187, *by Mr.* Tho. Simpſon.

By reading your di'ry a croud of ſtrange notions
Crept into my head, of your rules, laws, and motions;
Your extravagant fancies my ſenſes confound;
Can the unwieldy earth at the ſun caper round?
But you ſay, ſhe's an atom, each ſtar a huge ſun,
And attendant worlds with their moons round 'em run.
Such a tott'ring ſtrange whirligig you've ſet's upon,
We wonder ere now we're not ſhak'd off and gone:
If what eyes ne'er ſaw you ſo ſoon can diſcloſe,
Then pray ſolve this queſtion: The earth, we'll ſuppoſe,
Round her axis in thirty-eight minutes to roll;
Shou'd we, who're degrees thirty-eight from the pole,
Be hurl'd thro' the air; where ſhould we deſcend?
How long wou'd it be ere our circuit did end?
How far from the center in ſix hours time
Wou'd they be, who live in the midſt o'th' hot * clime?
Kind artiſt, be pleas'd theſe things to let's know?
We'd rather believe you, than e'er find them ſo.

N. B. We ſuppoſe the earth ſole actor, and to continue inviolate, and that we ſhall acquire the ſame velocity as the place of our reſidence. 52 deg. lat.

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* *The equator.*

II. QUESTION 188, *by Mr.* Eumenes Pamphilus.

I pray be ſo kind as the third ſide to ſhow
Of a three-corner'd field, by theſe data below.
Two hundred ninety-ſix poles, and no more,
Is one ſide; another two hundred and four.
The content of it likewiſe, I know, has been found
One hundred and thirty-eight acres of ground;
The ſought ſide, to free you from errors, I know,
Is longer than either of the given two.
If to ſolve me this queſtion, your aid you will lend,
You'll highly oblige a kind brother and friend.

III. QUESTION 189 *by Mr.* Rob. Beighton.

Solomon, we are aſſured in holy writ, was a man of the moſt extenſive judgment, and the wiſeſt of all mankind in his days. How great a philoſopher he was, may be gathered from his writings; and amongſt them are ſome indications that he was acquainted with the circulation of the blood, (not rightly deſcribed to us before Dr. Harvey wrote of it in 1628) the compoſition and frame of the human bodies, as well as others. His ſkill in architecture is ſufficiently evident from the account we have of his building that moſt magnificent, beautiful, and perfect pile, the temple, and its furniture: but I cannot learn from the divine writings, or from Jeſephus's fabulous hiſtory, how perſons can form an idea in what orders, and in what manner exactly the workmanſhip was performed, as to give us exact draughts and models thereof: whether of the Corinthian or Ionic orders, with their members and moldings, ſuch as we have had tranſmitted down to us from the Greeks and Romans, and copy at this day. Solomon, no doubt, would have rejoiced to have added to him one bleſſing which we have enjoyed, the acquaintance of (the glory of the age) Sir Iſaac Newton, who, we may ſay, has exceeded all men ſince Solomon's time. It may perhaps be thought a preſumption too nice and curious, to examine any of Solomon's great works mathematically: but as nothing is intended, nor can be ſaid to leſſen his wiſdom or ſtupendous work. I ſhall only ſuppoſe a queſtion was taken from that deſcription the inſpired writer has given us of his molten ſea, 1 Kings vii. 23. 'And he 'made a molten ſea ten cubits wide from brim to brim, 'round in compaſs, and five cubits high, and a line of thirty 'cubits did *compaſs it about*.' It is evident it could not be a circle; for then the line that compaſſed it muſt have been more than thirty cubits, viz. 31·4159265359. Therefore we may ſuppoſe it to be elliptical, (or that the workmen were not very curious in their meaſures, or not ſkill'd in geometry) and the tranſverſe diameter ten cubits, and its periphery thirty: What then would be the leaſt triangle that could circumſcribe the ſame? and how analytically to inveſtigate the ſolution?

N. B. A cubit is equal to 21·888 inches.

IV. QUESTION 190, *by Mr.* John Bulman.

There is a caſk, ſuppoſed the fruſtum of a parabolic conoid, or a caſk of the 2d variety. The bung diameter is 38·4 inches; the head diameters are unequal, the greater 33·5, the leſſer 28·8; the length of the caſk 54·27. Required the content of the caſk in ale gallons, the diſtance of each head diameter from the bung; it being ſuppoſed that there had been a decay in the caſk, and cut off, and a new and larger head put in at one end; and to let us know the diameter, length, and content of the greateſt cylinder that can be inſcribed therein, the circumference of each baſe touching that of the caſk around?

V. QUESTION 191, *by Mr.* John Turner.

Within a maſon's yard one day,
A ſtone of ſize immenſe there lay;
The form of which (it was agreed on)
To be a parallelopipedon.
The depth, and breadth, and length, I here declare
In true arithmetic progreſſion are:
The ſtone's content below is * ſhown,
With what is needful to be known,
In order to inveſtigate
All the dimenſions ſeparate.

* *The ſolidity is* = 5184 *cubic feet, and the common difference of the depth, breadth, and length is equal to one forty-eighth part of the rectangle of the depth into the length. The depth of the ſtone being the leaſt dimenſion, and the length thereof the greateſt.*

VI. QUESTION 192, *by Mr.* C. Maſon.

Find three ſuch cube numbers, whoſe ſum may be both a ſquare and cube number; and if that ſum be ſquared, to be a cube; alſo if cubed, ſhall be a ſquare.

The PRIZE QUESTION, *by Mr.* Tho. Simpſon.

Young Strephon, long bleſs'd with his charming fair,
In happy conſort liv'd devoid of care;
Till cruel fate call'd the fair nymph away
From his kind arms, to croſs the raging ſea;
Where horrid tempeſts, in thick darkneſs, roar,
And toſs'd his deareſt to an unknown ſhore.
He mourns, is reſtleſs, wanders day and night,
In ev'ry clime to find his dear delight.
Her lovely aſpect his wing'd ſoul inſpires;
He longs, ſighs, wiſhes, melts in ſoft deſires.
Propitious Venus pities his ſad moan,
Glides ſhining down from her etherial throne,
And ſmiling ſays, (in a majeſtic tone)
'Thrice ten degrees north of th' equator lies
'This place, from which as Sol due eaſt doth riſe,
'Set out; and keep him always in your face;
'Move not too faſt, but ſuch an equal pace,
'To be eight miles more ſouth at four hours end,
'And you'll arrive to th' arms of your dear friend.'
Thus ſaid, ſhe vaniſh'd from his wond'ring ſight:
But ſtill the ſwain is in a mournful plight,
Unleſs, fair nymphs, you'll vouchſafe to explain,
And ſhew the place, that muſt the fair regain.
i. e. *the diſtance required to move in an hour.*

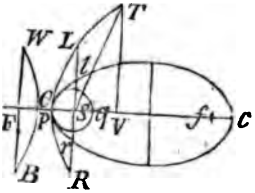
A Geographical Paradox by Mr. C. Maſon.

If Briſtol from London be exactly due weſt;
Obſerve what I ſay; you may think it a jeſt,
That London, from Briſtol, I can make appear,
From what I've obſerv'd, due eaſt will not bear.
This ſeeming abſurdity pray reconcile,
When Briſtol's from London one hundred mile.
And in your next diary make it appear,
What rhomb 'tis that London from Briſtol does bear.

1737.

Queſtions anſwered.

* I. QUESTION 187 *anſwered by Mr.* J. Turner.



THE circumference of the earth being 25080 miles, at the equator, if it revolves round its axis in 38 minutes, each point of the equator will go through 11 miles each ſecond, with this velocity the body is projected from P. Let $SP = 1$: then the velocity of a body moving in a circle Pq, at the diſtance of one ſemidiameter from the earth's center, is ſuch, as would make it go 4·92 miles each ſecond uniformly; and according to Mr. Abr. de Moivre, in Philoſ. Tranſact. Abridg'd, p. 5. vol. 4.

Putting $R = 11$, $Q = 4{\cdot}92$; $RR - 2QQ : RR :: SP : PF$. i.e. $72{\cdot}6 : 121 :: 1 : 1{\cdot}666$, and the point F will be the other focus; and becauſe $FC = PS = 1$, therefore $CP = {\cdot}666$ the tranſverſe axis of the ſection, which is an hyperbola.—— $QQ : RR :: 2SP : LR$; that is, $24{\cdot}2 : 121 :: 2 : 10$; hence the latus-rectum of the hyperbola is 10 ſemidiameters of the earth.

Again,

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* I. QUESTION 187.

The above computations of Mr. Turner's are rightly made as far as they are carried, but he has left ſome part of the queſtion undetermined, which we ſhall here ſupply.

The body projected from the latitude of 52° will deſcribe an ellipſe whoſe focus is the center of the earth, its tranſverſe axe $=$ 19·13 (ſemidiameters of the earth), and its parameter 3·791, as determined above. So that the ellipſe, no where touching the earth but in the place from whence the body is projected, the body will

Again, each point in the parallel of 52 degrees goes through 6·773 miles each ſecond, put this $= R$; $2QQ - RR : RR :: SP : Pf$; that is, $2·53 : 45·87 :: 1 : 18·13$, and the point f will be the other focus: and becauſe $fc = PS = 1$, therefore $cP = 19·13$, the tranſverſe axis of the ſection, which in this caſe is an ellipſis, and $QQ : RR :: 2SP : lr = 3·791$ its latus-rectum.

Merones

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will revolve about the earth without falling upon it again or touching it except in the place from whence it was projected, ſuppoſing the earth's revolution to ceaſe at the inſtant when the body is projected; but if the revolution be continued, the body will touch ſome other part of the ſame parallel of latitude, or will return to the vertex of its orbit again when ſome other point of the ſaid parallel paſſes under it in revolving; and which point will be thus found: Putting $s = 16\frac{1}{12}$ feet, $v = 6·773 \times 5280$ feet velocity in the vertex, and $f = SP = 3992 \times 5280$ feet; then the periodic time, or time of one revolution, is known to be $3·1416 \times \frac{4sff}{\overline{4fs - vv}|^{\frac{3}{2}}}$ $= 132277$ ſeconds; this being divided by ($38'$ or) $2280''$, the quotient 58 ſhews the number of compleat revolutions the earth makes in the ſame time, and the remainder 37 ſhews what part of another revolution is made; wherefore as $2280 : 37 :: 360° : 5° 50' =$ the difference of longitude required from the point of projection.

Again, with regard to the body projected from the equator, the path will be an hyperbola whoſe focus is alſo the earth's center, its tranſverſe axe $\frac{2}{3}$, and latus-rectum 10, as determined above; and conſequently its conjugate axe $= 2\sqrt{\frac{5}{3}}$. Then to find at what diſtance from the focus it will be in 6 hours $= 21600$ ſeconds: Since the body deſcribes equal areas in equal times, and the rate of deſcription being $\frac{11 \times 1}{2 \times 3992} = \frac{11}{7984}$ ſquare ſemidiameters of the earth per ſecond, therefore $\frac{21600 \times 11}{7984}$ will be the area deſcribed in 6 hours, or the area included by the curve, and the focal diſtances from the vertex and the other end of the curve, viz. the ſpace SPT, and in which $PS = 1$.

Now

Merones ſays that the body in 6 hours will be 8 of the earth's diameters from the center. Mr. *Lowe* ſays the equator will go 10·9 miles, the parallel of 52°, 6·7; and in 6 hours the moving body would be 1486709 miles from the earth's center. Mr. *Abr. Lord* 246321, Mr. *Geo. Brown* 204660, and Mr. *Hauxley* 400589 miles. I have not the ingenious author's ſolution by me, to the intricate and difficult queſtion, ſo ſhall ſay no more of it till I can procure it.

II. QUES-

Now if the ordinate TV be drawn, and PV be put $= x$, the tranſverſe $PC = \frac{2}{3} = t$, the conjugate $2\sqrt{\frac{5}{3}} = c$, and the area $PTS = \frac{21600 \times 11}{7984} = A$; then $SV = x - 1$, $TV = \frac{c\sqrt{tx + xx}}{t}$, and the $\triangle STV = \frac{x-1}{2} \times \frac{c\sqrt{tx+xx}}{t}$; conſequently the hyperbolic ſegment will be expreſſed by $A + \frac{x-1}{2} \times \frac{c\sqrt{tx+xx}}{t}$. But by Rule IV. p. 376 Menſuration, the area of the ſame ſegment is expreſſed by $\frac{21\sqrt{tx + \frac{5}{7}xx} + 4\sqrt{tx}}{75} \times \frac{4cx}{t}$; theſe two expreſſions being made equal to each other, and reduced, we have $168x\sqrt{tx + \frac{5}{7}xx} + 32x\sqrt{tx} = \frac{150At}{c} + \overline{x-1} \times 75\sqrt{tx+xx}$; or, by reſtoring the values of t and c, it will be $168x\sqrt{2x + 2\frac{1}{7}xx} + 32x\sqrt{2x} = 30A\sqrt{5} + \overline{x-1} \times 75\sqrt{2+3xx}$; and the root x is eaſily found $= 3{\cdot}083 = PV$. Then $ST = \sqrt{TV^2 + VS^2} = \sqrt{cc \times \frac{tx+xx}{tt} + \overline{x-1}|^2} = 4x + 1 = 13\frac{1}{3}$ ſemidiameters $= 53227$ miles, the diſtance required.

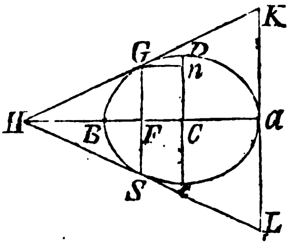
* II. QUESTION 188 *anſwered by the Propoſer.*

Let x = the ſide ſought, then will $\frac{x}{2} + 250$ = half the ſum of the ſides; each ſide being ſubtracted from it, and the three remainders multiplied into it, will give the ſquare of the content, and will produce $16154\,xx - \frac{x^4}{16} - 132250000 = 487526400$ poles; which, reduced, gives $x = 460$ poles, the ſide required.

Mr. *Turner*'s ſolution is omitted, as being on the ſame principles.

* III. QUESTION 189 *anſwered by Mr.* Turner.

Let $Ba = 10$ cubits $= b$; x = the conjugate axis De; the periphery $= 30$ cubits $= d$; therefore a quadrant $BGD = 7·5$.



The gentlemen of the Weekly Oracle have given us a theorem, by which nearly to find the periphery of an ellipſis, viz. To twice the ſquare root of the ſum of the ſquares of the two principal diameters, add one-third of the conjugate diameter, and it will give the circumference within leſs than one-hundredth part of the whole. Hence $2\sqrt{bb + xx} + \frac{1}{3}x = d$; the root $x = 9·056$ cubits $= 198·22$ inches.

But

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* II. QUESTION 188.

This queſtion may be eaſier ſolved thus:

Let the two given ſides be a, b, and the given area A. Then $\frac{2A}{ab}$ = the ſine of the included angle, whoſe coſine call c; then, by trigonometry, $\sqrt{aa + bb - 2abc}$ or $\sqrt{aa + bb - 2\sqrt{aabb - 4A^2}}$ = the ſide required.

But by fluxions, put $BC=a=5$; $DC=c$; $b=7{\cdot}5$; $x=Gn=FC$: Then, by the property of the ellipſis, $FG=y=\frac{c}{a}\sqrt{aa-xx}$, and $\sqrt{\dot{x}^2+\dot{y}^2}=\frac{\dot{x}}{a}\sqrt{\frac{a^4-aaxx+ccxx}{aa-xx}}$. Throw this into an infinite ſeries, and the fluent will be $x+\frac{c\ x^3}{6a^4}+\frac{c^2x^5}{10a^6}-\frac{c^4x^5}{40a^8}+\frac{c^2x^7}{14a^8}-\frac{c^4x^7}{28a^{10}}+\frac{c^6x^7}{112a^{12}}+\frac{c^2x^9}{18a^{10}}$, &c. which will be equal to the length of the arch GD; and ſuppoſing x to flow till it becomes $x=a$, then $a+\frac{c^2a^3}{6a^4}+\frac{c^2a^5}{10a^6}-\frac{c^4a^5}{40a^8}$, &c. $=b=a+\frac{c^2}{6a}+\frac{c^2}{10a}-\frac{c^4}{40a^3}$, &c.

Now I find the powers of c above c^2 are very ſmall, and may be ſet aſide at preſent, and the law of the progreſſion in c^2 being viſible, we have $a+\frac{cc}{6a}+\frac{cc}{10a}+\frac{cc}{14a}+\frac{cc}{18a}+\frac{cc}{22a}+\frac{cc}{26a}$ &c. $=b$.

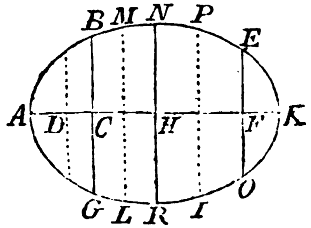
And by reduction $c^2=21{\cdot}17$, from which ſubtract the negative powers of c^4 which are equal to about $\cdot52$. And then $c=\sqrt{20{\cdot}05}=4{\cdot}544$, and the conjugate $=9{\cdot}088$ cubits, nearly the ſame as before.

For the leaſt circumſcribing triangle.

Put $b=Ba=10$; $c=De=9{\cdot}056$; $z=BF$; $aF=b-z$; then the ſubtangent $HF=\frac{bz-z^2}{\frac{1}{2}b-z}$; and by the property of the ellipſis, $b^2:c^2::bz-zz:GF^2$; therefore $2GF=\sqrt{\frac{4ccbz-4cczz}{bb}}$. Now the Δs HGS, HKL are ſimilar, whence $HF:(GS)\sqrt{\frac{4ccbz-4cczz}{bb}}::(HA)\frac{bb-bz}{b-2z}:(KL)=\frac{b}{2z}\times\sqrt{\frac{4ccbz-4cczz}{bb}}$: but $\frac{1}{2}KL\times Ha=$ the area of the $\Delta\,HKL$ which is to be a minimum, i. e. $\frac{b}{4z}\times\frac{bb-bz}{b-2z}\sqrt{\frac{4ccbz-4cczz}{bb}}=$ area; which reduced is $=\frac{b-z}{b-2z}\sqrt{\frac{b-z}{z}}=$ minimum; this in fluxions is $\frac{b\dot{z}}{\overline{b-2z}|^2}\sqrt{\frac{b-z}{z}} - \frac{b-z}{b-2z} \times \frac{b\dot{z}}{2z^2\sqrt{\frac{b-z}{z}}}$ $= 0$; which equation, reduced, gives $z = \frac{1}{4}b = 2{\cdot}5$. From whence the area of the $\Delta\, HKL$ is $= 117{\cdot}6$ cubits $= 39{\cdot}135$ ſquare feet.*

IV. QUESTION 190 *anſwered by the Propoſer.*

Put $x = DC =$ the difference of the diſtances of the two heads from the bung; $a = NH = 19{\cdot}2$; $b = BC = 16{\cdot}75$; $c = EF = 14{\cdot}4$; $d = CF = 54{\cdot}27$; then $\cdot 5d + \cdot 5x = DH = HF$; and $\cdot 5d - \cdot 5x = CH$: Per property of the curve, $aa - cc : \cdot 5x + \cdot 5d :: aa : \frac{\cdot 5aax + \cdot 5aad}{aa - cc} = AH$, and $\frac{aax + \cdot 5ccd - 5ccx}{aa - cc} = AC$



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* III. QUESTION 189.

Put $a = 10 =$ tranſverſe, $z =$ conjugate, and $c = 30 =$ circumference. Then, by Rule VI. p. 233 Menſuration, $\frac{a+z}{4} \times p + \frac{1}{2}p\sqrt{\frac{aa+zz}{2}} = c$, where p is $= 3{\cdot}1416$; hence $z = a - \frac{4c}{p} + 4\sqrt{\frac{2c}{p} - a \times \frac{c}{p}} = 9{\cdot}0876 =$ the conjugate axe very true.

Again, for the leaſt triangle; ſince $CF : FB :: aF : FH$ by the nature of the ellipſe, and $aF = FH$ in all curves, $\because CF = FB = \frac{1}{4}aB$. Then $aH = 2aF = \frac{3}{2}aB$, and $aK = 2FG = \frac{2DC}{aB}\sqrt{aF \times FB} = \frac{2DC}{aB}\sqrt{\frac{3}{4}aB \times \frac{1}{4}aB} = \frac{DC\sqrt{3}}{2}$; conſequently $Ha \times aK = \frac{3}{4}DC \times aB\sqrt{3} = 118{\cdot}05 =$ the leaſt triangle HKL required.

The propoſer *alſo ſolved* this queſtion, but the ſolution is omitted *as it was ſo very falſely* printed.

$= AC$, and per prop. $\frac{aax + {\cdot}5ccd - {\cdot}5ccx}{aa - cc} : bb ::$ $\frac{{\cdot}5aax + {\cdot}5aad}{aa - cc} : aa$; then, per 16 Eucl. 6, $a^4x - {\cdot}5aaccd - {\cdot}5aaccx = {\cdot}5aabbx + {\cdot}5aabbd$; hence $x = \frac{bbd - ccd}{2aa - bb - cc}$ $= DC = 15{\cdot}93174$. Hence the leſſer head from the bung $=$ $FH = 35{\cdot}10087$ inches, and $CH = 19{\cdot}1691$.

The content of $\left\{\begin{matrix} NEOR \\ NBGR \end{matrix}\right\} = \left\{\begin{matrix} 112{\cdot}6191 \\ 69{\cdot}3193 \end{matrix}\right\}$ ale gallons, or $\left\{\begin{matrix} 137{\cdot}483 \\ 84{\cdot}623 \end{matrix}\right\}$ wine gallons.

The whole content 181·9384 ale gallons or 222·106 wine gallons. The diameter of the greateſt cylinder inſcribed $=$ $BG = 33{\cdot}5$, length 38·33, and its content 119·8304 ale gallons or 146·2842 wine gallons.

Mr. *R. Dunthorne*, in a curious and conciſe manner, has wrought this queſtion, the content being the ſame as above, but he makes the diameter 30·88, and length 46·587 inches, for the greateſt inſcribed cylinder.

Mr. *Turner* ſays the queſtion refers to p. 444 of Ward's Math. and the ſecond variety is the parabolic ſpindle, and therefore makes the content greater, viz. 195·28 ale gallons, and the cylinder's length 45·5, the diameter 23·4 inches.

Mr. *Pilgrim* obſerves on this queſtion, That in all fruſtums of a parabolic conoid, as have their abſciſſa to their leaſt diameter more than half of the abſciſſa to the greater, can be inſcribed no greater cylinder than that to the fruſtum's leaſt diameter and height. Hence the diameter $= 30{\cdot}223$, length $= 49{\cdot}699$, content in ale gallons $= 126{\cdot}436$.

V. QUESTION 191 *anſwered by Mr.* Dunthorne.

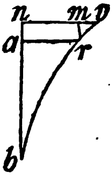
Put x, y, and $z =$ the depth, breadth, and length of the ſtone, $b = 5184$, and $c = 48$: then $\frac{b}{y} = xz$, and $\frac{b}{cy} = \frac{xz}{c}$ $=$ the common difference per queſtion. Whence $y - \frac{b}{cy} =$ x; and $y + \frac{b}{cy} = z$; conſequently $y^3 - \frac{bb}{ccy} = b$; reduced $y^4 - by = \frac{bb}{cc}$. Solved, $y = 18$; whence $x = 12$, and $z = 24$.

Mr. *Joſ. Roſſer*, Mr. *Robert Heath*, and Mr. *Paul Sharp*, have anſwered this queſtion in a very methodical manner.

VI. QUESTION 192 *anſwered by Mr.* J. Hill.

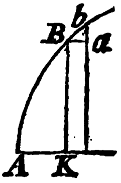
The three numbers are $\frac{x^6}{8}$, $\frac{8x^6}{27}$, and $\frac{125x^6}{216}$; x being any number at pleaſure: Hence if $x = 216$, we have three whole numbers for the anſwer.

The PRIZE QUESTION *anſwered by Mr.* Tho. Simpſon *the Propoſer.*



Let the radius be $= 1$; then the ſine of the latitude $= \frac{1}{2}$; for the verſed ſine of the hour from 6 put x; then the ſine of the ſun's azimuth will be $= \frac{1-x}{\sqrt{1-\frac{3}{2}x+\frac{3}{4}xx}}$; now whilſt x flows $\dot{x}$ the arch of time flows $\frac{\dot{x}}{\sqrt{2x-xx}}$; which (becauſe the man's motion is equal) muſt be in a conſtant ratio to the lineola (rv) = the diſtance moved by the man in the ſame time: therefore by putting (v) for the ſaid ratio, we have $(rv) = \frac{v\dot{x}}{\sqrt{2x-xx}}$; and becauſe the angle (mrv) is the complement of the ſun's azimuth, it is as radius : ſine of (mrv) :: $(vr):(mv) = \frac{v\dot{x}}{2\sqrt{1-\frac{3x}{2}+\frac{3x^2}{4}}}$: the fluent of which is $= \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{32} + \frac{27x^4}{512}$, &c. $\times v$, and when x becomes $\frac{1}{2}$, is the diſtance given $= {}^{\cdot}308v = 8$, therefore $v = \frac{8}{{}^{\cdot}308}$. But the fluent of (vr) is the arch of 4 hours; $60^\circ \times v = 1{}^{\cdot}0472v = 27{}^{\cdot}2 =$ the diſtance moved in 4 hours; therefore 6·8 miles is the required diſtance.

The ſame anſwered by Merones.



Let AB be the line he deſcribes, put $a =$ the ſpace moved in an hour, $r =$ radius $= 1$; $s =$ ſine, $c =$ coſine, $t =$ tangent, $ſ =$ ſecant, $q =$ arch of 60°; $AB = z$, $AK = x$; $y =$ the ſine of the hour from 6; $u =$ hour arch from 6: then will $\frac{ry}{\sqrt{rr-yy}} =$ tangent of hour. Let $\dot{z}$ and $\dot{x}$ be paſſed over in the ſame ſmall part of time, then by uniform motion $q : 4a :: \dot{u} : \dot{z} = \frac{4a\dot{u}}{q}$; and by the nature of the circle $\sqrt{rr-yy} : r :: \dot{y} : \frac{r\dot{y}}{\sqrt{rr-yy}} = \dot{u}$. And by trigonom. $r : s :: \frac{ry}{\sqrt{rr-yy}} : \frac{sy}{\sqrt{rr-yy}} =$ tang. azim. and $\frac{ty}{\sqrt{ſſ-yy}} =$ s. azim. from 6. The angle $Bba = \odot$ azimuth from 6. And $r : \frac{ty}{\sqrt{ſſ-yy}} :: (Bb)\ \dot{z} : (Ba)\ \dot{x} = \frac{ty\dot{z}}{r\sqrt{ſſ-yy}}$, &c. whence $a = \frac{ax}{4t \times 5329} = \frac{8q}{4t \times 5329} =$ 6·8072 miles required.

The prize of 10 Diaries was won by *Merones*.

The Geographical Paradox anſwered.

Under the artic pole we can look no way but ſouth; as ſoon as you change the pole from being your zenith, you have vertical circles, and conſequently eaſtings and weſtings.—— No two places in an oblique ſphere can have their vertical circles perpendicular to the plains of both meridians, conſequently no two places can be due E. and W. of each other, except they both lie under the equator. London will bear from Briſtol, from E. to N. 1 deg. 18 min. vide Gordon's Geogr. paragr. 42 prob. 43.

Mr.

Mr. Rob. Heath's *Anſwer to the ſame.*

To determine the bearing of two places from each other, there is commonly given the diſtance of each place from the pole, and their difference of longitude, (or angle at the pole) to find the bearing angles. Whence it is evident, that when the diſtances from the pole are equal, the bearing angles are equal. When each place is 90 degrees from the pole, each bearing angle is 90: conſequently thoſe places muſt bear in a contrary direction, viz. E. and W. of each other, which can only be when both places are under the equinoctial. For if the diſtances from the pole are equal, and not equal to 90 degrees, then one place bears as far from the weſt of the other, as that bears from the eaſt of it; and if the diſtances are unequal (as of Briſtol and London) the bearing angles muſt be unequal, and conſequently thoſe places can't bear contrary.——The ſolution.

Given the diſtance from the pole to Briſtol equal to 38 deg. 30 min. from the pole to London equal to 38 deg. 28 min. the angle at London equal to 90 deg. the angle at Briſtol will be found by a ſingle proportion equal to 87 deg. 49 min. which is 2 deg. 11 min. E. to N. London bears off Briſtol.

Of the Eclipſes in 1737.

Within the ſphere of the earth's orbit will happen four eclipſes this year. Twice will the moon in her wandering courſe interpoſe, and hide the ſplendour of the ſun from falling on the earth, or its atmoſphere: And twice will the earth in its courſe ſo fall in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection. The firſt is the greateſt eclipſe of the ſun that has happened here ſince the year 1724, or that will be before 1748, on Friday the 18th of February, at three in the afternoon. The ſeveral calculations for divers places follow.

Computed by	Beg. h.m.	Mid.	End	Dur.	Dig.
By Aſtron. Carolina, Coventry	2 25	3 46	5 0	2 35	10 5
Mr. Chattock, London	2 33	3 58	5 13	2 39	10 4
Mr. Chattock, Coventry	2 25	3 50	5 6	2 41	10 5
Mr. Leadbetter, London	2 1	3 24	4 42	2 41	9 45
Mr. T. Williams, Oxon	1 56	3 18	4 38	2 32	9 48
Mr. T. Withnall, Namptwich	2 3	3 27	4 44	2 40	10 32
Mr. Sam. Evans, Cornwall	2 50	3 40	4 30	1 40	10 37

Mr.

		Beg. h. m.	Mid.	End	Dur.	Dig.
Mr. A. Lord at Leiceſter, by	Shackerley's Tab.	2 11	3 37	4 59	2 43	11 17
	Harm. Cœleſt.	2 15	3 39	5 1	2 46	10 4
	Leadbetter's	1 32	2 59	4 21	2 49	11 23
	Newton's	1 47	3 16	4 33	2 46	11 12
Mr. T. Cowper, Wellingborrow		2 11	3 34	4 53	2 41	11 8
Mr. E. Hauxley,	Kirkleatham	1 31	2 59	4 21	2 50	10 24
	York	1 56	3 24	4 38	2 42	9 45
	Newcaſtle	1 55	3 23	4 37		
Mr. W. Mobbs, Mixbury Oxon		2 12	3 35	4 53	2 41	9 58
Mr. Forſters, Branſpath		1 53	2 17	4 40	2 47	10 30
Mr. C. Forreſt, Newcaſtle		2 21	3 45	5 4	2 42	10 23
Mr. Tho. Wade, Leiceſter		1 30	2 57	4 19	2 49	11 18
Philoſtratus, Leeds		1 47	3 10	4 32	2 45	11 27
Mr. Wm. Brown,	Cleobury	2 21	3 41	4 56	2 35	10 11
	Barbadoes	8 5	9 39	11 23	3 18	6 48
Mr. T. Sparrow,	Nottingham	2 21	3 41	4 55	3 34	10 16
	Hitchin	2 26	3 46	5 0	3 34	9 56
Mr. Rob. Cooke, Newcaſtle		1 41	3 7	4 34	2 53	10 50
Mr. J. Wilſon, Morpeth		2 13	3 37	4 50	2 37	10 13
Mr. Peter Barons, Landſend		1 23	2 50	4 3	2 40	8 15
Mr. J. Bulman,	London	2 29	3 51	5 5	2 36	10 0
	Carliſle	2 22	3 44	5 3	2 41	10 23
	Deptford	2 30	3 53	5 7	2 37	10 0
Mr. J. Dorking,	Coventry	2 25	3 46	5 0	2 35	10 5
	London	2 27	3 49	5 4	2 37	10 9
	Norwich	2 28	3 50	5 6	2 38	10 11
	Wrentham	2 29	3 52	5 10	2 41	10 11

2. Eclipſe

The above Eclipſe was obſerved thus:

At	By	Begin. h. m. s.	End h. m. s.	
Fleet Street, Lond.	Geo. Graham	2 25 3	— — —	ap. t.
Greenwich	Dr. Bevis and Dr. Halley	2 25 39	5 3 29	ap. t.
Cambridge	Cha. Maſon	2 38 30	5 16 2	tr. t.
Rome	D. de Ravillas	3 43 2	— — —	
Bologna		3 33 34	— — —	
Edinburgh	Col. Mac Laurin	2 5 55	4 44 51	ap. t.
ibid.	Hon. Sir J. Clerk, Burt.	2 5 36	4 44 50	ap. t.

This eclipſe was obſerved to be Annular from about Morpeth *in Northumberland* to beyond Inverneſs in North Britain.

2. Eclipſe of the moon March 5, at noon, inviſible.
3. Sun eclipſed Auguſt 15, at 1 morning inviſible.
4. Moon eclipſed on Monday Auguſt 29, at 8 morning.

Computed by		Beg.	Mid.	End	Dur.	Dig.
By Aſtron. Carolina, Coventry		2 11	3 20	4 28	2 17	4 30
Mr. Chattock, London		2 9	3 34	4 59	2 50	6 25
Mr. Leadbetter, London		2 34	3 44	4 54	2 20	4 34
Mr. Sam. Evans, Cornwall		3 51	4 55	6 0		5 2
Mr. A. Lord forLeiceſter, by	Shackerley's Tab.	2 27	3 51	5 15	2 48	6 3
	Harm. Cœleſt.	2 25	3 52	5 19	2 55	6 24
	Leadbetter's	2 8	3 22	4 36	2 28	5 1
	Newton's	2 11	3 39	4 47	2 36	5 33
Mr. T. Cowper, Wellingborough		2 44	3 55	5 6	2 22	4 41
Mr. W. Mobbs, Mixbury		2 30	3 40	4 50	2 20	4 25
Mr. T G. Forſter, Branſpath		2 29	3 40	4 47	2 17	4 51
Mr. Tho. Wade, Leiceſter		2 0	3 15	4 30	2 30	5 10
Mr. Wm. Brown,	London	2 17	3 25	4 34	4 30	2 16
	Cleobury	2 7	3 15	4 24		
	Dublin	1 49	2 57	4 6		
	Rome	3 9	4 17	5 29		
Endymion, London		1 57	3 32	5 7	3 10	6 17
Mr. T. Sparrow,	Nottingham	2 7	3 16	4 25	2 18	4 36
	Hitchin	2 10	3 19	4 28	2 18	4 30
Mr. J. Wilſon, Morpeth		2 29	3 40	4 51	2 22	4 41
Mr. E. Hauxley,	Kirkleatham	2 30	3 40	4 50	2 20	4 34
	York	2 29	3 39	4 49		
	Newcaſtle	2 28	3 38	4 48		
	Hexham	2 27	3 37	4 47		
Mr. J. Bulman,	London	2 31	3 40	4 50	2 19	4 32
	Carliſle	2 20	3 39	4 39		
	Deptford	2 32	3 41	4 51		
Mr. Mark Rafton	London	2 8	3 25	4 42	2 34	5 7
	Yarmouth	2 14	3 31	4 48	2 34	5 7
	Durham	2 4	3 21	4 38	2 34	5 7
	Rotterdam	2 24	3 41	5 58		

New

A COMET

Was alſo obſerved this year by ſeveral perſons, particularly by *J. Bradley*, Sav. Prof. Aſtron. in the months of January, February, and March; and who from his obſervations, on the ſuppoſition of a Parabolic Orbit, computes theſe Elements:

In its Perihelion Jan. 19, 8 h. 20 min. Temp. Equat. Lond.

Inclination of its path to the Ecliptic —		18°	20′	45″
Place of the Deſcending Node — —	♉	16	20	—
Place of the Perihelion — — —	♒	25	55	—
Diſt. Perihel. from the Deſcending Node		80	27	—

Log. Perihel. Diſt. from the Sun 9·347960
Log. *of the Diurnal Motion* — 0·938188

New Queſtions.

I. QUESTION 193, *by* Merones.

Ye bright ſons of art, that a rule did impart
In the * Diary 1735, to obtain
The ſolid content of a conic ſegment,
Cut off by a vertical plane.†
Since ye're ſo expert, proceed in like ſort,
To compute us the ſurface convex
Of a ſegment the leſſer, and all ſuch to meaſure
A general theorem annex.

* *Queſtion 177, propoſed in the Diary 1734, and anſwered in 1735.*
† *Perpendicular to the baſe.*

II. QUESTION 194, *by Mr.* J. Turner.

Ye diarian train, whoſe genius and parts,
In the various branches of the liberal arts,
Have ſo long been experienc'd; pray, lend us a while
Your aſſiſtance, two brethren to reconcile.
The caſe it ſtands thus: Their father, at's death,
An elliptical field did unto them bequeath,
Whoſe axis tranſverſe is juſt ſixty chains,
The conjugate forty preciſely contains.
Within this encloſure a pond there was made,
Whoſe juſt ſituation may thus be diſplay'd:
If you from the vertex chains fifteen direct
On the axis tranſverſus do count, and erect
A ſemiordinate there, as it's eaſy to do,
Then i'th' middle o'th' ſame the pond you may view:
Thro' this pond (for the father ſo order'd the matter)
A fence it muſt paſs, but of ſuch a nature,
As to be, of all other, the ſhorteſt that can
Be drawn thro' the ſaid point, and terminate in
The elliptic periphery at both ends, which will
Cut the mead in two parts. Exert now your ſkill,
To find the length of the fence, and alſo what ground
Each brother muſt have, which from thence may be found.
Methinks, with the firſt, C. Maſon, I hear
Say, this will do; prepare more for next year.

III. QUESTION 195 *by Mr.* Rich. Lycett.

There is an oak tree (the fruſtum of a cone) whoſe length *is ten yards,* the diameter at the top one foot, at the bottom three feet; and an ivy twiſteth round it in the manner of a ſpiral ſcrew, that each twiſt is ten inches diſtant; three-ſevenths of the ivy is eat into the tree; the diameter of the ivy is at the bottom one foot, at the top three inches. Quere the length of the ivy, and the content of both?

IV. QUESTION 196, *by Mr.* Robert Heath.

Three ſhips, *A*, *B*, and *C*, ſailed from a certain port in north latitude, until they arrived at three different ports, all lying under the equinoctial; *A* ſailed on a direct courſe, between the ſouth and the weſt 175·62 leagues; *C* ſailed 133 leagues between the ſouth and eaſt; *B* ſailed a courſe between *A* and *C* 102 leagues, making the angle or rhumb with *A*, equal the angle that *C* made with the equinoctial. Hence it is required to find the port ſailed from, each ſhip's courſe, and diſtance from each other, and their reſpective ports? and to ſolve it by an equation not higher than a quadratic.

V. QUESTION 197, *by Mr.* Ant. Thacker.

Let $AB = 1000$, $BC = 2000$, $CD = 3000$, $DE = 4000$, and AE variable; ſhew how to find the greateſt area that can poſſibly be included by theſe five lines.

See the fig. to the ſolution.

VI. QUESTION 198, *by Mr.* Chr. Maſon.

On Albion's Auſtral bounds, on Suſſex ſtrand,
A range of rocks defend th' adjacent land
From vile invaders, and inſulting waves;
Indulgent nature, whom ſhe loves ſhe ſaves.
Erſt Seven Cliffs, but now three Charles's call'd,
The fatal place where once the Dutch were maul'd.
The furrow'd front with viſage ghaſtly pale,
Frowns at the billows of each boiſt'rous gale;
Friendly informs afar, of Beachy Head,
A ſhoal of rocks to mariners a dread.
O ghaſtly ſight! a ſpeedy death to touch,
Full oft experienc'd by the found'ring Dutch,
Who vent'rouſly look o'er the bending brow;
Pigmy-like they ſeem to thoſe below.
Seafaring fowl of num'rous ſorts here throng,
Both for their refuge, and to brood their young;
And when ſurpriz'd, each have their diff'rent cry,
Altho' in diſcord, yet in harmony.
On the broad ſhoulders of theſe cliffs there lie
The faireſt downs e'er fac'd the azure ſky;

Where a rich carpet o'er the ſame is ſpread,
And num'rous flocks thereon are yearly fed;
Whoſe ſilver fleece, and ſweeter fleſh exceed
Banſted, or Bagſhot, or fam'd Coleſwold breed.
Near, on the eaſt, there is a grateful ſoil,
Which well rewards the tiller's care and toil.
Fair ſmiling meads, where once a briny flood;
Fine glad'ing fields, where a fair city ſtood,
Eaſt Bourn now call'd, whilom Anderida.
But mouldring time the hardeſt flints decay;
Here's ſcarce one mark where the old ruins lay;
The footſteps dim, the hiſt'ry dark to trace,
The ſea, the downs, the wield concur that it's the place.
But not to trifle with ſo old a tale,
Here's what will more the reader's ear regale;
This fertile place in plenty doth produce
All the ſubſtantials fit for human uſe.
Fowl, fiſh, and fruit, the ſeaſons ſtill ſupply,
Their luxury, not want, to gratify.
If from the top of this ſtupend'ous height,
You dextrouſly let fall a pond'rous weight,
The time obſerv'd, one-eleventh of that ('tis plain)
The ſound requires, for to return again.
Now from the data draw your conſequence,
You'll find the height of this great eminence.

The PRIZE QUESTION *by Mr.* Turner.

Could I (unſkill'd in verſe) ſuch lines indite
As would the fair ones pleaſe, I'd oft'ner write,
And unto B--gh--n (as I now do here)
Send a prize queſtion for th' enſuing year:
If he approves, ten diaries are the claim
Of him or her, who truly ſolves the ſame.
[*] In the annext triangle BAD,
The ſides are given, as below you ſee;
Upon the angular point A as center,
A ſemicircle, whoſe diameter
Is thirty feet exact, deſcribed there.
Now, 'tis required to draw two lines, that each
From both the given points B, D, may reach,
To ſome one point in the circumference
Of the ſaid circle (put F for pretence) *See the fig. to the ſolution.*
And ſo, as that the angle BFD
The greateſt that is poſſible, may be.
To find this, various methods may ſuffice;
But he who purpoſes to win the prize,
Muſt prove himſelf ſkill'd in arts mathematic,
And ſolve it by equation call'd quadratic.

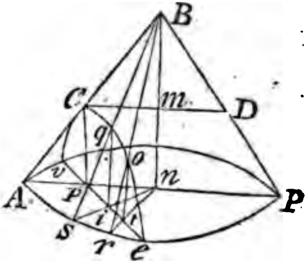
[*] $BA = 40$, $AD = 53$, $DB = 55$ feet.

1738.

Queſtions anſwered.

I. QUESTION 193 *anſwered by the Propoſer.*

THrough the axis Bn, and any two points infinitely near each other as r, s, draw two planes Brn, Bsn, cutting the baſe in the lines rn, sn, and the plane of the hyperbola in the lines ot, qi, parallel to the axis Bn.



The ratio of the $\triangle$s nti, Boq, to each other, is compounded of the ratios of $\triangle nti$ to $\triangle nrs$, and $\triangle nrs$ to $\triangle Brs$, and of the $\triangle Brs$ to $\triangle Boq$, that is, of the ratio of nt^2 to nr^2, and of nr to Br, and of (Br^2 to Bo^2, or of) nr^2 to nt^2. Whence $\triangle nti : \triangle Boq :: nr : Br$. And ſince $\triangle nrs : \triangle Brs :: nr : Br$. Therefore the $\triangle nrs - \triangle nti : \triangle Brs - \triangle Boq :: nr : Br$. That is, area $trsi$: area $orsq :: nr : Br$. And ſince this always holds for any ſmall correſpondent parts of the circle and the conic ſurface, it follows that the wholes are in the ſame ratio. Hence this theorem,

As the radius of the baſe : ſide of the cone :: ſegment of the baſe : ſurface of the ſegment of the cone.

And the area of the ſegment $eAvCe$ will be found to be 904·38 ſquare inches. The anſwer.

Mr. Ri. Dunthorne'*s Anſwer to the ſame.*

The curve ſurfaces of cones may be conſidered as made up of an infinite number of infinitely narrow annuli, whoſe radii are in arithmetical progreſſion, and the coſines of their ſegments equal.

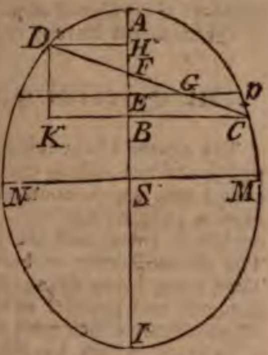
Let $r =$ radius, $d = 3{\cdot}14159$ &c. and $b =$ the coſine of any indefinite ſegment, then by the arithmetic of infinites, $dr - 2b - \frac{b^3}{3rr} - \frac{3b^5}{20r^4} - \frac{5b^7}{56r^6} - \frac{35b^9}{576r^8} -$ &c. $=$ the arch of that ſegment.

Put $R = An$, $B = Cm$, $A = Ap$, $C = AC$; and let $e =$ the infinitely ſmall breadth of one annulus. Then will $dre - 2Be - \frac{B^3e}{3rr} - \frac{3B^5e}{20r^4} - \frac{5B^7e}{56r^6} - \frac{35B^9e}{576r^8} -$ &c. $=$ ſuperficies of any indefinite annular ſegment, conſtituting the curve ſurface of ſuch conic ſegment.

Therefore ſuch curve ſurface will be compoſed of an infinite number of ſuch ſeries, having B conſtant, and r increaſing in arithmetic progreſſion, from B to its greateſt R, and $\frac{C}{e} =$ the number of terms. Conſequently $\frac{dR^2C}{2A} - \frac{dB^2C}{2A} - 2BC + \frac{B^3C}{3RA} - \frac{B^3C}{3BA} + \frac{B^5C}{20R^3A} - \frac{B^5C}{20B^3A} + \frac{B^7C}{56R^5A} - \frac{B^7C}{56B^5A} + \frac{5B^9C}{576R^7A} - \frac{5B^9C}{576B^7A} +$ &c. $=$ the curve of ſuch conic ſegment, which from the numbers in queſtion 177, gives 905 ſquare inches, the anſwer nearly the ſame as the propoſer's above.

If we ſuppoſe $r =$ radius, and $a =$ verſed ſine of any indefinite ſegment of a circle; we ſhall, by the arithmetic of of infinites, have $2\frac{1}{2} + \frac{4R^{\frac{1}{2}}A^{\frac{1}{2}}C}{3} + \frac{7A^{\frac{3}{2}}C}{15B} - \frac{71A^{\frac{5}{2}}C}{840B^{\frac{1}{2}}} + \frac{319A^{\frac{5}{2}}C}{10080B^{\frac{5}{2}}} - \frac{5419A^{\frac{9}{2}}C}{354816B^{\frac{7}{2}}} +$ &c. $=$ the curve ſuperficies of of ſuch conic ſegment, in numbers above.

II. QUESTION 194 *anſwered by the Author* J. T.



Let CD be the line ſought. Put $AS = 30 = t$, $MS = 20 = n$, $AE = 15 = a$, $GE = 8\ 66 = b$; $EF = z$, $SB = x$, $AB = t - x$, $BC = y$, $FB = t - x - a + z = m - x + z$.

As $b : z :: y : \frac{yz}{b} = FB$. And, by the property of the ellipſe, $tt : nn :: tt - xx$ $\frac{ttnn - nnxx}{tt} = yy$. Ergo $x = \sqrt{\frac{ttnn - ttyy}{nn}}$. Put $r = tt$, $s = \frac{tt}{nn}$. Then $x = \sqrt{r - syy}$. Or, putting for x its value, and tranſpoſing, $\frac{mb + bz - yz}{b} = \sqrt{r - syy}$; and, ſquaring the parts, $mmbb + 2mbbz - 2mbzy + bbzz - 2bzzy + zzyy = bbr - bbsyy$. Put $q = bbs + zz$, $2p = 2mbb + 2bzz$; $g = bbr - mmbb - bbzz - 2mbbz$. Then we have $qyy - 2py = g$. And extracting the root, $y = \frac{p}{q} \mp \sqrt{\frac{g}{q} + \frac{pp}{qq}}$. Hence it is evident that $\sqrt{\frac{4g}{q} + \frac{4pp}{qq}}$ is equal to the difference of the double value of y; that is, of the lines $+CB$ and $-DH$, or CK. Laſtly, $GE^2 : GF^2 :: CK^2 : CD^2$ which is to be a maximum; which in ſymbols will be, when the value of q, p, and g are reſtored,

$$= \frac{bb + zz + 4b^4rs - 4mmb^4s - 4b^4szz - 8mb^4sz + 4bbrzz}{b^6ss + 2b^4szz + bbz^4}$$

And, dividing by bb, and ſubſtituting letters for the known quantities, $\frac{h - kz + lz^2 - fz^3 + \varphi z^4}{a + \delta zz + z^4}$, in fluxions, $= 0$. It is evident that this will only produce an equation of the ſixth power, by which the value of z comes out $= EF$.

Mr. *Lord* finds the length of the fence 32·03 chains: One ſhare 161 a. 14 p. The other 27 a. 1 r. 20 p.

Merones, by a curious and ſhort proceſs, finds *CD* the fence = 33·84; the greater ſegment 1561·45; the leſſer 323·511.

From a geometrical conſtruction I find the numbers nearly as follow: *EF* = 2·65 chains; *GE* = 8·6; *SB* = 12·12; *BE* = 2·88; *CD* the ſhorteſt fence 33·25; *CB* = 18·15; *Ep* = 17·2; *AB* = 18·0; *FB* = 5·38. And the content of the ſegment *DApCD* = 28 a. 3 r. 38 p. The greater *CMINDC* = 159 2 1, the ſhares; and the whole 188 1 39.

III. QUESTION 195 *anſwered by the Propoſer.*

The center of the ivy will be found to lie without the periphery of the oak at the bottom ·055, and at top ·015 parts of a foot; which doubled and added to the diameters, at bottom 3·11, at top 1·03. Then the fruſtum's length on the outſide is 30·01, and the angle with the baſe is 88° 0′ 52″. The circumference at bottom 9·77, and at top 3·235 feet. Then a line being drawn through the extremity of the top diameter and parallel to the central line is a mean arith. proportional; which muliplied by the number of twiſts 36·02 gives 234·2, which let be one leg of a triangle, 30·01 the other, with the angle between them 88° 0′ 52″. The other ſide will be found 235·3 feet the length of the ivy: Hence the content of the ivy is 80·87 feet, three-ſevenths of which is 34·6; the content of the tree with the ivy growing into it is = 102·1; conſequently 102·1 — 34·6 = 67·5 feet, the real content of the tree.

Merones's *Anſwer to the ſame.*

In the cone compleated, let s = the length of the ſide, c = the circumference at the baſe, d = the diſtance of the ſpiral threads, z = the ſpiral line, and x = the diſtance from the vertex to top of the firſt helix or turn.

Then $d : c :: \dot{x} : \frac{c\dot{x}}{d}$, and $s : \frac{c\dot{x}}{d} :: x : \frac{cx\dot{x}}{ds}$ = a ſmall ſpace the ivy moves round. And let the hypothenuſe line the ivy moves round in be $= \dot{z}$; then $\dot{z} = \sqrt{\dot{x}^2 + \frac{ccx^2\dot{x}^2}{d^2s^2}}$ $= \frac{c\dot{x}}{ds}\sqrt{\frac{ddss}{cc} + xx}$. And, finding the fluent, $z = 237·746$ the length of the ſpiral ivy. Hence the content of the oak = *102·102; of* the ivy = 81·6925 feet.

IV. QUES-

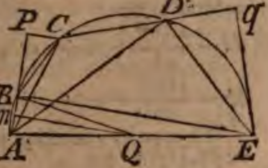
* IV. QUESTION 196 *anſwered by the Propoſer.*

As 102 : 175·62 :: 133 : 228·99 leagues: The port ſailed from is in 5° 5′ north latitude.

$$\text{The courſe of }\left\{\begin{array}{lll} A & 54^\circ\ 38' & \text{weſt.}\\ B & 4\ \ 47 & \text{weſt.}\\ C & 40\ \ 10 & \text{eaſt.}\end{array}\right\}\text{ diff. long. }\left\{\begin{array}{ll} 7^\circ & 9' \\ 0 & 25 \\ 4 & 17\end{array}\right\}\text{ Req.}$$

V. QUESTION 197 *anſwered by Mr.* Rob. Heath.

Let $AB = 1000 = b$, $BC = 2000 = c$, $CD = 3000 = d$, $DE = 4000 = f$; and let $a =$ AE the diameter, radius $= 1$.



Then as

$a : 1 :: b : \frac{b}{a} = \text{s.} \angle BEA$;

$a : 1 :: f : \frac{f}{a} = \text{s.} \angle DAE$.

Now as $b : \frac{b}{a} :: c : \frac{c}{a} =$ $\text{s.} \angle BAC = \angle BEC$ by the nature of a circle.

And as $f : \frac{f}{a} :: d : \frac{d}{a} = \text{s.} \angle CED = \angle CAD$.

The

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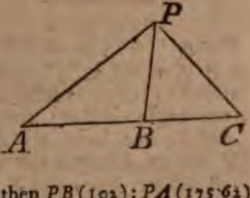
* IV. QUESTION 196.

The reaſon of the proportion, in the above original ſolution, for finding the diſtance (228·99) between the extreme ports, will clearly appear from the annexed figure, in which P repreſents the port ſailed from, and A, B, C the ports arrived at by the reſpective ſhips. For, ſince the $\angle C = \angle APB$ by the queſtion, and the $\angle A$ common to the two $\triangle$s APB, APC, therefore the third angles are equal, and thoſe triangles equiangular. Hence then $PB\,(102) : PA\,(175{\cdot}62) :: PC\,(133) : AC = 228{\cdot}99$. And in like manner as $AC : AP :: AP : AB$. Hence the perpendicular and the angles are eaſily found.



Hence alſo the problem will be eaſily conſtructed. For it is only taking AC a *fourth* proportional to the three given lines.

The $\angle BEA+\angle CEB=\angle AEC$ whoſe ſine $=\frac{b}{a}\sqrt{\frac{aa-cc}{aa}}+\frac{c}{a}\sqrt{\frac{aa-bb}{aa}}$; and the $\angle CAD+\angle DAE=\angle CAE$ whoſe ſine $=\frac{d}{a}\sqrt{\frac{aa-ff}{aa}}+\frac{f}{a}\sqrt{\frac{aa-dd}{aa}}$; then ſay as $1:a::\frac{b}{a}\sqrt{\frac{aa-cc}{aa}}+\frac{c}{a}\sqrt{\frac{aa-bb}{aa}}:b\sqrt{\frac{aa-cc}{aa}}+c\sqrt{\frac{aa-bb}{aa}}=AC$. And as $1:a::\frac{d}{a}\sqrt{\frac{aa-ff}{aa}}+\frac{f}{a}\sqrt{\frac{aa-dd}{aa}}:d\sqrt{\frac{aa-ff}{aa}}+f\sqrt{\frac{aa-dd}{aa}}=CE$. But $AC^2+CE^2=AE^2$ by 31 Eucl. 3, which is in algebraic terms,

$$\left.\begin{aligned}&dd-\frac{2ddff}{aa}+ff+2df\sqrt{\frac{a^4-a^2dd-a^2ff+ddff}{a^4}}\\&bb-\frac{2bbcc}{aa}+cc+2bc\sqrt{\frac{a^4-a^2bb-a^2cc+bbcc}{a^4}}\end{aligned}\right\}=aa.$$

Whence by reduction and the converging ſeries a is found $=6646'316$, &c.

But the diameter AE or radius AQ may be more expeditiouſly found by a table of natural ſines (which is an invention of my own) by finding out four chords, in the proportion of the chords given AB, BC, CD, DE. By a few trials, I find the angle AQm, under half the chord $=8^\circ\ 39'$ the next leſs to a minute; and $8^\circ\ 40'$ the next greater. The operation is thus:

	° ′ ″ ‴	*Nat. ſines*		
Take	8 39 0 0 =	4503'981	8 40 0 0 =	
angles	17 30 19 33 =	2×1503'981	17 32 23 57 =	2×
corre-	26 49 13 11 =	3×1503'981	26 52 32 40 =	3×
ſpond.	36 59 2 30 =	4×1503'981	37 3 59 42 =	4×
	89 57 35 14		90 8 56 19	*Nat. ſines* 1506'857 =
Comp.	0 2 24 46 diff.		Sub. 89 57 35 14	dif. 11 21 5 (′ ″ ‴)

Say, as 11′ 21″ 5‴ to 60″, ſo is 2′ 24″ 46‴ to 12″ 45‴ to be added to the next leſſer angle: Whence 8° 39′ 12″ 45‴ is the true angle AQm; conſequently 17° 18′ 25″ 30‴ = angle AQB; and ſince AQB is an iſoſceles triangle, the angle $QAB=ABQ=81^\circ\ 20'\ 47''\ 15'''$. By trigonometry, ſay 17° 18′ &c. (2974'929 N. S.) : 1000 : : 81° 20′ &c. (9886'161) : 3323'158 $=AQ$. Whence $AE=6646'316$ the diameter required.

Conſequently the greateſt area of the polygon $ABCDEA$ is $=14567'640$ &c. very near the truth.

The

The ſame anſwered by Merones.

In the laſt figure produce the lines *AB*, *CD*, and let fall perpendiculars on them from *C* and *E*.

When the area is the greateſt poſſible, 'tis plain the angles *ACE*, *ABE*, and *ADE* muſt be right angles, therefore the figure *ABCDE* is to be inſcribed in a ſemicircle whoſe diameter is *AE*. Let *AB* be called a, $BC = b$, $CD = c$, $DE = d$, $BP = x$, $AE = u$. The $\triangle$s *ACE*, *BCP*, *DEq* are ſimilar; therefore $b : x :: u : \frac{xu}{b} = EC$. And $b : \sqrt{bb - xx} :: d : \frac{d}{b}\sqrt{bb - xx} = Dq$. And by Eucl. II. 12, $\frac{xxuu}{bb} = cc + dd + \frac{2cd}{b}\sqrt{bb - xx} = CE^2$. And $aa + bb + 2ax = AC^2$. And by Eucl. I. 47, $AC^2 + CE^2 = AE^2$. Or $uu = aa + bb + cc + dd + 2ax + \frac{2cd}{b}\sqrt{bb - xx} = ss + 2ax + \frac{2cd}{b}\sqrt{bb - xx}$, putting ss for the known quantities. Hence we have $ss + 2ax + \frac{2cd}{b}\sqrt{bb - xx} = \frac{bbcc + bbdd + 2bcd\sqrt{bb - xx}}{xx}$. Which reduced gives $x = 1795{\cdot}0355$; and $AE = 6646{\cdot}31856$; and the greateſt area 14567·718¼.

Mr. *Walter Trott* gives the diameter 6646·3; area 1456740. Mr. *Robinſon* (not conſidering it in a ſemicircle) 14247150·54: and Mr. *Bird* but 14202859. Mr. *Tatton* 13692100. *Arithmeticus* 14567·64. Mr. *Colburn* ſays 'tis 5920875, agreeable to M. Oracle. Mr. *Dorking* 14567642. Mr. *Forſter* 14567638. Mr. *Lovatt* alſo ſolved it. Mr. *Thacker*'s number is 14567·560, and he ſhews that the figure muſt be in a circle.

VI. QUESTION 198, *by Mr.* Maſon.

Let a be the time a heavy body requires in falling from the top of the cliff; then $\frac{a}{11} =$ the time that ſound requires to move that ſpace; $b = 16$ feet; $c = 968$ feet.

Then $baa = \frac{ca}{11}$ per queſt. and $11baa = ca$, which divide by a, and it is $11ba = c$; therefore $a = \frac{c}{11b} = 5{\cdot}5$ ſeconds, the time ſought. Then $5{\cdot}5 \times 5{\cdot}5 \times b = 484$ feet, the height ſought. But if $b = 16\frac{1}{12}$ the anſwer is 481·417.

Nat. Percival. If ſound move 1142 feet in a ſecond, as by late authors; then the height ſought is 611·434 feet.

Col. Dagger's *Anſwer.*

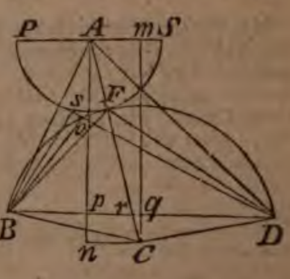
The deſcent of a weight,* and motion of † ſound,
If agreed in one ſecond of time to be found;
By the numbers annex'd, as the moderns allow,
Then the anſwer to Maſon hereunder we ſhew.
And the height of the rock will be found to appear,
Two hundred and twenty two yards very near.
The proceſs let Chriſtopher ſhew if he pleaſe,
I'll not ſcare the ladies with *A*'s and with *B*'s.

* Weight falls 16 feet 1 inch. † Sound moves 1142 feet.

Mr. *Powle* uſing the laſt data gives the height 670·148 feet.
Mr. *Mitchell* (ſound's mot. 1140) anſwers 667·8 feet.
But taking the modern data, moſt of my correſpondents give the true anſwer 670 feet = 223 yards 1 foot.

The PRIZE QUESTION *anſwered by Mr.* Rob. Heath.

In the triangle ABD, there is given $AB = 40 = b$, $AD = 53 = c$, $BD = 55 = 2d$, $PS = 30 = 2f$.



Make PS parallel to BD; draw mC perpendicular. In the center C deſcribe a circle, to touch the given one at F, thro' the points B, D; the point of contact F, from which lines muſt be drawn to B and D to conſtitute the greateſt angle, which is a maximum. Draw CA, CD and Ap the perpendicular of the triangle. In order to inveſtigate the center C, and the lines BF and DF, find the perpendicular $Ap = 36{\cdot}4342 = mq = h$; $qp = Am = 10{\cdot}991 = g$; and let $qC = a$.

Then per 47 Eucl. 1, $\sqrt{aa + dd} = CD = CF$; $\sqrt{aa + dd} + f = AC$; and $\sqrt{gg + hh + 2ha + aa} = AC = \sqrt{aa + dd} + f$. Whence $gg + hh + dd - ff + 2ha = 2f\sqrt{aa + dd}$ by involution and tranſpoſition, or $k + 2ha = 2f\sqrt{aa + dd}$ by ſubſtitution. Whence $aa + \frac{kk}{hh - ff}a = \frac{4ffdd - kk}{4hh - 4ff}$. And $a = 5{\cdot}10647$: conſequently $AC = 42{\cdot}97$. Now the $\triangle ACm$ and Apr are ſimilar, $mC : AC :: Ap : Ar = 37{\cdot}687$; $AC - Ar = rC = 5{\cdot}283$; whence $Fr = 22{\cdot}687$; $Br = 26{\cdot}139$; $\angle CAm = 75^\circ\ 11'$; $\angle rBF = 47^\circ\ 9'$; $BF = 22{\cdot}91$; $DF = 41{\cdot}02$; $\angle BFD = 100^\circ\ 30'$ a maximum.*

The ſame anſwered by the Propoſer.

In the laſt figure draw An perpendicular to BD, and Cn perpendicular to An, and Cq perpendicular to BD. Put $Bq = 27{\cdot}5 = m$, $AF = 15 = b$, $Ap = 36{\cdot}4 = c$, $pq = nC = 11{\cdot}1 = d$; and $CB = CF = x$, $Cq = np = y$. Then is $CA = b + x$, and $An = c + y$.

And by 47 Eucl. 1, $\begin{cases} xx = yy + mm, \\ bb + 2bx + xx = cc + 2cy + yy + dd. \end{cases}$

Subſtitute for xx its value $yy + mm$, and then $bb + 2b\sqrt{yy + mm} + yy + mm = cc + 2cy + yy + dd$. Put $n = cc + dd - bb - mm$, then $2b\sqrt{yy + mm} = n + 2cy$; and, involving, $4bbyy + 4bbmm = nn + 4ncy + 4ccyy$. Laſtly put $2p = \frac{4nc}{4cc - 4bb}$, and $q = \frac{4bbmm - nn}{4cc - 4bb}$; then $yy + 2py = q$. And, extracting the root, $y = \sqrt{pp + q} - p = 5{\cdot}1$; from whence the angle $BFD = 100^\circ\ 30'$.

Anſwered

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* PRIZE QUESTION.

This problem is very eaſily conſtructed in all its caſes, viz. when the angle is to be either a maximum, or a minimum, or a given quantity. For when it muſt be a given quantity, it is only deſcribing on the given baſe BD a ſegment of a circle to contain the given angle, which ſegment will cut the given circle in two points, either of which will do. When the angle is to be a maximum or a minimum, the circle BFD muſt touch the given circle PFS reſpectively below or above the vertex A; which is very well done in the 12th prob. of LAWSON's APOLLONIUS on TANGENCIES.

Calculated by		Beg.	Mid.	End	Dur.	Dig.
From Aſtron. Car. Coventry		10 7	11 7	12 8	2 4	3 35
By Mr. Chattock, Scien.	London	9 48	10 55	12 2	2 14	4 13
	Coventry	9 42	10 46	11 52	2 9	4 0
Mr. Leadbetter, London		9 57	11 2	12 8	2 10	4 8
Mr. Hulſe, Coventry		10 8	11 7	12 8	2 0	4 0
— Fr. Leadbetter's Tab. Lond.		9 57	10 41	11 24	2 2	4 6
Mr. Bamfield, Devonſhire		9 56	10 57	11 58	2 2	3 40
Mr. Cooper,	Northampton	9 50	10 51	11 53	2 3	3 34
	York	9 54	10 51	11 48	1 53	3 4
	London	9 53	10 56	12 0	2 7	3 45
Mr. Hughs, Flint		9 50	10 45	11 40	1 50	3 48
Mr. Sparrow, Bury S. Edmond's		10 9	11 9	12 11	2 2	3 45
Mr. Lord, Leiceſter		9 55	11 1	12 0	2 5	3 31
Mr. Barons, St. Ives, Cornwall		9 48	10 53	11 59	2 11	4 18
Mr. Wilſon, Morpeth		10 1	11 0	12 0	1 59	3 30
Mr. Facer, Wattlington		10 6	11 7	12 8	1 58	3 48
Wr. Williams, Middleton Sto.		9 50	11 1	12 6	2 16	4 10
Mr. Hampſon, Leigh, Lancaſh.		11 0	12 1	1 2	2 2	4 30
Mr. Wade,	Leiceſter	10 18	11 20	12 0	2 6	3 16
	Jeruſalem	2 2	3 15	4 4	2 12	4 42
	Gibraltar	9 20	10 2	12 34	3 14	3 42
Mr. Dorking,	London	10 9	11 10	12 12	2 3	3 36
	Coventry	10 7	11 7	12 8	2 1	3 35
	Norwich	10 11	11 12	12 13	2 2	3 35
	Yarmouth	10 13	11 16	12 19	2 6	3 37
	Yoxford	10 11	11 13	12 14	2 3	3 36
Mr. Forſter, Branſpath		9 59	11 0	12 5	2 6	3 11
Mr. Brown,	Cleobury	10 1	11 1	12 2	2 1	3 37
	London	10 13	11 15	12 17	2 3	3 45
Mr. J. Bulman,	London	10 12	11 13	12 18	2 6	3 37
	Carliſle	10 2	11 0	12 6	2 4	3 0
	Deptford	10 12	11 13	12 18	2 6	3 38
Mr. Withnalt, Namptwich		9 48	10 45	11 45	1 57	3 5

New

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* This Eclipſe was obſerved thus:

At	By	Beginning	End
		h. m. s.	h. m. s.
Fleet Street, London	Mr. Graham & Mr. Short	9 59 20	11 59 36 a. m.
Upſal	And. Celſius	12 18 52	12 42 12 tr. time
Bologna	Euſt. Manfredi	10 51 25	1 18 1 tr. time

Anſwered by Merones.

Draw the lines Ds, Bs, Bo; then the angle BFD or $BoD = BsD + sBo$; whence BFD is greater than BsD. To find the radius, let $Bq = b$, $pq = c$, $Ap = p$, $AF = r$, $Cq = x$. Then will $BC = CF$, or $\sqrt{bb + xx} = \sqrt{cc + pp + 2px + xx} - r$. Thence $cc + pp + 2px = bb + rr + 2r\sqrt{bb + xx}$. And (putting $ss = cc + pp - bb - rr$) $ss + 2px = 2r\sqrt{bb + xx}$. And laſtly $\frac{4pp}{4rr}xx + 4pssx = 4bbrr - s^4$. Whence $x = 6{\cdot}38716$, and $FC = 29{\cdot}19$, and the $\angle BFD = 100^\circ\ 32'$. *Q. E. I.*

The prize of 10 Diaries was won by Mr. *Ri. Gibbons*.

Of the Eclipſes in 1738.

There will happen but two eclipſes this year.

The firſt of the ſun on the 7th of February, at ſix at night, the ſun being then ſet, and inviſible to us.

The ſecond on friday the 4th of Auguſt, in the forenoon; the ſeveral calculations for divers places are theſe following.

Calculated

Calculated by		Beg.	Mid.	End	Dur.	Dig.
From Aſtron. Car. Coventry		10 7	11 7	12 0	2 4	3 35
By Mr. Chattock,	London	9 48	10 55	12 2	2 14	4 13
Scien.	Coventry	9 42	10 46	11 52	2 9	4 0
Mr. Leadbetter, London		9 57	11 2	12 8	2 10	4 8
Mr. Hulſe, Coventry		10 8	11 7	12 8	2 0	4 0
— Fr. Leadbetter's Tab. Lond.		9 57	10 41	11 24	2 2	4 6
Mr. Bamfield, Devonſhire		9 56	10 57	11 58	2 2	3 40
Mr. Cooper,	Northampton	9 50	10 51	11 53	2 3	3 4
	York	9 54	10 51	11 48	1 54	3 4
	London	9 53	10 56	12 0	2 7	3 45
Mr. Hughs, Flint		9 50	10 45	11 40	1 50	3 48
Mr. Sparrow, Bury S. Edmond's		10 9	11 9	12 11	2 2	3 45
Mr. Lord, Leiceſter		9 55	11 1	12 0	2 5	3 31
Mr. Barons, St. Ives, Cornwall		9 48	10 53	11 59	2 11	4 18
Mr. Wilſon, Morpeth		10 1	11 0	12 0	1 59	3 30
Mr. Facer, Wattlington		10 6	11 7	12 8	1 58	3 48
Mr. Williams, Middleton Sto.		9 50	11 1	12 6	2 16	4 10
Mr. Hampſon, Leigh, Lancaſh.		11 0	12 1	1 2	2 2	4 30
Mr. Wade,	Leiceſter	10 18	11 20	12 0	2 6	3 16
	Jeruſalem	2 2	3 15	4 4	2 12	4 42
	Gibraltar	9 20	10 2	12 34	3 14	3 42
Mr. Dorking,	London	10 9	11 10	12 12	2 3	3 36
	Coventry	10 7	11 7	12 8	2 1	3 35
	Norwich	10 11	11 12	12 13	2 2	3 35
	Yarmouth	10 13	11 16	12 19	2 6	3 37
	Yoxford	10 11	11 13	12 14	2 3	3 36
Mr. Forſter, Branſpath		9 59	11 0	12 5	2 6	3 11
Mr. Brown,	Cleobury	10 1	11 1	12 2	2 1	3 37
	London	10 13	11 15	12 17	2 3	3 45
Mr. J. Bulman,	London	10 12	11 13	12 18	2 6	3 37
	Carliſle	10 2	11 0	12 6	2 4	3 0
	Deptford	10 12	11 13	12 18	2 6	3 38
Mr. Withnalt, Namptwich		9 48	10 45	11 45	1 57	3 5

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* This Eclipſe was obſerved thus:

At	By	Beginning	End
		h. m. s.	h. m. s.
Fleet Street, London	Mr. Graham & Mr. Short	9 59 20	11 59 36 a. m.
Upſal	And. Celſius	12 18 52	12 42 12 tr. time
Bologna	Euſt. Manfredi	10 51 25	1 18 1 tr. time

New Queſtions.

I. QUESTION 199, *by Mr.* Tho. Bird.

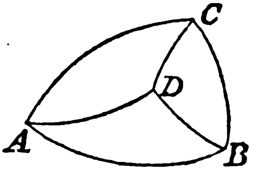
A ſpark having gain'd a young damſel's conſent,
He unto her father ſubmiſſively went,
Deſiring the favour to make her his bride:
Unto which petition her father reply'd,
'I have a nice garden, whoſe beautiful frame
'The form of a rhombus exactly does claim:
'In which is a ſquare inſcrib'd ſo by art,
'To touch the four fences alike on each part.
'Each ſide of this garden, ſo pleaſant and fair,
'Eight chains and three-fourths of a chain does declare;
'And alſo the ſquare in the middle inclos'd,
'Each ſide of ſix chains is exactly compos'd.
'Now tell us the garden's true area from hence,
'And I with your project of love will diſpenſe.
'But if your beſt judgment defective appear,
'In marriage you never muſt compaſs your dear.'
Now therefore kind ladies, come lend us your aid,
And ſhew Cupid's captive to compaſs the maid!
Produce the equation quadratic with care,
Nor ſuffer poor Strephon to bleed in deſpair,
For if your aſſiſtance you long do neglect,
His hanging or drowning you ſoon may expect.

II. QUESTION 200, *by Mr.* Ri. Lovatt.

It was the ninth of June my friend and I
One evening late went forth to ſpy
Thoſe heav'nly bodies which to us appear,
Like blazing tapers, wand'ring in the air.
Then ſtraightway we a ſuppoſition made,
Thro' boundleſs æther, and it was convey'd
Round the celeſtial orbs as ſwift as thought,
Viewing thoſe wond'rous works by nature wrought.
Three ſtars among the reſt, their rays of light
ſeem'd to enlighten the dull ſhades of night;
Their diſtance from each other as we took,
Inſerted are i'th' margin of this book.
Their angles alſo, at the zenith made,
From thence we did their altitudes invade.
But here we ſtopp'd, no farther could advance,
Our theorems fail'd us, and our works were chance.

The altitude's therefore fair ladies give
In the next Diary: and your fame ſhall live
While the terraqueous globe her orb ſhall keep,
Or Adam in the ſilent grave ſhall ſleep.

Given $AC = 67° 28'$, $BC = 60° 0'$, $AB = 40° 0'$.
The angles $ADC = 148° 6'$, $CDB = 121° 54'$.
Required AD, CD, and BD?



III. QUESTION 201, *by Mr.* Chriſ. Maſon.

Suppoſe a ſhip ſet ſail from the latitude 51° north, and ſhape a north-weſt courſe, and ſail without interruption. Where will it at laſt arrive; and how many leagues run?

IV. QUESTION 202, *by Mr.* Tho. Cooper.

In a northern clime, two * hills ſublime
Attract the diſtant ſight;
Th' exceſs of one, i'th' margin's † ſhown,
Above the other's height.
If from the ſummit of each mount
A line extended be,
The length thereof you muſt account
Juſt ſixty miles and three;
Their ſite is ſuch, This line will touch,
And reach the earth's ſurface;
And the contact, it is of fact,
I'th' intermediate ſpace.
I pray diſcloſe the height of thoſe?
And alſo at what time
The radiant ſun, on each of them,
Will firſt begin to ‡ ſhine.

* *The higheſt hill is in lat. 65 deg. the other in 64 deg. 30 min.*

† *Diff. perp. height 119 yards; ear. ra. 6980000.*

‡ *On the winter ſolſtice.*

V. QUESTION 203, *by* J. B. S.

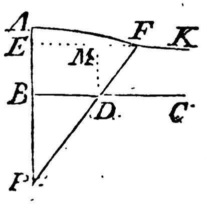
A ball of lead hanging from the top of a hall by a ſtring, drawn over a pulley, which is 20 foot long between the center of the ball and pulley, is ſet a ſwinging: The moment it begins to ſwing, a perſon, holding the other end of the ſtring, begins to pull it, and draws up the ball, and continues ſo to do, at an uniform ratio of 5 feet in a minute, until he has pull'd the ball quite up to the pulley. Quere, How many oſcillations will the ball make before it reach the top?

VI. QUESTION 204, *by Mr.* Maſon.

Soon after a hard gale of wind, ſome perſons ſtrolling along ſhore, upon the look-out, found a large caſk juſt driving aſhore, which proved a piece of old Jamaica rum: They ſoon boarded it, and racked off forty-one gallons, and filled up the caſk again with water, and acquainted ſome more of their party with their ſucceſs, who went and racked off the ſame quantity of the mixture, and filled it up again with water: In like manner it was ſerved ſo twice more; and at laſt, by the proof, there was found $25\frac{214}{720}$ gallons of rum remaining, the reſt water. How much did the whole piece contain, and how many gallons of rum was drawn out at each evacution?

The PRIZE QUESTION, *by Mr.* Ri. Dunthorne.

Let *AFK* be the conchoid of Nicomedes, which is continually approaching nearer to the line *BC*, yet if continued *ad infinitum* could never meet; which at firſt ſetting out to generate the line *AFK*, the diſtance *BA* is 16 inches; and the diſtance *BP* is 24 inches from the pole *P* to the aſymptote *BC*. It is required to find the diſtance of *F*, the point of inflection of the curve, from the line *BC*?

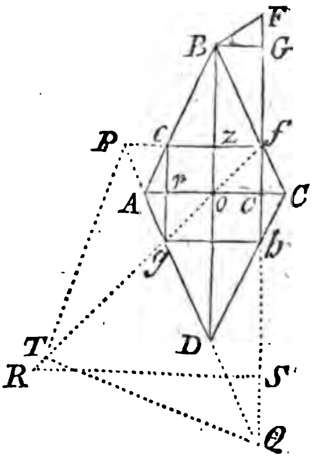


1739.

Queſtions anſwered.

* I. QUESTION 199 *anſwered by Mr.* Rob. Heath.

LET $b =$ the ſides $AB = BC$, &c. $= 8{\cdot}75$ chains, $2c = ef = fh$, &c. $= 6$ chains; then it is evident $ez = zf = 3$ chains $= c$: Let $Bz = x$. By ſimilar Δs, $x : c :: x + c : \frac{x+c}{x} \times c = Ao$. But $Ao^2 + Bo^2 = AB^2$ by 47 Eucl. 1, which is in algebraic terms $\frac{xx + 2xc + cc}{xx} \times cc + xx + 2xc + cc = bb$. By reduction $\overline{xx + cc}|^2 + 2cx \times \overline{xx + cc} = bbxx$: by comp. $\square$ and extraction $\overline{xx + cc} + xc = x\sqrt{bb + cc} = 9{\cdot}25x$; whence the equation becomes $xx - 6{\cdot}25x = -cc$. By comp. $\square$ and extraction a ſecond time, $x = \pm\sqrt{9{\cdot}765625 - cc} + 3{\cdot}125 = 4$; or $2{\cdot}25$ (according to the ambiguity of the queſt.) whence the area of the rhombus or garden $ABCD = 73{\cdot}5$ chains $= 7$ a. 1 r. 16 p.



The ſame anſwered by Mr. Hen. Travis *of Wirkſworth.*

Erect the line BE perpendicular to CB, continue the line bf till it cuts BE at F, let fall the perpendicular BG, which is equal to $zf = a$; call fF, x; and fB, y; then will $cf = b - y$: Now it is evident that the rectangled triangles, Ccf, FBf, and FGB are ſimilar, and that Ccf and FGB are equal, becauſe $cf = GB$; hence per 47 Eucl. 1, $fB^2 + BF^2 = fF^2$; that is, $yy + bb - 2by + yy = xx = bb - 2by + 2yy$; and $fc : fC :: fB : fF$, or $a : b - y :: y : x$, $\therefore ax = by - yy$, and $-2ax = -2by + 2yy$. $bb - 2by + 2yy = xx$. Now if for $-2by + 2yy$ we put $2ax$, its value found, we have $bb - 2ax = xx$, or $xx + 2ax = bb$. Hence $xx + 2ax + aa = bb + aa$; $\therefore x + a = \sqrt{bb + aa} = 9{\cdot}25$; $ax = by - yy$, or $yy - by = -ax$, and $yy - 8{\cdot}75y = -18{\cdot}75$; then $yy - 8{\cdot}75y + 4{\cdot}375^2 = -18{\cdot}75 + 4{\cdot}375^2$; $y - 4{\cdot}375 = \sqrt{-18{\cdot}75 + 4{\cdot}375^2}$; $\therefore y = 5$. And the area of the rhombus $= 7$ a. 1 r. 16 p.*

II. QUES-

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* I. QUESTION 199.

Produce the ſide AD of the rhombus both ways to meet the oppoſite ſide fe, fb, of the ſquare, produced, in P and Q; then will PQ be $= 2AD$: For, $ef = 2ro$, and $Pe = 2Ar$ (becauſe $ge = 2gr$), $\therefore Pf = 2Ao$, and by ſim. $\triangle$s, $PQ = 2AD$. The problem is therefore reduced to this, To apply a given line PQ (or $2AD$) between two lines fe, fb, given in poſition, and to paſs through a given point g; which is one of the problems of APOLLONIUS *Concerning Inclinations*, and which may be thus eaſily conſtructed.

Conſtruction.

In fg produced take $fR = 2AD$ the given line, and draw $RS \parallel gb$; take alſo gT ſuch that the rectangle fTg be $= RS^2$; then will the center T and radius RS croſs fe and fb in P and Q, which will be the points required.

Demonſtration.

Since $fTg = RS^2 = TP^2$, $\therefore Tf : TP :: TP : Tg$, and consequently the $\triangle$s TPf, TPg are ſimilar by Eucl. VI. 6, $\therefore$ the $\angle TPg =$ the $\angle TfP = \angle RfS$ or $\angle fRS$; in like manner the $\angle TQP = \angle RfS$; conſequently the $\triangle TPQ$ is ſimilar to the $\triangle fRS$; but RS or $Sf = TP$ or TQ, $\therefore$ alſo $PQ = fR = 2AD$.

II. QUESTION 200, *by Mr.* Robert Heath.

$$\text{Given sides}\left\{\begin{array}{l} AC = 67^\circ\ 28' = a \\ BC = 60\quad 0 = b \\ AB = 40\quad 0 = c \end{array}\right\}\text{Nat. s.}\ \Bigg|\ \text{Given angles}\left\{\begin{array}{l} ADC = 148^\circ\ 6' = d; \\ CDB = 121\ \ 54 = e; \\ ADB = 90\quad 0 = 1. \end{array}\right.$$

Whence by trigonometry is found the angles $CAB = 69^\circ\ 39'$, $ABC = 90^\circ\ 00'$, $ACB\ 44^\circ\ 7'$. Let $x = BD$. Then by ſpheric trigonom. $c : 1 :: x : \frac{x}{c} = \text{s.} \angle DAB$.

Let $m =$ ſine angle CAB, and n its coſine.

Then $m\sqrt{\frac{cc-xx}{cc}} - \frac{nx}{c} = \text{s.} \angle CAD$. Say $d : a ::$ $m\sqrt{\frac{cc-xx}{cc}} - \frac{nx}{c} : \frac{am}{dc}\sqrt{cc-xx} - \frac{nax}{cd} = DC$. Now the ſine $\angle ABD = \sqrt{\frac{cc-xx}{cc-ccxx}}$: Then ſince $\angle ABC$ = a right angle, therefore the complement of $\angle ABD = \frac{x}{c}\sqrt{\frac{1-cc}{1-xx}} = \angle DBC$. By ſpherics, $e : b :: \frac{x}{c}\sqrt{\frac{1-cc}{1-xx}} : \frac{bx}{ec}\sqrt{\frac{1-cc}{1-xx}} = DC = \frac{am}{dc}\sqrt{cc-xx} - \frac{nax}{cd}$. Which equation reduced, and turned into numbers, will be

$$2{\cdot}549528\sqrt{{\cdot}4131758987 - xx} - {\cdot}945629x = \frac{1{\cdot}215692852x}{\sqrt{1-xx}}.$$

Solved, $x = {\cdot}47477$, &c.

$$\text{Whence}\left\{\begin{array}{l} DB = 28^\circ\ 21' \\ CD = 40\ \ 58 \\ AD = 29\ \ 29 \end{array}\right\}\text{whoſe complements}\left\{\begin{array}{l} 61^\circ\ 39' \\ 49\quad 2 \\ 60\ \ 31 \end{array}\right\}\text{are the alt. of the ſtars required.}$$

N. B. *This method is univerſal, when the given angle exceed a right angle.*

The Propoſer Mr. Ri. Lovatt's *Anſwer.*

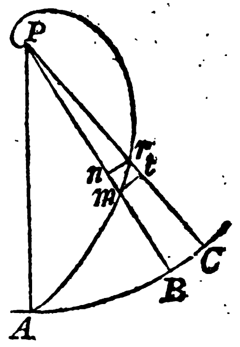
Put $b =$ ſine of $AB \div$ by ſine of the $\angle ADB$; $p =$ coſine AC; $d =$ ſine $BC \div$ by s. $\angle CDB$; s. $ABC = n$; and $m =$ its coſine; $a =$ s. $\angle ABD$. Then $ab =$ s. alt. AD. And $m\sqrt{1-aa} - am$ will be $=$ s. $\angle DBC$; and $nd\sqrt{1-aa} - amd =$ s. CD. Hence this analogy, as radius $1 :$ coſine $AD = \sqrt{1-aabb}$ :: coſine $CD = \sqrt{2anmd^2\sqrt{1-aa} - aamm + aann - nn + 1} : p$.

Therefore $\overline{1 - aabb} \times 2anmd^2 \sqrt{1 - aa} - aamm + aann - nn + 1 : = pp$. Which gives $AD = 29° 29'$, $BD = 28{\cdot}21$, $CD = 40{\cdot}50$; whoſe complements are the altitudes.

* III. QUESTION 201 *anſwered by Mr.* J. May, *jun. of Amſterdam.*

The ſhip will arrive at the north pole, and the leagues run, will be the length of the loxodromic, beginning at the latitude of 51 north, and ending at the pole, after having performed infinite revolutions about the ſaid pole; the length of the ſaid loxodromic will be 1278·56 leagues, ſuppoſing the earth to be a true ſpherical figure; and the circumference according to Norwood's obſervations, likewiſe the ſhip infinite ſmall.

The ſame anſwered by Mr. Walter Trott, *per Corollary to Prob.* XII. Stone's *Appendix.*



Suppoſe a line drawn infinitely near to PB; then may the triangle made thereby be looked on a rectilinear; and making nm radius, mr will be the ſecant of the courſe, and $a = PA = PB$; $Am = \dot{y}$; and $mn = \dot{x} =$ fluxion of the diff. latitude; then per plain trigonometry, R : ſec. courſe :: $\dot{x} : \dot{y}$. Hence, taking the fluents, $y =$ ſecant of the courſe multiplied by x, divided by the radius. The ſhip's diſtance ſailed is 1103·08 leagues, and then coincides with the pole.

Mr.

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* III. QUESTION 201.

A Complete Inveſtigation of this queſtion may be ſeen at Queſt. 18 *of our* MATH. MISCELLANY.

Mr. Tho. Bird *anſwers thus:*

The ſmall decrements of the ſpiral, or the diſtance the ſhip runs while it paſſes through ſmall, but equal angles of longitude (as at every 0·01 deg.) are a ſeries of geometrical proportionals continued decreaſing and ending in 0, in the pole: whoſe ratio (1·0001745) together with the firſt or greateſt term (0·57736) and laſt or leaſt (0) being known; the ſum of all the ſeries or length of the ſpiral is eaſily found = 3309 minutes of a degree, or 1103 leagues; which may more eaſily be done by fluxions. However it may be ſolved to a geometrical exactneſs by this eaſy and known analogy.

As the coſine of the courſe co. ar. 0·150515
To diff. lat. 780 leagues — 2·192095
So radius — — — 10·000000
To diſt. run 1103 leagues 3·042610

After the ſhip has paſſed through all the degrees of longitude, and is arrived to the ſame meridian itſelf, it will be but 4·632 minutes from the pole, according to artificial tangents for Wright's meridional parts being made from falſe ſecants, are not to be truſted) after which it will be no deviation from mathematical exactneſs to ſuppoſe the remainder to the pole a plane, in which caſe the proportion of the velocity of the ſhip's approach towards the pole, is nearly as 10000000 to 18151, ſhe in the next round will be but 0·00841 min. from the pole, and the next revolution falls into the pole itſelf, contrary to what the ingenious Oughtred ſuppoſed in his Cir. Prop. of Nav. p. 37.

Mr. Chr. Maſon *the Propoſer anſwers in this manner.*

Let P denote the north pole, A the place whence the ſhip ſet ſail, AmP its courſe, AP the meridian, and BP, CP two other meridians infinitely near. Nr a parallel of latitude. Let AP = co. lat. $= b$; $r =$ rad. $c =$ coſine of the courſe, $Bm = a$; $AmP = x =$ diſt. run. Then by plain trigonometry, $a : \frac{ra}{c} :: b : \frac{bra}{ca} = \frac{br}{c} = x = 1277·73$ leagues.

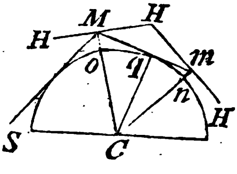
The ſame anſwered by Mr. R. Dunthorne.

Since parts of rhumbs are every where to their correſponding parts of the meridians, as radius to the coſine of the courſe, it will be S. 45 : rad. :: 900·25 leagues (the diſt. of 51° from the pole) : 1273·14 leagues the anſwer.

This queſtion was alſo curiouſly ſolved by Mr. *George Brown*, jun.

* IV. QUESTION 202 *anſwered by Mr.* Rob. Heath.

Let r = earth's radius = $Co = Cq = Cn = 6980000$ yards; d = diff. mountains heights = 119 yards, x = height of the loweſt mountain + earth's radius = Cm? Then $\sqrt{xx-rr}=qm$, by 47 Eucl. 1.



Again $\sqrt{xx+2dx+dd-rr}=Mq$; but $Mq+qm=63$ miles = 110880 yards = b. Conſequently $\sqrt{xx-rr}-b=-\sqrt{xx+2xd+dd-rr}$. Reduced, $xx+dx=\frac{bb-dd}{4}+\frac{bbrr}{bb-dd}$. In numbers $xx+119x=48723529707626{\cdot}7$, &c. Solved, $x=6980164{\cdot}68$ yards; conſequently the height of the loweſt mountain = 164·68 yards; that of the higheſt = 283·68 yards.

The time when the ſun firſt begins to ſhine on the higheſt mountain (M) is when he cuts the tangent to the earth's ſurface (SM) below the horizon (HMH) the angle HMS (found by trig.) = 31 min. alſo he firſt begins to ſhine on the top of the loweſt mountain (m) when he cuts the tang. (Mm) below the horizon (HmH) the $\angle MmH$ = 23 min. Now if the ſun's refraction be allowed for near the horizon = 33′, the angle HMS will be = 1° 4′; and angle HmM = 56 m. which the ſun is below the horizon of each mountain when he firſt ſhines on their tops.

The hour or times will be found by ſpheric trigonometry; thus, at 10h. 8.m. 38s. on the higheſt mountain, at 10h. 2m. 4s. on the loweſt.

N. B. Sun riſes in lat. 65, at 10h. 35m. 16s. in 64° 30′, at 10h. 22m. 56s. This ſhews how much thoſe are miſtaken, who ſuppoſe the ſun would firſt ſhine on them, nearly at his riſing.

Merones

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* IV. QUESTION 202.

In this problem we have given the baſe Mm, the perpendicular Cq, and the difference of the ſides $CM-Cm$; whence the $\triangle CMm$ may be eaſily conſtructed.

Meroner gives the higheſt hill 283·69 yards, the lower 164·69; ſun riſes on the higher at 10h. $21\frac{1}{4}$min. on the lower at 10h. $14\frac{1}{5}$ min.

The propoſer, Mr. *Tho. Cooper*, ſays the ſun appears on the higheſt at 10h. 21m. 12s. on the lower at 10h. 13m. 12s.

V. QUESTION 203 *anſwered by Mr.* Ri. Dunthorne.

Let $a = 20$ feet, $b = 4$ minutes, $c =$ number of vibrations which the pendulum whoſe length is a makes in the time b, and $e =$ a ſmall particle of time. Then $b : c :: e : \frac{ce}{b} =$ number of vibrations which the pendulum a makes in the time e; and $b : a :: e : \frac{ae}{b} =$ portion of the ſtring drawn up in the time e; then will $a - \frac{ae}{b} =$ length of the pendulum after the firſt time e; $a - \frac{2ae}{b} =$ that after the ſecond time e, &c. And $\frac{ccee}{b \times \overline{b-e}} = \square$ of the number of vibrations in the ſecond time e. Conſequently its ſquare root = number of vibrations in ſecond, e. In like manner $\frac{ce}{\sqrt{b} \times \sqrt{b-2e}} =$ number of vibrations in the third time e. $\frac{ce}{\sqrt{b} \times \sqrt{b-3e}} =$ number in the fourth, &c. Whence 'tis manifeſt that the number of vibrations in the ſeveral times e, as above, are a ſeries of fractions, whoſe numerators are equal, and their denominators are ſquare roots, whoſe ſides are ſingle powers, decreaſe in arithmetical progreſſion from b, and $\frac{b}{e} =$ the number of terms. So that by the arithmetic of infinites, $2c$ will be the ſum of all the terms in the ſeries. But $c = 97$, whence $2c = 194$, the number of vibrations required.

The ſame anſwered by the Propoſer I. B. S. Tycho.

I obſerve, that the anſwer reſults to this, viz. To find the length of a pendulum, which remaining invariable, ſhall make the ſame number of vibrations in a given time, as one does, which is continually lengthening or ſhortening, in ſome given ratio.——This I find to be 56·25 inches nearly; wherefore the number of vibrations before the ball reach the pully will be about 200.

Note, The ball is ſuppoſed to be a point, and the ſtring a mathematical line, and the oſcillations performed in ſimilar arcs.

Merones ſays, the ball will make 194 oſcillations, being twice the number which a pendulum of the whole length of 20 feet would have made in the ſame time.

Mr. *Tho. Bird* anſwers 194·49 vibrations.

VI. QUESTION 204 *anſwered by Mr.* Rob. Heath.

Let $b = 25\frac{214}{729} = 25{\cdot}2935528$ gallons of rum left in the caſk.
$q = 41$ gallons, the liquor drawn off each time.
$n =$ the number of times of drawing. And
$x =$ the quantity of neat liquor the caſk held.

Say $x : x - q :: x - q : \frac{\overline{x - x}|^2}{x} =$ rum left at 2d drawing.

$x : \frac{\overline{x - q}|^2}{x} :: x - q : \frac{\overline{x - q}|^3}{xx} =$ rum left at 3d drawing.

$x : \frac{\overline{x - q}|^3}{xx} :: x - q : \frac{\overline{x - q}|^4}{xxx} =$ rum left at 4th drawing.

Whence it is evident, that the quantity of neat liquor, left at any number of times drawing off, will be univerſally $\frac{\overline{x - q}|^n}{x^{n-1}}$; conſequently $\frac{\overline{x - q}|^4}{xxx} = b$. Reduced $x - q = b^{\frac{1}{4}} x^{\frac{3}{4}}$. In numbers $x - 41 = 2{\cdot}1426\,x^{\cdot 75}$; ſolved, [according to a new method of managing exponentials] $x = 124{\cdot}671$, &c.

Now, if each quantity left in the caſk, at any time of drawing off, be ſubtracted from the quantity left the time preceding, the neat rum at each time drawn off will be found; and is as follows:

Rum drawn off	1.	41·000	gallons Total 124·67.
	2.	27·516	
	3.	18·467	
	4.	12·394	
add gallons left		25·293	

Mr. *T. Robinſon*, Mr. *J. May*, *Merones*, Mr. *W. Rubins*, Mr. *El. Colbourn*, Mr. *J. Badder*, Mr. *Waller Trott*, Mr. *Tho.*

Tho. Cooper, Mr. *T. Woodward*, Mr. *J. Parminter*, Mr. *J. Prichard*, Mr. *Paul Sharp*, Mr. *E. Verrall*, Mr. *J. Bir* , Mr. *Powle*, Mr. *W. Spicer*, Mr. *Rob. Cooke*, Mr. *Forſter*, A. B. Mr. *Hitton*, Mr. *Mitchel*, Mr. *Young*, Mr. *Williams*, Mr. *G. Brown*, Mr. *Ballard*, Mr. *Dunthorne*, Mr. *Wade*, Mr. *Lord*, and others have given true anſwers to this queſtion.

The PRIZE QUESTION *anſwered by Mr.* Hen. Travis.

Let FB and AB (in the ſcheme to the queſt.) be called b and c, the abſciſſa and ordinate x and y; then $bbcc + 2bccy + ccyy - bbyy - 2byyy - y^4 = yyxx$, will be the equation, expreſſing the nature of this curve, &c. The firſt in fluxions, $2bcc\dot y + 2ccy\dot y - 2by\dot y - 6byy\dot y - 4y^3\dot y = 2y\dot yxx + 2x\dot xyy$; again by an uniform velocity $ccy\ddot y - 6byy\ddot y - 6yyy\ddot y = y\ddot yxx + 2yy\ddot xx + 2yy\ddot xx + \ddot xxyy$; and from the 1, 2, and 3 ſteps we have $y^3 + 3byy = 2bcc$. Therefore $y = 12{\cdot}0885$ the anſwer.

Anſwered by Mr. J. May.

Put AB or $DF = 16 = a$, $BP = 24 = b$, BE or $DM = x$, and $EF = y$. Then by the ſimilar triangles DMF and PEF, we have $DM = x : MF = \sqrt{aa - xx} :: PE = b + x : EF = y$. Multiplying extreams and means, and dividing by x, have $y = \frac{\overline{b+x}\times\sqrt{aa-xx}}{x}$; in fluxions $\dot y = \frac{\overline{x^3 + aab}\times\dot x}{xx\sqrt{aa-xx}}$; again $\ddot y = \frac{\overline{2a^4b - aax^3 - 3aabxx}\times\dot x\dot x}{\overline{aax^3 - x^5}\times\sqrt{aa-xx}} = 0$. Then is $x^3 + 3bxx - 2aab = 0$; by means of this equation the value of x may be found by the ſection of a parabola and circle. But if brought into numbers, and reduced, will give $x = 12{\cdot}0885$ inches for BE; being the diſtance of the point of inflection of the curve from the aſymptote.

This prize queſtion was taken from *Infiniments Petits par Mar. de l'Hopital*, p. 65, Paris edit. 1696, or *Hay*'s Fluxions, p. 91, and is in Stone's Fluxions, p. 87. *Simpſon*'s Fluxions, p. 87.

The prize of 10 Diaries fell to the lot of Mr. *John Withered*.

Of the Eclipſes in 1739.

Within the ſphere of the earth's orbit will happen five eclipſes this year; three times will the moon in her wandering courſe interpoſe and hide the ſplendour of the ſun from falling on the earth or its atmoſphere; and twice will the earth in its courſe, ſo fall in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

The firſt eclipſe is of the moon, Jan. 13, at night.

Calculated by	Beg. h. m.	Mid.	End	Dur.	Dig.
From Aſtronomia, Coventry	9 59	11 20	12 41	2 42	6 40
Mr. Chattock, London Coventry	9 24	11 0	12 35	3 11	8 9
Mr. Leadbetter for London	9 31	10 47	12 15	2 42	6 22
Mr. T. Robinſon, London	9 57	11 26	12 54	2 57	7 1
Newcaſt.	9 51	11 20	12 48		
Mr. Ra. Hulſe, Cheſhire	9 32	10 47	12 15	2 42	6 22
Mr. T. Cooper, Wellingbor.	9 40	11 5	12 30	2 50	6 52
Mr. J. Canton, London	9 36	11 1	12 26	2 50	6 51
Stroud	9 27	10 52	12 17		
Mr. W. Brown, Cleobury	9 54	11 15	12 36	2 42	6 39
London	10 4	11 25	12 46		
Jeruſalem	12 26	1 47	3 8		
Barbadoes	6 5	7 26	8 47		
Iſpahan	1 37	2 58	4 19		
Mr. J. Wilſon, Morpeth	9 28	10 48	12 8	2 40	6 21
Mr. Sam. Bamfield, Devonſh.	9 39	11 5	12 31	2 50	6 52
Mr. T. Glaſpool, Wincheſter	9 29	10 51	12 12	2 43	6 21
Norwich	9 35	10 56	12 17	2 42	6 22
Mr. Rob. Cooke, Newcaſtle	9 58	11 22	12 46	2 46	7 0
Mr. N. Oats	9 18	10 45	12 12	2 54	6 33
Mr. Geo. Forſter, Branſpath	10 10	11 26	12 43	2 32	6 41
Mr. Ed. Dale, Sunderland	10 10	11 26	12 43	2 32	6 41
Mr. Schoolcroft, Hovingham	9 55	11 22	12 49	2 53	7 7
Mr. E. Silborow, Buntingford	10 2	11 26	12 49	2 47	7 6
Mr. Jo. Benwell, Highworth	9 15	10 42	12 8	2 50	7 4
Mr. Tho. Williams	9 24	10 40	12 9	2 45	6 30
Mr. Cha. Facer, per Hamſtead	9 32	10 56	12 21		6 36
Mr. Sparrow, Edmundſbury	9 35	10 56	12 17	2 42	6 31
Mr. Tho. Wade, Leiceſter	10 55	11 44	12 53	2 18	7 7

The ſecond eclipſe is of the ſun, Jan. 28, at 4 morn. inviſible.

The third eclipſe of the moon, July 9, 4 aftern. inviſible.

The fourth eclipſe is of the ſun, July 24, at 4 afternoon.*

Calculated by	Beg.	Mid.	End	Dur.	Dig.
Aſtron. Caroline, Coventry	3 23	4 33	5 37	2 14	7 12
Mr. Chattock, London	3 4	4 19	5 27	2 23	7 26
Mr. Chattock, Coventry	2 54	4 10	5 19	2 24	7 28
Mr. Leadbetter, London	3 10	4 22	5 29	2 18	7 6
Mr. Robinſon, Newcaſtle	3 20	4 34	5 42	2 22	8 35
Mr. Hulſe, London	3 10	4 25	5 30	2 20	7 6
Mr. Cooper, Wellingborrow	3 12	4 25	5 33	2 2[illegible]	7 43
Mr. W. Brown, Cleobury	3 18	4 28	5 32	2 14	7 10
Mr. W. Brown, Copenhagen	4 16	5 25	6 27	2 11	[illegible] 42
Mr. W. Brown, Paris	3 49	4 56	5 58	2 9	7 0
Mr. Wilſon, Morpeth	3 15	4 27	5 35	2 2[illegible]	7 30
Mr. Bamfield, Honiton	3 37	4 48	5 55	2 18	7 18
Mr. Glaſpool, Wincheſter	3 8	4 20	5 26	2 18	7 5

The fifth eclipſe is of the ſun, on wedneſday December 19, in the morning, beginning before ſun-riſing.†

Calcu-

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* This Eclipſe was obſerved at *Wittemberg* by JOH. FRIED. WEIDLER.

The Beginning — — 4h. 15m. 30s. } P. M.
The End — — — 6 27 20 } P. M.

† The End of this Eclipſe was obſerved in *Surry-ſtreet*, *London*, at 9 h. 1 m. 45 s. Ap. time, by Mr. SHORT.

A COMET

Was obſerved this year at *Bologna* by EUST. ZANOTTI, and from his obſervations he determines its Elements thus for a Parabolic Orbit.

Place of the Perihelion — ♋ 5° 11′
———— Deſcending Node ♈ 25 18
Inclin. Orbit to the Ecliptic 53 25
Was in the Perihelion June 9d. 9h. 59m.
———— Deſc. Node July 18 4 57

The Comet's motion in its proper orbit was Retrograde; and its Perihelion was between the Orbits of Mercury and Venus, Its diſtance from the Sun being ·69614 parts of the Earth's Mean Diſtance.

Calculated by	Beg.	Mid.	End	Dur.	Dig.
Mr. Leadbetter, London	8 10	8 48	9 30	1 19	2 10
Mr. Robinſon, Newcaſtle	7 42	8 9	9 50	1 7	2 7
Mr. Cooper, Wellingborrow	7 59	8 38	9 18	1 19	2 15
Mr. W. Brown, Cleobury	7 56	8 32	9 11	1 15	2 4
Mr. T. Glaſpool, Wincheſter	8 7	8 46	9 27	1 20	2 9
Norwich	8 12	8 52	9 33	1 21	2 9
Mr. Rob. Cooke, Newcaſtle	8 18	8 48	9 38	1 19	2 18
Mr. T. G. Forſter, Branſpath	8 8	8 47	9 28	1 20	3 26
Mr. Ed. Dale, Sunderland	8 8	8 47	9 28	1 21	3 26
Mr. W. Schoolcroft, Vienna	8 41	9 24	10 11	1 29	2 3
Mr. J. Benwell, Highworth					
Mr. T. Williams	8 4	8 44	9 24	1 20	2 18
Mr. Cha. Facer	8 2	8 39	9 17	1 14	2 3
Mr. T. Sparrow, Edmondſbury	8 11	8 50	9 32	1 21	2 7
Mr. Tho. Wade, Leiceſter	7 36	8 20	8 35	1 4	1 50

New Queſtions.

I. QUESTION 205, *by Mr.* John Turner.

Where Derwent's ſtreams with gentle murm'rings glide,
Kiſſing the flow'ry banks on either ſide,
A lovely meadow lies, whoſe fertile ſoil
Amply rewards the painful lab'rer's toil.
Fenc'd with tall ſhady trees, a cool retreat
From the meridian ſun's intenſer heat.
Here, to relax my weary mind, I ſtray'd,
And, as I walk'd, theſe obſervations made:
The form of the aboveſaid piece of ground,
A triangle obtuſe, I quickly found:
The lengths of whoſe two ſhorter ſides I knew,
AC fourteen, *CB* chains twenty-two.
From *C*, a drain was carried on to *D*,
A circle's arch exact, (as in the figure ſee) } *See the Fig. to the Solution.*
Its center *A*, the radius is *AC*.
Laſtly, the analyſt I muſt inform
That the third ſide *AB*, as yet unknown,
Was to the arch *CE* as ten to four;
From whence the unknown ſide he may explore.

II. QUESTION 206, *by* Merones.

If a cannon ball be projected upwards in a direction perpendicular to the horizon, half a mile high, and in the latitude of 53 degrees; where will it fall?

III. Question 207, *by Mr.* John May, *jun.*

There came three Dutchmen of my acquaintance to ſee me, being lately married; they brought their wives with them. The men's names were Hendrick, Claas, and Cornelius; the women's, Geertruii, Catriin, and Anna: but I forgot the name of each man's wife. They told me they had been at market to buy hogs; each perſon bought as many hogs as they gave ſhillings for each hog; Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewiſe, each man laid out 3 guineas more than his wife. I deſire to know the name of each man's wife.

IV. Question 208, *by Mr.* Hen. Travis.

Given this equation, viz. $Axy + Bu + z - C = 0$; expreſſing the relation of the ſides of a trapezium inſcribed in a circle whoſe diameter is known to be 75 feet (or d): Required the ſides ſeparately, and area, by a general method that will reſolve all ſuch problems?

N. B. $A = 100$; $B = 5$; $C = 432246$.

V. Question 209, *by Mr.* Robert Heath.

Ingenious ladies of the Britiſh iſle,
Whoſe minds are fraught with ſcientific arts;
Renown'd for virtue, wit, and excellence;
The admiration of the learned world:
Whoſe bright reflections, ſwift, like lightning, pierce
The ſecret pow'r, and hidden depth of things:
Diſcloſe to light * this dark myſterious truth,
And diſtant nations ſhall your praiſe reſound.

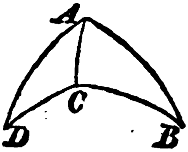
* $\overline{x^{\frac{1}{3}}}|^{x^{\frac{2}{3}}}$ *a minimum. Quere* x^2.

VI. Question 210, *by Mr.* Rich. Dunthorne.

Suppoſe a caſk in form of the middle fruſtum of an hyperbolic ſpindle, whoſe length is 24 inches, bung diameter 30, head diameter 20, and tranſverſe axis of the generating hyperbola 100 inches. Required its content in ale gallons?

VII. QUESTION 211 *by Mr.* Ri. Lovatt.

Suppoſe that in the ſpherical triangle ABD, there is given $AB = 80° 3'$, $AD = 60° 10'$, $AC = 40° 21'$, the angle $BAD = 73° 0'$, and $DCA = BCA$: What are the ſides DC and BC.



VIII. QUESTION 212, *by Mr.* Chr. Maſon.

There is a triangular piece of ground, whoſe center of gravity meaſures from each angle 12, 16, and 20 chains: It is required to find the periphery of the greateſt inſcribed ellipſis; and alſo the content of each angular piece without the ellipſis?

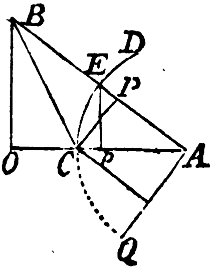
The PRIZE QUESTION, *by Mr.* R. Heath.

Given the latitude of three places, Moſcow 55° 30', Vienna 48° 12', Gibraltar 35° 30', all lying directly in the ſame arc of a great circle: The difference of longitude between Vienna (ſituated in the middle) and Moſcow, eaſterly, is equal to that between Vienna and Gibraltar, weſterly: It is required to find the true bearing and diſtance of each place from the other, and the difference of longitude, according to the convexity of the globe?

1740.

Queſtions anſwered.

I. QUESTION 205 *anſwered by the Propoſer,* J. Turner.



LET $AB = x$, $AC = 14 = b$, $CB = 22 = c$, $cc - bb = 288 = m$. Then the coſine of the angle $BAC = \frac{xx - m}{2x}$: but the arch $CE = {\cdot}4x = nx$; and the radius $= b$; conſequently the coſine of the ſaid arch $= b - \frac{nnxx}{2b} + \frac{n^4x^4}{24b^3} - \frac{n^6x^6}{720b^5}$ &c. Therefore $\frac{xx - m}{2x} = b - pxx + qx^4$, &c. From which equation by converging ſeries, the value of x comes out $= 28{\cdot}9443$: And the angle $BAC = 47$ degrees 17 minutes.

Merones *anſwers this Queſtion thus :*

Let the radius $AC = r$, $CB = s$; take an arch p, as near CE as poſſible, let $a =$ its ſine, $b =$ its coſine to the radius r; and let $p + z = CE$: and per queſt. $\frac{5}{2}p + \frac{5}{2}z = AB$; alſo the coſine $AP = b - \frac{az}{r} - \frac{bz^2}{2rr} + \frac{az^3}{6r^3} +$ &c. And $BP - AP = \frac{5}{2}p + \frac{5}{2}z - 2b + \frac{2a}{r}z + \frac{b}{rr}z^2 - \frac{az^3}{3r^3} -$ &c. different ſegments of the baſe. But by Ax. 4th of plain trigonometry, $\frac{5}{2}p + \frac{5}{2}z : s + r :: s - r :$ different ſegments. Whence we have

$$\left.\begin{matrix} +\frac{5ap}{r} \\ +\frac{25p}{2} \\ -5b \end{matrix}\right\} z + \left.\begin{matrix} +\frac{5bp}{2rr} \\ +\frac{5a}{r} \\ +\frac{25}{4} \end{matrix}\right\} z^2 \left.\begin{matrix} -\frac{5ap}{6r^3} \\ +\frac{5b}{2rr} \end{matrix}\right\} z^3 = \left.\begin{matrix} +ss \\ -rr \\ +5bp \\ -\frac{25}{4}pp \end{matrix}\right\} = R.$$

Now aſſume $p = 11\frac{1}{2}$: and then $\frac{180p}{r \times 3{\cdot}141592}$ = the degrees of the arch P; from whence is had $a = 10{\cdot}249675$; $b = 9{\cdot}536464$; all which ſubſtituted in the foregoing ſeries, and putting A, B, C, &c. for the known coefficients, and reverſing the ſeries, we ſhall have $z = \frac{R}{A} - \frac{BR^2}{A^3} + \frac{2BC - AC}{A^5}R^3$, &c. $= 0{\cdot}704043$, and the arch $CE = 11{\cdot}5704043$; and $AB = 28{\cdot}926011$.

Mr. Robert Heath

Says, by a table of natural ſines and a few trials, I find the angle $CAB = 47° 21'$; whence the required ſide $AB = 28{\cdot}925$, &c. The method of ſolving this by infinite ſeries, which converges ſo ſlow, renders it more tedious than uſeful.

Anſwered by Mr. Hen. Travis.

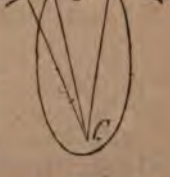
Let $CB = b$, $CA = a$, $AP = y$; and as in Simpſon's Flux. p. 121, we have the arch $QC = y + \frac{y^3}{2.3a^2} + \frac{3y^5}{2.4.5a^4} + \frac{3.5y^7}{2.4.6.7a^6}$ &c. and $\frac{2a \times 3{\cdot}1416}{4}$ = the ſide QE $21{\cdot}991 = q$; from which take QC, leaves $q - y - \frac{y^3}{2.3a^2} - \frac{3y^5}{2.4.5a^4} - \frac{3.5y^7}{2.4.6.7a^6}$ &c. $= CE$; which multiplied by $\frac{5}{2}$ gives $\frac{5q}{2} - \frac{5y}{2} - \frac{5y^3}{2.2.3a^2} - \frac{3y^5}{2.4a^4}$ &c. $= AB$, and per Eucl. 1. 47, $aa - yy = CP^2$; alſo $bb - aa + yy = BP^2$, or $288 + yy = BP^2$: Call $288 = rr$; then $PB = \sqrt{rr + yy}$, or $r + \frac{yy}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ &c. $= PB$; to which add y, gives $r + y + \frac{y^2}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ &c. $= AB$; hence $r + y + \frac{y^2}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ &c. $= \frac{5q}{2} - \frac{5y}{2} - \frac{5y^3}{2.2.3a^2} - \frac{3y^5}{2.4a^4}$; therefore $y + \frac{5y}{2} + \frac{y^2}{2r} + \frac{5y^3}{2.2.3a^2} - \frac{y^4}{8r^3} + \frac{3y^5}{2.4a^4}$ &c. $= \frac{5q}{2} - r$; multiplying by 2, and dividing by 7, gives $\frac{5q - 2r}{7} = z$;

$= z$; ∵ $y = z - \frac{zz}{7r} + \frac{2z^3}{49r^2} - \frac{5z^3}{7 \times 2a^3} + \frac{5 \times 5z^4}{7 \times 7 \times 2ra^3} - \frac{5z^3}{7 \times 7 \times 7r^3} + \frac{z^4}{7 \times 4r^3}$ &c. ∵ $y = 9{\cdot}46$ and $AB = 28{\cdot}94$.

Mr. Nich. Farrer's *Anſwer.*

Let fall the perpendicular BO, on AC produced, alſo Ep from the point E; then let $r = AE = 14$, $m = BC = 22$; and put $x =$ the arch of a circle, whoſe radius is unity, ſimilar to the arch EC; then $rx =$ arch EC per ſimilar figures; and its ſine $= r \times x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ &c. $= Ep$; and $\frac{5rx}{2} = AB$ per queſtion, and $\frac{25rxx}{8} - \frac{mm + rr}{2r} = OC$ per 12 Eucl. 2. ſubſtitute $a = \frac{25r}{8}$, and $b = \frac{mm + rr}{2r}$; then $axx - b = OC$; and per 47 Eucl. 1, $\sqrt{mm - aax^4 + 2baxx - bb} = BO$; and per 4 Eucl. 6, $AE : Ep :: AB : BO$, i.e. $r : r \times x - \frac{x^3}{6} + \frac{x^5}{120}$ &c. $:: \frac{5rx}{2} : \sqrt{mm - aax^4 + 2baxx - bb}$; therefore $\frac{2}{5rx}\sqrt{mm - aax^4 + 2baxx - bb} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ &c. and the value of $x = {\cdot}82646$; and $AB = 28{\cdot}9211$ chains required in anſwer.

II. QUESTION 206 *anſwered by* Meronès.



Let the ball be projected from A, the time of its flight will be 25 ſeconds; in which time the point A will be carried to B, through a ſpace AB, of 23666 feet, by the earth's rotation. Now the ball (carried by a compound motion of its projection and the earth's rotation) will deſcribe an ellipſis whoſe focus is in the center of the earth; in which the elliptic area $ArtC =$ the circular ſector ABC: Or the area $ArtA$ = area BCt; but by reaſon of the ſmall ratio of rs to AC the portion Art may be taken for a parabola. Let $AC = b = 21000000$ feet, AB

= d

$= d = 23666$ feet, $rs = b = 2640$, $a = Bt$; then will $\frac{ba}{2} = \frac{2}{3}b \times \overline{d-a}$; and $3ba + 4ba = 4bd$; whence $a = \frac{4bd}{3b+4b} = \frac{4bd}{3b}$ nearly $= 3{\cdot}967$ feet. Near 4 feet to the weſt.

Mr. Hen. Travis's *Anſwer.*

The time of the ball's aſcending is equal to the time of its deſcending, according to the writers on projectiles; which time call (x) and the number of feet a heavy body will fall or deſcend freely, by the force of its own gravity, in one ſecond of time $= n$. Then will $nxx = 2640 =$ the feet in half a mile; $\because x^2 = \frac{2640}{n} = \frac{2640}{16{\cdot}1} = 163{\cdot}98$ nearly, $\because x = 12' \; 48'' =$ the time the ball is aſcending or deſcending; and conſequently the ball will fall near 4 feet from the place it was projected.

Mr. *Bird* ſays the time of the projectile was $65{\cdot}32$ ſeconds.

III. QUESTION 207 *anſwered by Mr.* J. Hill.

Call the number of hogs any woman bought x; the number her huſband bought $x + n$; money laid out by the woman is xx ſhillings; money laid out by the huſband is $xx + 2nx + nn$ ſhillings. Equation $xx + 2nx + nn = xx + 63$. $\because x = \frac{63-n}{2n}$. If $n = 1$, then $x = 31$, and $x + n = 32$; hence ſome woman bought 31 hogs, and her huſband 32: If $n = 3$, then $x = 9$, and $x + n = 12$; therefore ſome other woman bought 9, and her huſband 12: If $n = 7$, then $n + x = 8$; $\because$ ſome woman bought 1, and her huſband 8. Conſequently

Hendrick bought	32,	and his wife Anna	31
Claas — —	12	— — — Catriin	9
Cornelius —	8	— — — Geertruii	1

Anſwered by Merones.

	Men.			Women.		
For the perſons put	A,	B,	C,	P,	Q,	R,
Hogs	a,	e,	y,	$e-c$,	$a-b$,	u,
Money	aa,	ee,	yy,	$\overline{e-c}^2$,	$\overline{a-b}^2$,	uu.

Let $b = 23$, $c = 11$. Compare B with Q, then per queſtion $ee - \overline{a-b}^2 = 63$ ſhillings; that is, putting $e = a + z$; $2az + zz$

$+ zz + 46a = 592$; therefore $a = \frac{23 - z}{2} + \frac{63}{2z + 46}$; now 'tis evident the laſt term cannot be a whole number; therefore z in the firſt term muſt be an even number, ſo the laſt term $\frac{63}{2z + 46}$ muſt be the half of a whole number; let $\frac{63}{z + 23} = v$. Whence $z = \frac{63}{v} - 23$; hence v muſt be either 1, 3, 7, 9, 21, or 63: From each of which is had $\left.\begin{matrix} a\ 54, & 32, & 14, & 22, & 24 \\ e\ 32, & 12, & 12, & 8, & 8 \end{matrix}\right\}$. And again comparing C with P, then $yy - ee + 22e = 184$; and we find $\left\{\begin{matrix} y & 12, & 8, & 8, & 12, & 52. \\ e & 2, & 10, & 12, & 20, & 42. \end{matrix}\right.$

Whence e muſt be the ſame in both ſuppoſitions; therefore 'tis 12, if the queſtion be poſſible in whole numbers. But ſince the other two perſons A, R, muſt be compared, therefore $aa - uu = 63$: From hence $a = 32$, $u = 31$, $e = 12$, and $y = 8$; but comparing the men and women in any other manner, it will appear there is no other anſwer in whole numbers. Therefore Hendrick and Anna, Claas and Catriin, and Cornelius and Geertruii, are man and wife.

The ſame anſwered by Mr. Rob. Heath.

Let $x =$ the hogs bought by either Hendrick, Claas, or Cornelius; then xx will be the ſhillings they coſt, and $xx - 63$ the ſhillings their wives hogs coſt, which (as whole hogs) muſt always be a ſquare number; becauſe the ſquare root of the ſhillings laid out for each parcel is equal to the number of hogs. Let $x - y =$ the ſide of that ſquare, then $xx - 63 = xx - 2xy + yy$. Couſequently, by reduction, $x = \frac{63 + yy}{2y}$; whence we find y may be

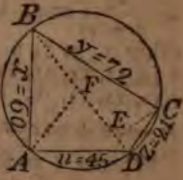
$$\left.\begin{matrix}1\\3\\7\end{matrix}\right\}\ \begin{matrix}\text{Conſeq.}\\ x =\end{matrix}\ \overset{\text{Hogs.}}{\left\{\begin{matrix}32\\12\\8\end{matrix}\right\}}\ \begin{matrix}\text{bought by the}\\ \text{men, coupled}\\ \text{with}\end{matrix}\ \left\{\begin{matrix}31\\9\\1\end{matrix}\right\}\ \text{bought by their}$$

Wives. Whence are joined Hendrick and Anna, Claas and Catriin, Cornelius and Geertruii.

Mr. N. Farrer

Obſerves, that the number of hogs the three men and their reſpective wives bought will be expreſſed by three pair of numbers, the difference of whoſe ſquares muſt be 63. Now all the whole numbers whoſe ſquares will produce this difference are 1 and 8, 9 and 12, 31 and 32; therefore 8, 12, 32, the men bought; 1, 9, 32 the women.

IV. QUESTION 208 *anſwered by the Propoſer.*



$d : y :: z : \frac{zy}{d} = CE$. Hence

$\frac{yy - zzyy}{dd}$, or $\frac{\sqrt{ddyy - zzyy}}{dd} = BE$,

$\frac{zz - zzyy}{dd}$, or $\sqrt{ddzz - zzyy} = ED$,

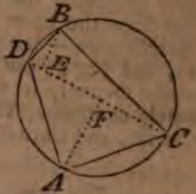
$\frac{\sqrt{ddzz - zzyy} + \sqrt{ddyy - zzyy}}{d} = BD$,

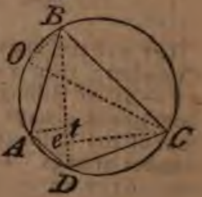
$\sqrt{ddxx - uuxx} + \sqrt{dduu - xxuu} = BD$.

$Axy + Bu + z = C$, per queſt.

$z = C - Axy - Bu$; which ſubſtit. for z.

$\sqrt{ddxx - uuxx} + \sqrt{dduu - xxuu} = \overline{C - Axy - Bu} \times \sqrt{dd - yy} + \sqrt{ddyy - yy \times \overline{C - Axy - Bu}^2}$.





Here we have one equation including three unknown quantities, and yet the queſtion is truly limited; and to be reſolved as the following queſtion is.

Given $2yx + \frac{2592}{uy} + 4ux = 48x - x^2$.

Quere x, y, and u? In fluxions,

$2y\dot{x} + 4u\dot{x} = 48\dot{x} - 2x\dot{x}$,

$2\dot{y}x - \frac{2592u\dot{y}}{uuyy} = 0$, and $-\frac{2592\dot{u}y}{uuyy} + 4\dot{u}x = 0$.

From the firſt, ſecond, and third ſteps, by common algebra we get $x = 12$, $y = 6$, and $u = 3$; and by the very ſame method of reaſoning, the ſides of the trapezium are found to be $x = 60$, $y = 72$, $u = 45$, and $z = 21$; and the area $= 2106$ ſquare feet. This trapezium may be placed 4 or 5 different ways in a circle, which I have proved by a large geometrical projection, and every way juſtly contain the remaining chords of the circle, and meaſured, amount to each the ſame area 2106 ſquare feet. (1) In the firſt fig. the ſeveral lines are obvious; and the diagonal $BD = 75$, multiplied by the half of the two perpendiculars ($CE = 19{\cdot}65$, $AF = 36{\cdot}52$) will give the area as above. (2) In the ſecond fig. $BC = y = 72$, $BD = z = 21$, $DA = x = 60$, and $AC = u = 45$. And the diagonal DC ($= 74{\cdot}96$) multiplied by half the perpendiculars ($AF = 35{\cdot}75$, and $BE = 20{\cdot}2$) gives nearly the ſame area. (3) In the third fig. y is the line CB, x is BA, z is AD, and u is x. The diagonal $BD = 70{\cdot}01$, the perpendicular $At = 16{\cdot}6$, $Ce = 43{\cdot}1$ nearly; and (4) in the ſame fig. z is repreſented by BO, u by OA, and x by AC; the diagonal OC is nearly $74{\cdot}95$, which by the perpendiculars give the area as before.

Mr. *Paul Sharp* has found the ſides 72, 60, 45, and 21, in anſwer.

Mr. *Tho. Robinſon* gives the ſides 72,·2, 59, 9, 45, 1, 20, 7, nearly true. In this queſtion it does ſeem to appear, that the number of quantities ſought exceed the number of given equations, and (as my ingenious correſpondents have obſerved) is unlimited. But I preſume ſince the numbers given in the queſtion, viz. $A = 100$, $B = 5$, and $C = 432246$; and the four numbers ſought are together obliged to extend the chords of 360 degrees, and the diameter of the circle is given; it may be ſaid to be limited; but I ſhall rather leave it to the ſpeculation of thoſe ingenious perſons who are pleaſed to appear in the emendata next year.

V. QUESTION 209 *anſwered by* Merones.

$\overline{x^{\frac{1}{3}}}|^{x^{\frac{2}{3}}}$ = a minimum. Therefore $x^{\frac{2}{3}} \times$ log. $x^{\frac{1}{3}}$ minimum; whence $\frac{2}{3}x^{-\frac{1}{3}}\dot{x} \times$ log. $x^{\frac{1}{3}} + \frac{1}{3}x^{-\frac{1}{3}}\dot{x} = 0$; or 2 log. $x^{\frac{1}{3}} + 1 = 0$. And therefore hyp. log. $x^{\frac{1}{3}} = -\frac{1}{2}$. And tab. log. of $x^{\frac{1}{3}} = -\frac{1}{2} \times {\cdot}434294 \doteq -1{\cdot}782853$. Therefore $x^{\frac{1}{3}} = {\cdot}60653$, and $x = {\cdot}22313$.

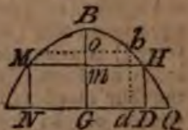
The Propoſer, Mr. Heath, *anſwers thus.*

Its log. is a minimum. Let $y^3 = x$, the expreſſion will become y^{y^2}; its log. $=$ l. $y \times yy$, fluxed, $\dot{y}y +$ l. $y \times 2y\dot{y} = 0$. Reduced, l. $y = -\frac{1}{2}$, conſequently l. $y^3 =$ l. $x = -\frac{3}{2}$; Whence the natural number correſponding thereto is $=$ ·223130, &c. and accurately, which is the value of x required.

N.B.

N. B. This problem ſhews the diſuſe of Mr. Simpſon's ſeries, (at p. 165 of his Fluxions) for finding the number anſwering to any hyperbolical logarithm: For inſtead of converging, it diverges in many caſes; as if $\overline{x^{\frac{1}{x}}}|^{x^{\frac{1}{x}}}$ were a minimum, $l.x = -6$; and $x = {\cdot}00247875$, &c. Here it diverges very ſwift; but converges very ſlow in the former caſe as to be uſeleſs.

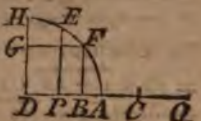
VI. QUESTION 210 *anſwered by Mr.* Rob. Heath.



In the hyperbola, there is given the tranſverſe axis in the figure $= 100$ inches $= p$, $BG = 15 = a$, $HD = mG = 10$, $Bm = 5 = b$, $GD = mH = 12 = c$; let $x = Bo$, and $y = ob$ required. By the property of the curve, $\overline{p+b} \times b : cc :: \overline{p+x} \times x : yy$; ∵ $xx + px = \frac{p+b}{cc} \times byy$, and $x = \sqrt{\frac{1}{4}pp + \frac{p+b}{cc} \times byy} - \frac{1}{2}p$; $a - x = a + \frac{1}{2}p - \sqrt{\frac{1}{4}pp + \frac{p+b}{cc} \times byy} = oG = bd$. (put $f = a + \frac{1}{2}p$, $g = \frac{p+b}{cc} \times b$); it is plain the fluxion of $Gd \times$ into the area of the circle whoſe radius is bd, is equal the fluxion of the indefinite ſolid generated by the rotation of the curve (Bb) about Gd; $ff + \frac{1}{4}pp + gyy - 2f\overline{\sqrt{\frac{1}{4}pp + gyy}} \times 3{\cdot}1416\dot{y}$; whoſe fluent is $3{\cdot}1416y \times \overline{ff + \frac{1}{4}pp} + 3{\cdot}1416y^3 \times \frac{g}{3}$ (let $n = 3{\cdot}1416$) $- fpny - \frac{2fngy^3}{3p} + \frac{2fng^2y^5}{5p^3} - \frac{4fng^3y^7}{7p^5} + \frac{10fng^4y^9}{9p^7} -$ &c. and if c be put inſtead of y, in the expreſſion, we have the ſolidity of half the caſk $= 6754{\cdot}8$ inches, true to a decimal; conſequently $13509{\cdot}6$ inches the whole content, or $47{\cdot}893$ ale gallons.

Merones's

Merones's *Anſwer*.



Let the ſemi-tranſverſe $CA = r = 50$, $CD = a = 65$, the rectangle $QPA = pp = 525$, $PE = q = 12$; $dd = aa + rr$, $c = 3{\cdot}14159$; $CB = x$, GF or $DB = a - x$, BF or $GD = y$. Then per conics, $qq : pp :: yy : xx - rr$; whence $x = \sqrt{rr + \frac{pp}{qq}yy}$; and $BD^2 = \overline{a-x}\rceil^2 = dd + \frac{pp}{qq}yy - 2a\sqrt{rr + \frac{ppyy}{qq}}$: Therefore $DB^2 \times c\dot{y}$ $cdd\dot{y} + \frac{cpp}{qq}yy\dot{y} - 2ca\dot{y}\sqrt{rr + \frac{pp}{qq}yy}$ = flux. ſolid $AFGD$ revolving round GD; put $L = 2{\cdot}302585 \times \log. \frac{p + \sqrt{rr+pp}}{r}$; and write q for y in the fluent, and we ſhall have $cddq + \frac{1}{3}cppq - caq\sqrt{rr+pp} - \frac{caqrr}{p}L$ = half the caſk = $6754{\cdot}88$ inches, and the whole $= 13509\frac{3}{4} = 47{\cdot}907$ ale gallons.

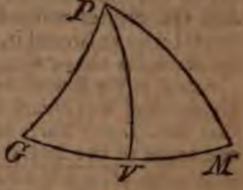
Mr. *Farrer*, Mr. *Turner*, and Mr. *Travis*, have alſo curiouſly wrought this anſwer by a different proceſs.

VII. QUESTION 211 *anſwered by Mr.* R. Heath.

Given	°	′	Sines	Coſines	
AD =	60	16	$\cdot 86747 = b$	$\cdot 49747 = c$	$\angle DCA$ =
AC =	40	21	$\cdot 64745 = d$	$\cdot 76210 = f$	AGB
AB =	80	3	$\cdot 98495 = g$	$\cdot 17278 = h$	Required DG
$\angle DAB$ =	73	0	$\cdot 95630 = s$	$\cdot 29237 = t$	and CB?

It muſt be noted that DC and CD are not two continued arches, for ſo the queſtion would be over-limited and abſurd, but ſides of different triangles. Let x = coſ. $\angle DAC$, $\sqrt{1 - xx}$ its ſine, radius = 1. Then (per Anderſon's Theor.) $bdx + cf$ = coſ. DC; its ſine $\sqrt{1 - \overline{bdx + cf}\rceil^2}$. By trigonometry, s. DC : s. $\angle DAC$ :: s. DA : s. $DCA = b\sqrt{\frac{1 - xx}{1 - \overline{bdx + cf}\rceil^2}}$: Now coſ. $\angle CAB = xt + s\sqrt{1 - xx}$, its ſine $xs - t\sqrt{1 - xx}$, and by the aforeſaid theorem

The PRIZE QUESTION *anſwered by Mr.* J. Turner.



Let P repreſent the pole of the world; M, Moſcow; V, Vienna; G, Gibraltar. Put $x =$ coſ. $\angle GPV = \angle VPM$; $\sqrt{1-xx}$ = its ſine; ſine $GP = b$; of $PM = c$; s. $\angle GVP = \frac{\sqrt{1-xx}}{z}$; the rectangle of the ſines of GP, $PV = d$; rect. of coſ. $= f$; rectangle of the ſines of VP, $PM = g$; rect. coſines $= h$. By Anderſon's Theorem, $dx + f =$ coſ. GV, and $gx + h =$ coſ. VM; as $\frac{\sqrt{1-xx}}{z} : b :: \sqrt{1-xx} : bz =$ ſine GV, and its coſ. $= \sqrt{1-bbzz}$; as $\frac{\sqrt{1-xx}}{z} : c :: \sqrt{1-xx} : cz =$ ſine VM, and its coſ. $= \sqrt{1-cczz}$. Conſe-

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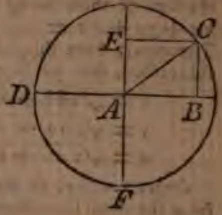
ſines of the three angles formed by thoſe three given lines are reciprocally as the products of each two about them; but thoſe ſides being 12, 16, and 20, are as 3, 4, and 5; therefore the ſines of the angles are as $\frac{1}{3 \times 4}$, $\frac{1}{3 \times 5}$, and $\frac{1}{4 \times 5}$; or as 5, 4, and 3; and ſo univerſally the ſines of the three angles are always as the three given lines. But the three angles about one point are equal to four right angles or 360°; whence the problem is to divide 360° or a circle into three ſuch parts, that their ſines ſhall obtain the given ratio. In the preſent caſe the ratios 5, 4, and 3 form a right-angled triangle; let it be ABC; then the ſupplements of the $\angle$s A, B, and C, of this triangle are the angles required to be formed by the given lines at the center of gravity. For, with the center A and radius AC deſcribe the circle; then it is evident that CB is the ſine of the arc DC or $\angle DAC$, that AB or CE is the ſine of CF or $\angle CAF$, and that AC is the ſine of the remaining quadrant FD or $\angle FAD$; and theſe are reſpectively the ſupplements of the three angles of the triangle ABC.



The other parts of this queſtion will be done as queſt. 430 propoſed in the year 1757, where the ſubject is reſumed, and to which therefore we refer.

Conſequently,

$$\left.\begin{aligned} ddxx+dfx+ff&=1-bbzz,\\ ggxx+2ghx+hh&=1-cczz;\end{aligned}\right\}\begin{aligned} zz&=\frac{1-ddxx-2dfx-ff}{bb}.\\ zz&=\frac{1-ggxx-2ghx-hh}{cc}.\end{aligned}$$

And therefore theſe two are equal to one another. But $\frac{dd}{bb}=\frac{gg}{cc}$; ſo the two terms wherein xx is found deſtroy each other. We have therefore $x=\frac{cc+bbhh-bb-ccff}{2ccdf-2bbgh}$ $=\cdot969343$ the coſine of 14° 13′ 27″, and the difference of longitude of Gibraltar from Moſcow 28° 26′ 54″.

Vienna and Gibraltar bears from Moſcow S. 56° 4′ weſterly.
Moſcow from Vienna, north 44° 50′ eaſterly.
Gibraltar from Vienna, ſouth 44° 50′ weſterly.
Viennna and Moſcow from Gibraltar, north 35° 16′ eaſterly.

Vienna is diſtant from Gibraltar 16° 29′ = 1146 Engliſh geom. miles; Vienna from Moſcow 11° 23′ = 791 miles; Gibraltar from Moſcow 27° 52′ = 1937 miles. This anſwer is performed by a ſimple equation ——The ſame was anſwered by Mr. *Rob. Heath* the propoſer, and by *Merones*.

Mr. *N. Farrer*, Mr. *Rob. Robinſon*, Mr. *H. Travis*, Mr. *J. Powle*, Mr. *Joſ. Young*, and ſome others, have alſo curiouſly inveſtigated the anſwer to the prize queſtion.

Of the Eclipſes in 1740.

Mr. *W. Schoolcraft* gives the Tranſit of Mercury over the Sun, April 21, 1740, inviſible at London, but may be ſeen in the weſtern parts of America. The beginning at 10 h. 22′ at night; middle 11 h. 41′; end 1 h. 1′ in the morning; duration 2 h. 39′; apparent time at London.

Venus over the Sun, May 26, 1761; apparent time at York, beginning at 2 h. 26′ morn. middle 5 h. 30′; end 8 h. 34′.*

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* *A* TRANSIT *of* MERCURY *over the Sun*

Was obſerved this year at *Cambridge*, in *New-England* by Mr. J. WINTHORP; who found the Internal Contact to be at 5 h. 1 m. *P. M.* April 21, app. time.

Within the ſphere of the earth's orbit will happen ſix eclipſes this year; three times will the moon in her wandering courſe interpoſe and hide the ſplendour of the ſun's rays from falling on the earth or its atmoſphere; and thrice will the earth in its courſe, ſo fall in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Moon eclipſed the 2d of January, at 10 at night.

Calculated by		Beg. h m.	Mid.	End	Dur.	Dig.
From Aſtron. Car. Coventry		8 34	10 28	11 18	3 48	20 51
Mr. Chattock, London		8 22	10 27	12 35	4 11	21 41
Mr. Leadbetter for London		8 30	10 25	12 20	3 50	20 29
Mr. Peachey, Mildenhall		8 9		12 5	3 56	21 0
Mr. Bamfield, Honiton		6 18	7 44	9 20	3 2	16 54
Mr. May, jun.	London	8 31	10 29	12 27	3 55	
	Paris	8 41	10 39	12 37		
	Amſterdam	8 50	10 48	12 46		
	Coventry	8 27	10 25	12 23		
Annus Amanuenſis, London		8 34	10 26	11 18	3 50	20 30
Mr. Robinſon,	Guiſbrough	8 7	10 6	12 5	3 58	21 12
	Whitby	8 9	10 8	12 7		
Mr. W. Brown,	Cleobury	8 30	10 24	12 18		20 52
	London	8 40	10 34	12 28		
	Paris	8 49	10 43	12 37		
	Madrid	8 26	10 20	12 14		
Mr. Ab. Donn,	London	8 42	10 40	12 38	3 58	22 30
	Bideford	8 24	10 22	12 20		
	Virginia	3 42	5 40	7 38		
Friend Montague, London		8 37	10 32	12 27	3 50	21 13

The above Eclipſe of the 2d of January was obſerved in *Fleet-ſtreet*, *London*, by Mr. SHORT.

Beginning about — — 8h. 25m. 0s. App. time.
Beg. Total Darkneſs — 9 31 10
End of Total Darkneſs 11 15 20
End of the Eclipſe about 12 42 0

			h. m.	h. m.	h. m.	h. m.	
Mr. Nic. Farrer.	Leadb. Tables	London	8 10	10 9	12 8	3 58	21 12
		Sunderland	8 5	10 4	12 3		
		Morpeth	8 4	10 3	12 2		
	Scien. Steta.	London	8 31	10 25	12 20	3 49	20 29
		Sunderland	8 26	10 20	12 15		
		Morpeth	8 24	10 19	12 13		
Mr. J. Bulman,		London	9 1	10 30	12 25	3 24	20 28
		Edinburgh	8 50	10 19	12 14		
		Dublin	8 33	10 2	11 57		
		Carliſle	8 50	10 19	12 14		
		Deptford	9 2	10 31	12 26		
Mr. T. Glaſpool,		Wincheſt.	8 2[illegible]	10 22	12 17	3 50	20 28
		Aylſham	8 31	10 26	12 21		
Mr. Jo. Taylor by Aſtr. Angl.		London	8 32	10 35	12 38	4 6	21 30
		Spaith	8 28	10 31	12 34		
		Coventry	8 26	10 29	12 32		
		Jeruſalem	1 34	13 37	15 40		
		Liverpool	8 2[illegible]	10 25	12 28		
Mr. Cockſon, Lambton, Du			8 29	10 22	12 16	3 46	20 31
Mr. R. Hughs, Pentrefioden			8 34	10 27	12 21	3 47	20 39
Mr. J. Wilſon, Morpeth			8 25	10 15	12 5	3 40	21 14
Mr. J. Hilton, ———			8 1	10 9	12 2	3 47	20 30
Mr. P. Pilbrow, ———			9 0	11 5	12 54	3 54	20 30
Mr. J. Canton, ———			8 22	10 20	12 18	3 56	20 56
Mr. Sparrow, Edmundſbury			8 32	10 27	12 22	3 50	20 27
Mr. W. Leighton,		London	8 29	10 24	12 18	3 48	20 25
		Arlingto	8 39	10 33	12 27	3 48	
Mr. Ch. Facer, Watlington			8 34	10 30	12 26	3 52	20 50
Mr. W. Schoolcroft at Hovington, Yorkſhire, by		Aſtr. Inſta.	8 17	10 23	12 28	4 10	21 37
		Aſtr. Carol	8 36	10 30	12 24	3 47	20 51
		Sci. Stella.	8 27	10 22	12 16	3 49	20 28
		Flamſt. T.	8 28	10 25	12 23	3 55	21 5
		Leadbett.	8 22	10 22	12 21	3 58	21 10
Mr. J. B. Smith, Oxford			8 26	10 24	12 22	3 56	
M. Jo. Benwell, Highworth			8 21	10 17	12 13	3 51	20 50
Mr. Cooper, Wellingborrow			8 24	10 21	12 18	3 54	20 37

The ſecond eclipſe is of the ſun, Jan. 17, at 8 at night, inviſible to us.

The third eclipſe is of the ſun, June 13, at 2 in the morning, inviſible.

The fourth eclipſe is of the moon, June 28, at 9 in the morning, inviſible.

The fifth eclipſe is of the ſun, Decemb. 7, at 11 at night, and inviſible.

The ſixth eclipſe is of the moon, at 11 at night.

Calculated by		Beg. h. m.	Mid.	End	Dur.	Dig.
Aſtron. Carolina,	Coventry	10 22	11 36	12 50	2 29	5 42
Mr. Chattock,	London	10 21	11 49	1 17	2 56	7 17
Mr. Leadbetter,	London	10 32	11 48	1 4	2 32	5 49
Mr. J. Peachey,	Mildenhall	10 16	11 36	12 56	2 40	6 0
Mr. S. Bamfield,	Honiton	9 43	11 5	12 27	2 44	6 32
Mr. J. May,	London	10 25	11 53	1 21	2 56	6 33
	Amſterdam	10 44	12 12	1 40		
	Peterſburg.	12 26	1 54	3 22		
Annus Amanuen.	London	10 34	11 48	1 7	2 36	5 46
Mr. Robinſon,	Guiſbrough	9 59	11 20	12 41	2 42	6 27
Mr. W. Brown,	Cleobury	10 17	11 32	12 46		
	Dublin	9 59	11 14	12 28		5 43
Mr. Ab. Donn,	Biddeford	10 25	11 44	1 3	2 38	6 15
	Virginia	5 43	7 2	8 21		
Friend Montague,	London	10 36	11 59	1 23	2 47	7 5
Mr. N. Farrer,	London	10 33	11 49	1 5	2 32	5 49
	Sunderland	10 28	11 44	1 0		
	Morpeth	10 27	11 43	12 59		
Mr. J. Bulman,	London	10 30	11 51	1 8	2 38	5 51
	Carliſle	10 19	11 40	12 57		
	Deptford	10 31	11 52	1 9		
Mr. Glaſpool,	Wincheſter	10 31	11 18	1 5	2 34	5 18
	Nottingham	10 36	11 52	1 7	2 31	
	Bungay	10 34	11 51	1 7	2 33	
Mr. Jo. Taylor,	London	10 18	11 43	1 9	2 51	7 9
	Snaith	10 14	11 39	1 5		
	Liverpool	10 8	11 33	12 59		
Mr. C. Cockſon,	Lambton	10 26	11 42	12 59	2 32	5 48
Mr R. Hughs,	Pentrefioden	10 30	11 44	1 5	2 25	
Mr. J. Wilſon,	———	10 27	11 42	12 57	2 33	5 54
Mr. J. Hilton,	———	10 16	11 30	12 45	2 29	5 43
Mr. J. Canton,	London	10 32	11 57	1 12	2 40	6 19
	Stroud	10 23	11 43	1 3		
Mr. Sparrow,	Edmundſbury	10 34	11 50	1 6	2 32	5 48
Mr. Cha. Facer,	Watlington	10 21	11 36	12 51	2 30	5 43
Mr. Schoolcroft,	York	10 23	11 44	1 5	2 41	6 24
	London	10 27	11 48	1 9		
	Jeruſalem	12 49	2 10	3 31		
Mr. J. B. Smith,	Oxon	10 27	11 46	1 5	2 38	6 2
Mr. J. Benwell,	Highworth	10 26	11 42	12 58	2 32	5 44
Mr. T. Cooper,	Wellingbor.	10 14	11 34	12 54	2 39	6 13

New Queſtions.

I. QUESTION 213, *by Mr.* Rob. Heath.

A miſer thus, fair ladies, makes requeſt;
What pounds are thoſe, at compound intereſt,
He muſt, for time, on theſe conditions lend,
To gain an equal value in the end?
Square root of years, ſquare root of pounds per cent,
Muſt equal ſquare root of the money lent:
To make it clear, the ſquare root of each three,
Compar'd with each, muſt equally agree;
Time, rate per cent, and principal unfold,
And wed him, fair one, for his bags of gold.

II. QUESTION 214, *by Mr.* Nich. Farrer.

Sometime in the ſpring quarter, in 1739, in the forenoon, an obſervation being made of the ſun, his altitude was found 33° 41′ 40″; and azimuth from the north 102° 40′ 52″; and ſometime after, on the ſame forenoon, his altitude was found 48° 46′ 53″, and azimuth 134° 39′ 56″. From whence the latitude of the place of obſervation, month, day, and hours of obſervation may be found, and are here required? With a general theorem for all queſtions of this nature.

III. QUESTION 215, *by Mr.* J. May.

Going to paſs a leiſure hour at billiards, I wondered to find the tables an irregular hexagon, when ſeeing the balls fly very ſtrangely in ſtriking the ſeveral gins, made me think, If two balls, *A* and *B*, lay on the ſaid table, and the ball *A* was ſtruck againſt the gin *RS*, from thence reverſing to *ST*, from thence to *TV*, then to *VW*, then to *WX*, thence to *XR*, thence reverſed, and ſtruck the ball *B*; to find geometrically the points in the ſeveral gins, where the ball *A* will ſtrike; and that by a general conſtruction for all polygons, ſuppoſing the balls to be geometrical points?

IV. QUESTION 216, *by Mr.* Henry Travis.

At Matlock, near the Peak in Derbyſhire, where are many ſurprizing curioſities in nature, is a rock by the ſide of the river Derwent, riſing perpendicularly to a wonderful height, which, being inacceſſible, I endeavoured to meaſure in a mathematical method. From a ſtation at ſome diſtance, (nearly level with the bottom of the rock) I took an angle of altitude to its top 47° 30′; and having deſigned a ſecond ſtation, I took an horizontal angle 87° 5′, between the foot of the rock and that ſtation; the meaſured diſtance between the ſtations was 4 chains and 29 links, (per Gunter) or 283·274 feet. At that place I had an angle of altitude 40° 12′, but forgot from hence to take an angle between my firſt ſtation and the foot of the rock; yet am in hopes ſome curious artiſt will, from this data, determine the perpendicular height of this ſtupendous rock.

V. QUESTION 217, *by Mr.* Ant. Thacker.

Given the * equation of the exponential curve, *MDSEB*, together with the axis $AB = b = 1000$; to find the greateſt ordinate (*SR*) and inſcribed parallelogram *DEQP*, and to give the analytical inveſtigation of the ſame?

* $PB^{AP} = PD^{PD}$; i. e. $\overline{b-x}|^{x} = y^{y}$.

VI. QUESTION 218, *by Mr.* Rich. Gibbons.

I will undertake, with 12 fair dice, to throw 42 once in 15 times; and between 37 and 47, at every throw. Quere, Whether I ſhall be a gainer or loſer by theſe chances, and the exact odds?

The PRIZE QUESTION, *by Mr.* Rob. Heath.

Near Twickenham's banks, the muſes ſeat, where Thames
Rolls thro' the valley his ſmooth clearer ſtreams,
A fabric does in peaceful order riſe,
Whoſe owner's virtues reach the lofty ſkies!
Secure of fame, he ſlights all c—rt renown
For Maro's glory, an immortal crown.
His generous fancy, free and unconfin'd,
Well ſuits the buſineſs of a noble mind;
Beholding flatt'ry with a pitying eye,
And, than be guilty, ſooner chuſe to die!
Wrapp'd in himſelf, he can his thoughts approve,
Of truth, of juſtice, poetry, or love;
Can, meditating on life's various ſcene,
See folly's rocks, and ſeas ingulph'd between;
And ſmoothly gliding down amuſements ſtream,
Make gardens, ſhady bowers, or grots, his theme.

Or, from aloft, tall ſpires, domes, waving woods,
Re-echoing hills, fair fields, and chryſtal floods;
Hear the wing'd choir in warbling conſort ſing,
The ſweet-tun'd praiſes of their heavenly king;
See ſwans below, boats, beauteous nymphs, and men,
All moving on ſerenely, and agen.
Who'd not refuſe the gaudy pomp of ſtate,
To live ſo bleſs'd, ſo nobly, good, and great.
T' enrich the proſpect, let it be ſuppos'd,
A park is purchas'd, thus to be inclos'd;
Two ſpreading trees, on Thames ſtreight other ſide,
(Three furlongs diſtance) ſhade the ſilver tide;
And from the muſe's ſeat do equally divide;
From whence a fence of pailing muſt ſurround,
(In length a mile) the yet unfaſhion'd ground;
On this condition carried from each tree,
To make the park the biggeſt that can be.
Again, ſuppoſe a line drawn from each tree,
To the contrary fartheſt boundary;
Theſe, and the fence, to touch two * circling ſhades
On right and left, each ſhelt'ring as it ſpreads:
Hemm'd with a range of trees, to ſcreen the deer,
The middle ſpace wide op'ning to the year.
Ingenious ladies, you are deſired to ſhew,
The park's true form, content, ſhades, area too.
Apollo thus — ſweet ladies, when you've done,
Bring all your harps, and taſte the veniſon.

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* *Circular encloſures touched by oppoſite ſides of the park, and interſections of the longeſt lines drawn from each tree a-croſs the park.*

1741.

Queſtions anſwered.

I. QUESTION 213, *by Mr.* J. Turner.

LET $x=$ principal, rate, and time; as $100 : x :: 1 : \cdot01x$. And putting $b=\cdot01$; what Ward in his Comp. Intereſt calls R, will be $=1+bx$; conſequently by his 1ſt prop. and per queſt. $x\times\overline{1+bx}^{x}=2x$; or $\overline{1+bx}^{x}=2$. Hence log. $\overline{1+bx}\times x=$ log. 2, $=\cdot693147$ (or c). But the log. of $\overline{1+bx}$ is $=bx-\frac{b^2x^2}{2}+\frac{b^3x^3}{3}-\frac{b^4x^4}{4}+\frac{b^5x^5}{5}$ &c.

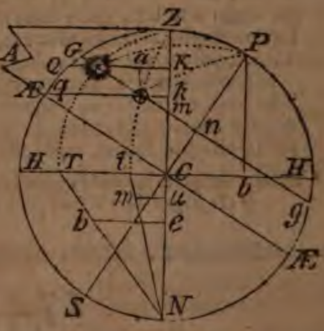
which multiplied by x is $= c$. Reverted, it gives $x = 8{\cdot}4983$ the time and rate, with which any ſum will gain the principal.

Anſwered by Mr. Peter Kay.

Let x be the number ſought; therefore $\overline{1 + \frac{x}{100}}\big|^{x} = 2$, per queſt. this in log. gives $x \times \frac{x}{100} - \frac{x^2}{20000} + \frac{x^3}{3000000}$ &c. $= {\cdot}693147$, whence $x = 8{\cdot}4983$ &c.

After the ſame manner Mr *Dunthorne*, *Merones*, Mr *Powle* have curiouſly anſwered this queſt. Alſo Mr *May*, Mr *Brown*, Mr *Sharp*, Mr *Clifford*, Mr *Story*, Mr *Gibbons*, &c.

II. QUESTION 214 *anſwered by Mr.* Ant. Thacker.



CONSTRUCTION. With radius = ſine of 90° deſcribe the primitive circle on the plane of the meridian $HZHN$; draw HH thro' the center for the horizon, and ZN the eaſt azimuth at right angles thereto; with the verſed ſines of 102° 40′ 52″ and 134° 39′ 56″, ſet off Ht and HT: And with the ſines of 33° 41′ 40″ and 48° 46′ 53″ lay down Ck and CK; then draw kq and KQ parallel to HH: make Nu and $Ne = kq$ and KQ; join TN and tN; and draw wu and be parallel to HH; then make k⊙ and K☼ $= wu$ and be; through ☼⊙ draw G☼⊙mng, and draw the equator $ÆCÆ$ parallel thereto; and Cnp the elevation of the pole at right angles to it. From whence Cn will be the ſine of the ſun's declination, and Pb the ſine of the latitude, &c.

1. From which projection, the inveſtigation of a general theorem will eaſily ariſe. For per ſimilar triangles, ſay as NC (radius $= 1$) : Ct (z) :: Nu (q) : $qz = uw = k$⊙. And NC (1) : CT (m) :: Ne (s) : $ms = be = K$☼; ∵ $ms - qz =$ ☼a. Alſo as ⊙a $(s - p)$: ☼a $(ms - qz)$ :: CZ (1) : $AZ = \frac{ms - qz}{s - p}$ = the tang. of the required latitude.

2. *For the Sun's Declination.* As $Pb\ (x) : Cb\ (y) :: \odot k\ (qz) : km = \frac{qzv}{x}$; and as $mC\ (p - \frac{qzy}{x}) : Cn\ (d) :: CP\ (1) : Pb\ (x)$; therefore $px - qyz = d$, the ſun's declination.

3. *Laſtly, for the hour of the day.* Put $c =$ coſine of ſun's declination; and s and e for the ſine and coſine of the hour from noon; then will $px - qyz = d$; and $dx + cye = p$;* and by ſubſtitution $xxp - xqzy + cye = p$; but $1 - yy = xx$; $\because ce = py + xqz$; and as $s : q :: n$ (ſine of $\odot$ azimuth) : c; $\because \frac{qn}{s} = c$; which ſubſtituted for c, gives $\frac{qne}{s} = py + xqz$; therefore $\frac{e}{s} = \frac{py + xqz}{qn} =$ cotangent of the hour from noon, at the firſt obſervation.

* N. B. If the ſun's azimuth is leſs than 90° (from the north) then tC muſt be taken on the contrary ſide of C, and therefore negative with reſpect to what it is (in this queſt.)

Hence $\frac{qz - ms}{s - p}$; $px + qyz = d$; and $\frac{s}{e} = \frac{py - xqz}{qn}$ are general expreſſions for the queſt. as above. Only whenever any angle or ſide exceeds 90°, its coſine muſt be expreſſed by a negative ſine.

This theorem $px + qyz = d$, will be found to anſwer all the ends propoſed by that of Mr. Anderſon, in the Diary, 1732. And is indeed more ſimple, and is better adapted to the uſes than his; tho' I freely own, that the above method of deriving it, was firſt hinted to me ſome years ago by my ingenious friend Mr. *T. Simpſon*.

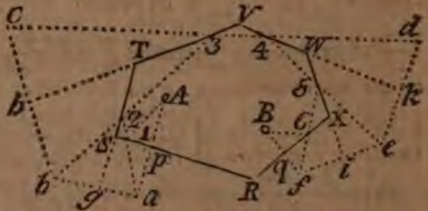
This theorem, viz. $\frac{e}{s} = \frac{py - xqz}{qn}$ is found to be more uſeful than the other, e. g. In the ſolution of the prize queſtion 1739, ſolved by it, comes out $\frac{st}{2bc} = x$; where the ſine $= s = GP + MP =$ ſine 89°; and $t =$ tangent PV; and all the others as by Mr. Turner, Diary 1740. Alſo queſt. 211 may readily be anſwered by this theorem, and will come out $xx + 2px = n$.

Theſe theorems above being brought out in numbers, by help of the artificial and natural ſines, to facilitate the labour; gave, 1. The latitude $= 54^\circ\ 51'$. 2. The ſun's declination $= 20^\circ\ 24'$ anſwering to the 10th day of May. And 3d, the hour $60^\circ = 4$ h. or 8 in the morning, and $30^\circ = 2$ h. or, 10 o'clock, agreeing preciſely with that of the ingenious propoſer's true anſwer.

This queſtion was very methodically and truly ſolved by Mr. *J. Turner*, Mr. *Dunthorne*, Mr. *R. Robinſon*, Mr. *Heath*, Mr. *Farrer*, Mr. *Rob. Price* in trigonometrical method; Mr. *Dan. Boote*, Mr. *H. Travis*, Mr. *May*, Mr. *Powle*, Mr. *Badder*, Mr. *Story*, Mr. *Ardon*; and the ſolution was given by Mr. *Brown*, Mr. *Webber*, and others.

III. QUESTION 215 *anſwered by* Merones.

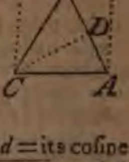
Produce all the ſides of the figure as in the ſcheme: on RS let fall the perpend. Apa, and make $ap = Ap$; draw $ab \perp ST$, and make $bg = ga$; draw $bc \perp TV$, and make $ch = bh$. Alſo $Bf \perp RX$, and make $qf = Bq$; draw $fe \perp WX$, and make $ei = fi$; draw $ed \perp VW$, and make $dk = ke$. Then draw the lines cd, $3b$, $2a$, $4e$, $5f$, and $6B$, cutting the ſides of the figure; then $A123456B$ will be the path of the moving body A.



After the ſame manner Mr. *Brown* and Mr. *Webber* have ſolved it, and Mr. *J. B. Smith* and Mr. *Betts*, by protracting the lines and angles, in a progreſſive form, have made the demonſtration not only curious but very eaſy and general, which is omitted here for another place, as will be ſhown farther on.

IV. QUESTION 216 *anſwered by Mr.* N. Farrer.

In the annexed figure, if BT, BT, repreſent the height of the rock perpendicular to the horizontal plane ABC; the points T and T (on the turning up the two triangles to which and the plane they are perpendicular) are ſuppoſed coincident; A and C the two ſtations, and BAC the horizontal angle given: by letting fall a perp. CD by plain trigonom. I find $CD = m$, and $AD = p$. Then put $q =$ ſine $\angle BAT$, (the angle at the firſt obſervation) and $s =$ its coſine; $h =$ ſine $\angle BCT$ ($\angle$ altit. at the ſecond obſervation) and $d =$ its coſine. Let $x = DB$, then (per 47 Eucl. 1.) $\sqrt{xx + mm} = CB$, and

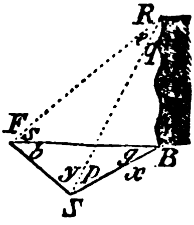


and $x+p=AB$; then as coſine $\angle BAT$: its ſine :: $AB : BT$; coſ. BCT : its ſine :: $BC : BT$; i. e $s : q :: x+p : \frac{qx+qp}{s} = BT$; $d : h :: \sqrt{xx+mm} : \frac{h\sqrt{xx+mm}}{d} = BT$. Then $\frac{h\sqrt{xx+mm}}{d} = \frac{qx+qp}{s}$; hence $sh\sqrt{xx+mm} = dqx+dqp$. This equation ſquared and brought into numbers, the ſquare compleated and root extracted, &c. I find $x = 311{\cdot}24$ feet; and BT the rock's height $= 118{\cdot}47$ yards, the anſwer to the queſtion.

N. B. By a miſtake in printing, the angle was 40° 12′ inſtead of 41° 12′; ſo the true height was more, as in the following anſwer.

The ſame anſwered by H. Travis.

Let FSB repreſent the ground lines, as ſuppoſed truly horizontal; in which is given FS the ſtationary line 429 links $= a$; and the horizontal angle $SFB = 87° \, 5' = b$; and in the vertical $\triangle \, BFR$ the angle of altitude at $F = 47° \, 30' = s$; in the other $\triangle \, BSR$, right-angled at B, the ſecond angle of altitude $BSR = 41° \, 12' = p$. Then ſay, as $q : x :: p : BR = \frac{px}{q}$; as $s : \frac{px}{q} :: e : BF = \frac{pex}{sq}$; and as $b : x :: y : BF = \frac{xpe}{sq}$; $\because yx = \frac{pebx}{sq}$, and $y = \frac{peb}{sq} = \angle S$.



The Operation by Logarithms.

$p = 9{\cdot}8186807$
$e = 9{\cdot}8296833$ $\quad s = 9{\cdot}8676309$
$b = 9{\cdot}9994370$ $\quad q = 9{\cdot}8764574$

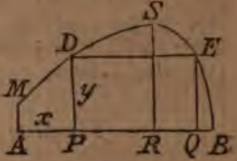
$peb = 29{\cdot}6478010 - qs = 19{\cdot}7440883$ gives $9{\cdot}9037927 = 53°$ 15′, which added to 87° 5′, then ſubtracted from 180, leaves 39° 40′ $= \angle B$, whoſe natural ſine call g; then as $g : a :: b : x = \frac{ba}{g}$. Which ſubſtitute for x in the value of BR, gives $\frac{bap}{gq} =$ the height of the rock. The operation:

$b =$

$b=9{\cdot}9994370$
$a=2{\cdot}6324573$ $g=9{\cdot}8050385$
$p=9{\cdot}8186807$ $q=9{\cdot}8764574$
$lap=22{\cdot}4505750-gq=19{\cdot}6815959=587$ links or 129 yards.

This queſtion was truly ſolved by *Marcellas Beighton*, Mr. *Will. Daniel*;-and the anſwer to it as printed, by Mr. *R. Dunthorne*, Mr. *D. Webber*, Mr. *Heath*, Mr. *Robinſon*, *Merones*, *J. B. Smith*, Mr. *J. May*, Mr. *Arden*, &c.

V. QUESTION 217 *anſwered by Mr.* Rob. Heath.



When an ordinate (or any quantity) is a maximum, the logarithm of it, or any power of it, is a maximum; conſequently the log. of $RS^{RS}=$ log. of RB^{AR} $=x\times$ log. of $b-x$ is a maximum.

In fluxions $-\frac{x\dot{x}}{b-x}+\dot{x}\times\log.\overline{b-x}=0$; where x is eaſily determined $=836{\cdot}0532$ &c. by a new method of ſolving exponential equations; $\because\ \overline{163{\cdot}9468}^{863{\cdot}052}=y^y$, and $y=657{\cdot}1442$ &c. very true to a decimal, the length of the greateſt ordinate RS required. By drawing the true figure, or trying the increaſe of the parallelogram, it appears that Ap is greater than 400, when the $\square\, QEDP=DP\times PQ$ is a maximum; and by the table of logarithms, it alſo appears that $DP=EQ$ is greater than 420, and AQ leſs than 987. This muſt be obſerved in order to have the following logarithmetic ſeries converge. Put $400+x=AP$, $420+y=DP=EQ$, and $987-z=AQ$; to find DP and PQ. By the equation of the curve, $PB^{AP}=PD^{PD}$ $=QE^{QE}=QB^{AQ}$, or $\overline{600-x}|^{400+x}=\overline{420+y}|^{420+y}$ $=\overline{13+z}|^{987-z}$, or $\overline{m-x}|^{n+x}=\overline{p+y}|^{p+y}=$ $\overline{q+z}|^{r-z}$ by ſubſtitution; in logar.

$$\left.\begin{array}{r}\text{l}.\,m\times n-\frac{n}{m}\\ +\text{l}.\,m\end{array}\right\}x\left.\begin{array}{l}-\frac{n}{2mm}\\ -\frac{1}{m}\end{array}\right\}xx\left.\begin{array}{l}-\frac{n}{3m^3}\\ -\frac{1}{2mm}\end{array}\right\}x^3\text{ \&c.}=\left.\begin{array}{r}\text{l}.\,p\times p+1\\ +\text{l}.\,p\end{array}\right\}y$$

$$+\frac{3}{2p}y^2+\frac{5}{6pp}y^3\text{ \&c.}=\left.\begin{array}{r}\text{l}.\,q\times r+\frac{r}{q}\\ -\text{l}.\,q\end{array}\right\}z\left.\begin{array}{l}+\frac{r}{2qq}\\ -\frac{1}{n}\end{array}\right\}zz\left.\begin{array}{l}+\frac{3}{3q^3}\\ +\frac{1}{2q^2}\end{array}\right\}z^3\text{ \&c.}$$

&c. Now the value of any two ſingle quantities may be found in terms of the other. The above in numbers (g) $21{\cdot}8648 + (a)\, 5{\cdot}730263x - (b)\, {\cdot}0022221x^2 - (k)\, {\cdot}000002x^3$ &c. $= (c)\, 7{\cdot}040254y + (d)\, {\cdot}0035714y^2 + (e)\, {\cdot}0000047y^3$ &c. and $= (c)\, 73{\cdot}358128z + (d)\, 2{\cdot}843195z^2 + (e)\, {\cdot}146791z^3$: I find the values of y and z in terms affected with x, by aſſuming an equation of this form, viz. y or $z = A + Bx + Cx^2 + Dx^3$ &c. Where by ſubſtitution for their aſſumed values, and making the coefficients of like terms on different ſides of each equation equal, we ſhall have the real reſpective values of the coefficients, viz. $A^3 + dA^2 + cA = g$; $A = 3{\cdot}1008$ and ${\cdot}36508$; $B = \frac{3eA^2 + 2dA + C}{a} = {\cdot}811364$ and ${\cdot}075904$; $C = \frac{-b - 2dB^2 - 3eAB^2}{3eA^2 + 2dA + C} = -{\cdot}0006478$ and $-{\cdot}0002586$; $D = \frac{-k - 2dBC - 6eABC - eB^3}{3eA^2 + 2dA + C} = -{\cdot}0000001005$ and $+{\cdot}000000686$. Conſequently $y = 3{\cdot}1008 + {\cdot}811364x - {\cdot}0006478xx$ &c. $z = {\cdot}36508 + {\cdot}075904x, - 0002586xx$ &c, wherefore $586{\cdot}63492 - 1{\cdot}075904x + {\cdot}0002586xx$ &c. $= PQ$, and $423{\cdot}1008 + {\cdot}811364x - {\cdot}0002586xx$ &c. $= DP$; the product of theſe two is a maximum, which fluxed and reduced, there reſults $20{\cdot}759 - 2{\cdot}28625x + {\cdot}003212xx$ &c. $= 0$; here $x = 9{\cdot}2$ nearly; whence $PQ = 576{\cdot}758$, $DP = 430{\cdot}5105$, and the area of the greateſt □ $248300\frac{1}{4}$. *Q.E.F.*

Anſwered by Mr. J. Turner.

The equation of the curve is $\overline{b-x}^{\,x} = y^y$; $AB = 1000 = b$; hence $x \times \mathrm{l}.\,\overline{b-x} = y \times \mathrm{l}.\,y$. In fluxions, $\dot{x} \times \mathrm{l}.\,\overline{b-x} - \frac{x\dot{x}}{b-x} = 0$; $\overline{b-x} \times \mathrm{l}.\,\overline{b-x} = x$. Or finding the logarithm thereof, $\overline{b-x} \times \mathrm{l}.\,b + \frac{x}{b} - \frac{x^2}{2b^2} + \frac{x^3}{3b^3} - \frac{x^4}{4b^4}$ &c. $= x$. This reduced gives $x = 836$ nearly; whence RS the greateſt ordinate is $= 657$, &c.

For the greateſt Parallelogram:

Put $z = PQ$, $DP = EQ = y$; the area $= A$.

By the nature of the curve, $\overline{b-x}^{\,x} = y^y$,

and $\overline{b-x-z}^{\,x+z} = y^y$,

and $zy = A$.

Which thrown into fluxions and reduced (by expelling z) gives $x = 407{\cdot}6$, $y = 429{\cdot}2$, $z = 578{\cdot}4$, $AQ = 986$, and the required area 248250.

The ſame anſwered by Merones.

1. For the greateſt ordinate: $\overline{b-x}|^{x} = y^{y}$, and $x \times$ log. $\overline{b-x} = y \times$ log. $y =$ max. whence $\dot{x} \times$ l. $\overline{b-x} - \frac{x\dot{x}}{b-x} = 0$; and $\overline{b-x} \times$ log. $\overline{b-x} = x$. Put $a+v=x$; $b-a=g$; and then $\overline{g-v} \times$ l. $\overline{g-v} = a+v$. Let $p =$ log. g: take $a = 836$; then $gp - a - pv - 2v + \frac{v^2}{2g} + \frac{v^3}{6g^2} +$ &c. $= 0$. And by reverſion of ſeries $v = \cdot 0531493$, $x = 836\cdot 0531493$, and $y = 657\cdot 02682$ the greateſt ordinate.

2. Then for the greateſt inſcribed parallelogram; let $AP = x$, $AQ = s$; then $y \times \overline{s-x} =$ max. put $d+u=s$, $f = b-d$, $r =$ log. f; then $\overline{d+u} \times$ l. $\overline{f+u} = \overline{d+u} \times$ l. $\overline{g-v}$ ($= y$ log. y). Theſe turned into ſeries, and reverted, will give the value of u: whence is had s. Alſo let $c+z=y$, then will $\overline{a+v} \times$ log. $\overline{g-v} = \overline{c+z} \times$ log. $\overline{c+z}$. This turned into ſeries, and the ſeries reverted, finds z, and conſequently y; then theſe values of s and y being ſubſtituted in the maximum, and its flux made $= 0$, and the ſeries reverted, gives v, x, and y, and thence the inſcribed parallelogram $= 247675$ nearly.

The ſame anſwered by the Propoſer Mr. An. Thacker.

Aſſume $AP = 407 + x$; $PD = EQ = 429 + y$; and $QB = 13 + v$: then will PQ or $DE = 580 - x - v$; and per nature of the curve we have $\overline{593-x}|^{407+x} = \overline{429+y}|^{429+y}$ and $\overline{13+v}|^{977-v} = \overline{429+y}|^{429+y}$. Theſe in log. are $\overline{407+x} \times :$ l. $593 - \frac{x}{593}$ &c. $= \overline{429+y} : \times$ l. $429 + \frac{y}{429}$ &c. And $\overline{987-v} \times :$ l. $13 + \frac{v}{13}$ &c. $= \overline{429+y} \times :$ l. $429 + \frac{y}{429}$ &c. And theſe equations ordered make $a + bx + cxx$ &c. $= A + By + Cyy$ &c. And $f + gv + hvv$ &c. $= A + By + Cyy$ &c. Conſequently $x = F + Gy + Hyy$ &c. and $v = K + Ly + myy$ &c. Hence the area of $PDEQ$ will be expreſſed by $\overline{O - Py - Qyy \text{ \&c.}} \times \overline{429+y}$, which per queſt. muſt be a maximum; then put into fluxions and reduced, gives $y = 1\cdot 5104$ &c. ∵ the greateſt area $= 248300\cdot 3749$ &c.

Again, for the greateſt ordinate, put $835+x=AR$, then is $\overline{165-x}|^{835+x}$ to be a maximum; therefore put into fluxions and reduced, gives $x=1\cdot0531$ &c. and $SR=657\cdot0268$ the true ſolution. *Q. E I.*

VI. QUESTION 218 *anſwered by Mr.* Peter Kay.

It is found from theorem 2, page 53, of Simpſon's Laws of Chance, that the odds between 37 and 47 coming up, at any aſſigned throw, with 12 dice, is as $1\cdot162$ &c. to 1, or as 7 to 6 very nearly; which is an anſwer to one part of the queſtion. And by theorem 1 in the ſame page, the probability of throwing juſt 42 comes out $=\frac{1}{15\cdot22}$, &c. Therefore $\overline{1-\frac{1}{15\cdot22}\ \&c.}|^{15}=\cdot3604$ is the probability of loſing in the other part of the queſtion; and conſequently the required odds for winning as 6396 to 3604, or as 16 to 9 nearly.

The ſame ſolved by Mr. J. May, *jun.*

Put the number of dice $=n$; the number of ſides on each dice $=m$; $42=p$; (but if p was greater than $\frac{1}{2}n+\frac{1}{2}mn$, ſubtract it from $n+nm$, and put the remainder $=p$). Then the chance the propoſer has to win in the firſt throw, or to throw juſt 42, will be $\frac{n\times n+1\times n+2\times n+3\times n+4}{1\times2\times3\ 4\times5}$ &c. The numb. of terms muſt be $p-n-\frac{n}{1}\times\frac{n+1\times n+2}{2\times3}$ &c. (in terms leſs than the foregoing) $+\frac{n\times n-1}{1\times2}\times\frac{n\times n+1\times n+2\times n+3\times n+4}{1\times2\times3\times4\times5}$ &c. (In terms leſs) $-\frac{n\times n-2\times n-3}{1\times2\times3}\times\frac{n\times n+1\times n+2\times n+3}{1\times2\times3\times4}$ &c. (in terms leſs) which put in numbers is 144840476; now all the chances in the ſaid dice will be m^n or 2176782336; ſo that there is 2031941860 chances to loſe, that is very near 3365 to win, againſt 47207 to loſe the firſt throw: Put $3365+\cdot47207=r$; $47207=s$; and $15=t$; the proportion which the propoſer has to throw 42 once in 15 times, is to the chances he has not, as $1 - \frac{s^{-t}}{r^t}$ to $\frac{s^{-t}}{r^t}$ (ſee Struyck p. 52) which is about 71199 to 128801. The ſecond query is eaſily deduced from the above ſeries, and the propoſer has 1400482914 chances to win, againſt 776299422 to loſe; i. e. if 37 and 47 are included; but if excluded, then 1194427194 to win, againſt 982355142 to loſe.

Mr. Rob. Heath *anſwers this Queſtion.*

The number of chances for 42 happening in one throw with 12 dice (by his theorem) will be 144840476; which taken from all the chances on all the dice, leaves 2031941860 chances for failing in one throw.——The advantage in wagering to throw 42, once in 15 times with 12 fair dice, will be 6440045 to 3559955; or nearly as 9 to 5. To find the number of throws to make an equal wager; make $\frac{b x}{\overline{a+b}|^{x}} = \frac{1}{2}$, which comes to $x = \frac{\log. 2}{\mathrm{l.}\, \overline{a+b} - \mathrm{l.}\, b}$ when reduced; and ſolved $x = 10{\cdot}0666$ throws by common logarithms: Mr. De Moivre ſays $\frac{b}{a} \times {\cdot}7$ ſhews the trials requiſite to that effect, when b is pretty large in reſpect of a; but in this caſe it ſhews it to happen in 9·82 throws; very near the truth: but his Table of Limits at p. 42 Doct. Chances, 2d edit. is not very exact, as being not deduced from exponential equations truly ſolved.

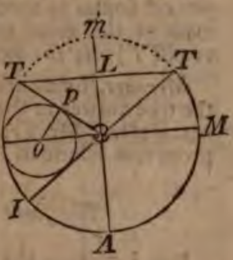
The ſum of the chances for each party, N. in one throw (by the ſeries) are found = 1194307074; which taken from all the chances on all the dice, leaves 982475262 chances for miſſing; therefore the odds are nearly 6 to 5. *Q. E. F.*

Mr J. Turner (in anſ. to 218 queſt.) by prob. 22. Mr Simpſon's Laws of Chance, the odds of throwing (in this queſt.) as 6439971 to 3560042; nearly as 8½ to 4½; and the latter part of the queſtion as 6 to 5 nearly.

Mr J. Hill ſays the odds are as 113156 l. 12s. 5¼d. to 105830 l. 6 s. 1¼d. And when he undertakes to throw between 37 and 47 (excluſive) he ought to lay 622097 l. 9s. 11¼d. to 511643 l. 6 s. 0¼ d. The propoſer Mr Gibbons has given tables for theſe queſtions, but as too long to inſert here, if he pleaſes to reviſe them, may be put in another place.

The

The PRIZE QUESTION *anſwered by Mr.* H. Travis.



Put $1 =$ radius of the circle; *TMAIT* the park; $q = 1{\cdot}570796$ &c. ($\frac{1}{4}$ periph. when rad. is 1) = quadrantal arch *AM*; $TM = x$; $TL = y$; then, as $8 : 3 :: q + x : y$; $\therefore 8y = 3q + 3x$; and by Sir Iſaac Newton's ſeries, $y = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \frac{x^6}{1.2.3.4.5.6}$ &c. $\therefore 8 - 4xx + \frac{x^4}{3} - \frac{x^6}{90}$ &c. $= 3q + 3x$. Solved, $x = {\cdot}0611928$ &c. $\therefore y = {\cdot}81849$ &c. and the area of the park = 89 a. 0 r. 14 p. For the area of the ſhades: Put $s =$ ſine arc *TM*; $z = po =$ radius of the circular ſhades; then, as $s : z :: 1 : 1 - z$; $\therefore z = \frac{s}{1+s}$; hence the area of each is 14 acres and 7 perches. *Q.E.I.*

The ſame anſwered by Mr. Heath *the Propoſer.*

Since the river *TT* (part of the boundary) is given ſtraight, therefore the neareſt the park can approach the form of a circle, is the ſegment *TMAIT* the true form; which comprehends for its area a maximum. Put $b = 320$ poles $= TMAIT$; $d = 60$ poles $= TLT$; $p = 6{\cdot}28318$ &c. $x =$ arch mT; then $\frac{b+2x}{p} =$ rad. $= \odot T$. And, by a ſeries for finding the ſine from the radius, and arch given, $d = x - \frac{x^3}{6 \times \overline{\frac{b+2x}{p}}|^2} + \frac{x^5}{120 \times \overline{\frac{b+2x}{p}}|^4} - \frac{x^7}{540 \times \overline{\frac{b+2x}{p}}|^6}$ &c. Where x is found $= 70{\cdot}287$ &c. whence rad. $= 73{\cdot}302$ &c. poles $= b$; and the area of the park $14255{\cdot}15$ poles, or 89 a. 0 r. 15 p.

Put $f = {\cdot}574459 =$ ſine $\angle o \odot p = \angle \odot TT$; $y =$ radius of each ſhade $= op$. Then $b - y = o\odot$; ſay rad. $1 : b - y :: f : y$; whence $y = \frac{fb}{1+f} = 26{\cdot}745$ &c. poles; conſequently the area of each ſhade = 14 a. 0 r. 7 p.

This queſtion may be more eaſily ſolved, by a table of nat. ſines (with rad. unity). Seek a chord and arc in the ratio of the given chord and arc, as 3 to 8, which are found by 2 or 3 trials = 1·6370516 and 250° 7′ 30″ ſimilar to the chord *TT*, and arc *TMAIT*; (for ·8185258 is ſine of 54° 56′ 15″ ſimilar to the ſine *LT*, and arc *Tm*, half the ſupplement of the given arc to 360°) ſay 1·6370516 : rad. 1 :: 120 (*TT*) : 73·302 &c. poles = *T*⊙, radius as found before.

Merones, by a curious and ſhort proceſs, has brought out the true anſwer to the queſtion. Mr. *Robinſon*, Mr. *Brown*, Mr. *Webber*, Mr. *Trott*, Mr. *Turner*, and others, have alſo given true ſolutions.

The ſame ſolved by Mr. J. Powle.

Put $b = 5400$ the min. in a quad. $c = ·0000000084616$; $d = 4 = \frac{1}{2}$ arc; $q = 3·14159$; $p = 1\frac{1}{2}$ = half the given chord; $x = \angle \odot TL$ in minutes; $y = T\odot L$; radius = 1. It will hold, $b + x : d :: 2b : \frac{2bd}{b+x}$ length of the ſemicircle; $q : 1 :: \frac{2bd}{b+x} : \frac{2bd}{qb+qx} = \odot T$; then, per ſim. $\triangle$s, $1 : y :: \frac{2bd}{qb+qx} : p$; i. e. $\odot a : ab :: \odot L : LT$; $\therefore \frac{2bdy}{qb+qx} = p$. Here $x = \frac{2bdy - qph}{qp}$: $xx = \frac{4bbddyy - 4dbbpqy + qqbbpp}{qqbb}$.

Then by Mr. Ward's equation for finding the ſine correſponding to a given arc (Math. Introd. p. 358) we have $y^4 + 28y^3 + 195y^2 + 36cxxyy + 108cxxy - 28y = 196 - 81cxx$; and exterminating xx, by means of its value before found, there will be produced this equation:

$$\frac{qqpp}{144cbbdd}y^4 - \begin{array}{c}28qqpp\\144dbbpqc\\432cbbdd\end{array}y^3 - \begin{array}{c}195qqpp\\36qqbbppc\\-432cdbbpq\\324cbbdd\end{array}y^2 \begin{array}{c}108qqbbppc\\-28ppqq\\-324cbbdqp\end{array}y$$

$$= 196qqpp - 81qqbbppc.$$

In numbers, $-5707·1022y^4 - 10979·01y^3 + 998·257y^2 + 9733·248y = 85·7091935$; ſolved, $y = ·8185287$ the ſine of 54° 56′ 16″; by trigonom. s. 54° 56′ : (*LT*) 1·5 :: rad. 90° : $CL = 1·83255$; then $\underline{1·8325 \times 4 = 7·3302} = 73$a. 1r. 8p. the ſector; and $\sqrt{3·3325 \times ·3325 \times 1·5 \times 1·5} = 15$a. 3r. 6p. = area $\triangle$. Their ſum 89a. 0r. 14p. the true area of the park. Shade 14a. 0r. 7p.

Of

Of the Eclipſes in 1741.

To the inhabitants of Great Britain or the adjacent iſlands, there will happen no viſible eclipſe this year, but at Fort St. George, the ſun will appear eclipſed the 27th of November, beginning at 8 h. 40′ morning. Middle at 10 h. End at 11 h. 14′. Duration 2 h. 30′. Digits eclipſed 2 h. 48′ [*John Skay*]. On the 2d of June the moon would be eclipſed, but by her depreſſion of latitude is rendered inviſible to us; yet will appear in the ſouthern parts of the world as follows, [*R. Hale.*]

	Beg.	Mid.	End	Dur.	Dig.
At Gibraltar in Spain	8 20	9 6	9 52	1 32	4 5
Alexandria in Egypt	11 18	12 9	12 59	1 41	5 24
Caluent in the Indies	2 5	3 5	4 5	2 0	9 0
Jeruſalem	0 16	1 9	2 2	1 46	5 31

Obſervations of Eclipſes at the Obſervatory.

At Trinity College, Cambridge, was took the beginning of the moon's eclipſe Jan. 2, 1740, at 8 h. 27′ apparent time, the total ſhadow entered the moon's N. E. limb about the 118th degree in Hevelius's chart, near Mount Audus, then clouds interpoſed. The emerſion was about 11 h. 15′ over-againſt M. Pherme.

At 3 miles from Beverley, Abr. Buncholot obſerved the moon's eclipſe Jan. 2, 1740, beginning 8 h. 20′. Begin. total darkneſs 9 h. 30′. End total darkneſs 11 h. 10′. End 12 h. 20′.

New Queſtions.

I. QUESTION 219, *by Mr.* J. May *of Amſterdam.*

Here in Holland, the land lying ſo very low, they are obliged to raiſe banks or dikes to keep the ſea from overflowing it; yet ſome time ago a great ſtorm, with a high tide, broke through one of the banks, and laid the country for ſome miles under water. Going with ſome friends to the place where this inundation happened, and walking upon one of theſe narrow banks, we ſaw at a diſtance three trees (*A*, *B*, *C*) ſtanding in the water, which we were told, ſtood at an equal diſtance from each other at the corners of a triangular field; and a pole equidiſtant from each tree, that formerly was uſed as a mark to ſhoot at with bows and arrows. Now one of our party propoſed to find the content of the inundated field by the help of a ſtaff, or meaſuring rod only, which we had with us. As we walked along this bank in a ſtraight line (*DP*) at *D* we came in a ſtraight line with the trees *C* and *A*; thence meaſuring 304 feet, at *E* we came in a right line with the trees *C* and *B*; thence continuing ſtraight 121·6 feet, at *R* we made a right line with the pole in the middle of the field, and the tree at the farther corner *B*; laſtly from *R* meaſuring right forward 159·6 feet, at *P* we came in the line of *BA*. From this, which was all we were then capable of doing, is required the content of the field *ABC*.

II. QUESTION 220, *by* Merones.

Being at ſea on the firſt of May, and a clear forenoon, I made two obſervations of the ſun, and found the difference of altitudes 16° 30′, the difference of azimuths 34 deg. and the difference of the times 2 h. 15 m. Required the latitude of the place and hours of the day.

III. QUESTION 221, *by Mr.* Peter Kay.

One with ſix dice undertakes to bring up four faces of a ſort at a throw; that is, either four aces, four duces, &c. in ſeven trials? What is the odds againſt him?

IV. QUESTION 222, *by Mr.* Daniel Boote.

Some time in the ſpring quarter 1740, an obſervation was made of the ſun, at 24 minutes paſt eight o'clock in the morning, and his altitude found 39° 7′. Alſo at 15 minutes paſt ten o'clock (on the ſame forenoon) the altitude was 55° 57′. From whence the latitude of the place of obſervation, month, and day may be found; and are required, with a general theorem for all queſtions of this nature.

V. QUESTION 223, *by Mr.* Peter Kay.

Suppoſing a body in 51° 32′ north latitude, to be projected with a velocity of ſix miles per ſecond, in a ſouth-eaſt direction, and an angle of 30° with the horizon: required the trajectory, the place of the earth where the body will fall, and the time it is in motion; allowing the earth to be ſpherical, its circumference = 25020 miles, that a heavy body deſcends 16·1 feet in a ſecond of time, and that the projectile moves in a non-reſiſting medium.

VI. QUESTION 224, *by Mr.* Nich. Farrer.

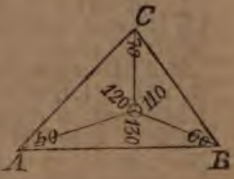
Two ports bear north and ſouth on Britiſh ſhore,
Their diſtance 50 leagues, nor leſs nor more;
A ſhip from the ſouth port ſets out to gain
The northern port, and plows the wat'ry main,
On larboard tacks a certain diſtance, then
Alters her courſe, ſo gains her port; but when
The diſtances compar'd, the firſt was more
By true ſea leagues exactly half a ſcore;
The diſtance of the ports and diſtance run,
Include a ſpace * as here below is ſhewn:
Now, artiſts, by theſe data it's requir'd,
To find the diſtance run, † and courſes ſteer'd:
To make it plainer, and require leſs art,
Give the ſolution by a plain ſea chart.

* *796 ſquare leagues.* † *Variations and lee-way allowed for.*

Now let us ſuppoſe the ſhip to have ſailed uniformly at the rate of four miles an hour, and at the ſame time that ſhe ſailed from the ſouth port, another ſhip ſails from the north port directly ſouth, at the rate of $5\frac{1}{2}$ miles an hour, till ſhe arrive at a port under the ſame meridian with the former ports, and is then known to be at the neareſt diſtance from the firſt ſhip, that ſhe could poſſibly have been during her whole paſſage, if ſhe had continued her courſe to the ſouth port: Required the bearing and diſtance of the middlemoſt port from the firſt ſhip, when ſhe brings her ſtarboard tacks on board.

VII. QUESTION 225, *by Mr.* J. Corbett.

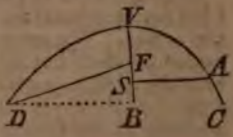
In order to ſurvey and divide the triangular field *ABC*, I bid my men meaſure the three ſides round, whilſt I took the angles. The owner would have it in three equal parts nearly divided, but as there was no water but one ſpring near the middle, in order to have the three fences run ſtraight from it to the three corners, I took the angles there, as well as thoſe at the ſpring to each corner, as in the ſcheme are ſet down; for he would have the content of each part ſeparate. Having thus much, as angles, ſides, &c. I thought ſurely I might from hence find the lines to the corners (and areas) as well as Mr. Beighton in his ſurvey of Warwickſhire could find the ſituation of Newbold, and diſtance to High Croſs, Tripontium, Eathorpe, &c. as he ſets forth in the compartment of his map; for that beſides angles, I ſhould have three ſides meaſured, whereas he had but one and a piece. As ſoon as we had done, night approaching, I repaired home; where I found they, blundering, had taken the three lines all in one ſum juſt 100 chains. And I, being ambitious to match this mapmaker, have been puzzling to find a theorem leſs complex than his: Now if any of you could aſſiſt me to do this by a ſimple equation, I doubt not but his *F. R. S.* might be torn away from his name, and more deſervedly put to mine. The angle at $A = 50$ deg. at $B = 60°$; and at $C = 70°$. Angles from the ſpring to A and C, 120°; A, B, 130°; C, B, 110°.



The PRIZE QUESTION, *by Mr.* J. Turner.

A certain 'ſquire, whoſe fertile grounds
The ſilver-ſtreaming Meſbrook bounds,
Met me the other day, and ſaid I ſhou'd
Survey for him a ſmall adjacent wood.
With his requeſt I ſtraight comply'd,
And when got there, my ſkill I try'd;
But all in vain; ſo therefore crave
The ladies' help: * Below they have
Such data, as my friend and I
Cou'd then procure, moſt accurately.
And for her pains, ſhe who unties
The knot † (with B—gh—n's leave) ſhall have the prize.

* DVF and CVF are two different Apollonian parabolas; V the vertex, and VB the tranſverſe diameter of both; DB and AS, ordinates rightly applied: There is given the area DFV equal to $1473\frac{1}{2}$ poles, and $VF = FB$; F being the focus of the parabola DVB.



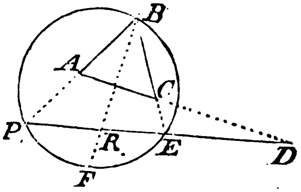
Again, VAC is the other Apollonian parabola, whoſe arch AV, is to the arch DV, as 1 to 2. And the ordinate AS (rightly applied) being let fall on the common axis VB, makes the area ASV a maximum. Required its parameter, and the area of the wood in ſtatute meaſure.

† By a ſimple equation.

1742.

Queſtions anſwered.

I. QUESTION 219 *anſwered by Mr.* R. Fall *of Dunbar.*



AS $PR+RE$ (281·2) is to $PR-RE$ (38), ſo is the tang. of half the ſum of the (oppoſite) angles P and E; to the tangent of half their difference (13° 10′); and half the ſum more $\frac{1}{2}$ the difference is equal to the greater angle ($PEB =$ 73° 10′); the half ſum (60) leſs $\frac{1}{2}$ difference is the leſſer angle ($EPB =$ 46° 50′.)

Then ſay, as ſine $\angle B$ (60°) : PE (281·2) : : s. P (46° 50′) : BE (236·9). Again, s. $\angle ECD$ (60) : ED (304) : : s. $\angle D$ (13·10) : EC (79·959); $\because BE - EC =$ 156·94 $=$ BC the ſide of the triangle required. Which ſquared and multiplied by $\sqrt{\frac{3}{4}}$ gives 10664·857 ſquare feet, the area of triangle ABC required.

There is a great many ways this problem may be ſolved, which I ſhall reſerve for another place, where the ſolutions to the queſtions in the Diaries ſhall be further diſcuſſed and illuſtrated. Only here obſerve, that by 3 Eucl. 6, the line BLR biſecting the $\angle B$, cuts the oppoſite ſide into two ſegments PR and RE, in proportion to the other two ſides (BP and PE), and conſequently the ſines of the angles they ſubtend in like ratio.

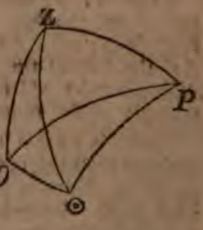
This queſtion was truly ſolved by Mr. *Da. Haſtings*, Mr. *F. Parrot*, *J. Corbet*, *G. Neal*, *R. Beighton*, *D. Boote*, *W. Kingſton*, *J. Jackſon*, *S. Bamfield*, *E. Croſs*, *J. Milbourn*, *T. Ramſay*, *Philoginus*, *J. Powle*, *R.* and *J. Robinſon*, *W. Spicer*, and Mr. *Tompſon*.

II. QUESTION 220 *anſwered by Mr. Ant. Thacker.*

As ſine 90° : coſine ſun's declin. (71° 47′) : : s. diff. of time (2 h. 15′) : 32° a fourth number = ☉*O* the diſtance in the parallel of declin. Then to the nat. coſ. of the diff. of azimuths (34° = ·829037) add radius, the ſum is 1·829037, which multiplied by the nat. coſ. 16° 31′ = ·958819) the difference of altitudes gives 1·753717082920, which made leſs by twice the coſine (32° = 2 × ·848048) = 1·696096, leaves ·057621082920; which divided by the verſed ſine of the difference of azimuths (34° = ·170963) quotes ·337433; which anſwers to the nat. ſine of 70° 18′; one half of which 35° 9′ made leſs by half the given difference of altit. (8° 15′) leaves 26° 54′ the altitude of the ſun at the firſt obſervation, and more by 8° 15′ is = 43° 24′ the altit. at ſecond obſervation. Whence there is now ſufficient given to find, by the common canons in trigonometry, the latitude = 57° 5′, and that the time of the firſt obſervation was at 7 h. 26′, and the ſecond at 9 h. 41′.

The analytical Inveſtigation of the aboveſaid Theorem.

Given ☉*P* = 71° 47′ comp. declination, the ∠ ☉*PO* = 33° 45′ diff. time, whence ☉*O* is found = 32°, whoſe coſine call m, put z = coſine ☉*ZO* the diff. azim. a and b for ſine and coſine of ½ diff. alt. then will $bb - aa$ or $1 - 2aa$, or $2bb - 1$ $(= n)$ be the coſine of the diff. of the altitudes. Alſo let x and y expreſs the ſine and coſine of half the ſum of the altitudes; then $yy - xx$ $(= v)$ will ſtand for the ſum, $xb + ya$ and $yb - ax$ the ſine and coſine of the greater altitude, $xb - ya$ and $yb + ax$ the ſine and coſine of the leſſer. Then by a theorem in page 176 of Simpſon's fluxions, $xxbb - aayy + yybbz - aaxxz = m$; or writing $1 - xx$ for yy, and reducing the equation we have $xx = \frac{m + aa - bbz}{1 - z}$



$$\therefore 1 - xx = \frac{1 - z - m - aa + bbz}{1 - z} = yy, \text{ and } \frac{1 - 2aa - z + 2bbz - 2m}{1 - z}$$

$$= yy - xx = \frac{1 - 2aa + \overline{2bb - 1} \times z - 2m}{1 - z} = \frac{n + nz - 2m}{1 - z}$$

=

$= \frac{n \times \overline{1+z} - 2m}{1-z} = v$ the coſine of the ſum of the altitudes. In numbers $\frac{{\cdot}9588 \times \overline{1 + {\cdot}8290} - 1{\cdot}6960}{1 - {\cdot}8290} = {\cdot}3375$, which anſwers in the tables to 70° 16′ for the ſum of the ſun's altitudes, whoſe half (35° 8′) added to the half diff. (8° 15′) gives 43° 23′) the greater altitude; whence the leſſer is = 26° 53′ and the latitude = 57° 7′. And the times of obſervation 7h. 27′ and 9h. 42′. *Q. E I.*

III. QUESTION 221 *anſwered by Mr.* Farrer.

Firſt (per p. 7 Simpſon's Laws of Chance) $\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times 6 = \frac{3125}{7776} =$ the probability that one face of a ſort comes up at one throw with 6 dice. Then $\frac{3125^4}{7776^4}$ is the probability of four faces of a ſort coming up at the firſt throw with ſix dice; and that of the contrary $\frac{7776^4 - 3125^4}{7776^4}$, and the odds $7776^4 - 3125^4$ to 3125^4, or as B to A. Then by page 13, $\frac{A^p \times B^{n-p}}{\overline{A+B}|^{n}} \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3}$ &c. to p, factors: here $n = 7$, and $p = 1$; which ſubſtitute in the above theorem, and it becomes $\frac{3125^4 \times \overline{7776^4 - 3125^4}|^6}{7776^7} \times 7 =$ the required probability $= {\cdot}8134$, and that on the contrary ·1866, and the odds 8134 to 1866, or as 4067 to 933, i. e. 2·1954 to 1.

Anſwered by Mr. Rob. Heath.

Raiſe the binomial a (1) + b (5) to the 6th power, the three firſt terms of which $(a^6 + 6a^5b + 15a^4b^2)$ will be the chances for 6, 5, and 4 aces to come up, which (in this caſe) being multiplied by 6, = 2436 chances for 6, 5, and 4 aces, duces, trays, &c. to come up at one throw; wherefore $\frac{44220^7}{46656^7}$ are the chances for failing to throw 6, 5, and 4 like faces in 7 trials; whence the wager's diſadvantage is 687033 to 312967, or 11 to 5 nearly, viz. 2·197 to 1.

The

The ſame anſwered by Merones.

The number of combinations of 4 aces out of 6 is 15; and in any one caſe of theſe 15, any two left out are capable of 25 variations where no ace is found; therefore 375 gives all the caſes where only 4 aces can be caſt. But ſince 5 or 6 aces may be caſt, the number of chances for theſe, which is 31, muſt be added; ∴ therefore all the caſes wherein 4 aces can be caſt with 6 dice, is 406. Now ſince there is the ſame variety for duces, trays, &c. therefore 2436 is the whole number of caſes wherein 4 points of any one ſort can be caſt: Let the whole number of chances $46656 = s$; $46656 - 2436 = q$; the odds againſt the thrower for 7 caſts will be as $\frac{q^7}{s^7}$ to $1 - \frac{q^7}{s^7}$, or as 2·1954 to 1.

Mr. *John May* has, from Mr. N. Struyck's method, given a very good ſolution to this problem; which are all the true ſolutions I have received.

IV. QUESTION 222 *anſwered by Mr.* J. May.

Let a be = ſine of ſun's alt. at the firſt obſervation; $b =$ that of the ſecond; $c =$ ſine of the hour angle from 6 o'clock at the firſt obſervation; $d =$ that at the ſecond; and let $r =$ radius; $b - a = p$; $d - c = q$; $r - c = h$; and $r + d = k$; then the ſine of the ſun's ſouthern altitude will be $a + \frac{hp}{q}$, the degrees of which put $= m$: Likewiſe the ſine of the ſun's depreſſion under the horizon in the north will be $\frac{kp}{q} - b$, whoſe degrees let be $= n$; then the ſun's declination will be $\frac{m - n}{2} = 20° 25'$ N. anſwering to the 11th day of May; and the latitude is $90° - \frac{1}{2}m - \frac{1}{2}n = 46° 58'$ north.

This queſtion was anſwered by Mr. *Boote* the propoſer, Mr. *Robinſon*, Mr. *N. Farrer*, &c. by a quadratic equation; and by *Merones*, Mr. *R. Robinſon*, Mr. *Ramſay*, Mr. *Heath*, Mr. *Turner*, Mr. *Poqule*, &c. by an equation of the fourth power. Alſo ſolutions were given by *Ens Rationalis*, Mr. *Bird*, Mr. *Pilgrim*, Mr. *Bamfield*, Mr. *Ant. Topham*, Mr. *Clarke*, Mr. *Howard*, and others.

But that we might ſhew what may be expected from the intended treatiſe of the diary queſtions, &c. where this and other queſtions will be ſolved by ſimple equations, which have their origin from the application of algebra to ſpherics, geometry, and all other branches of the mathematics; I will here give you the analytical inveſtigation of a theorem or two for ſolving this 222d queſtion.

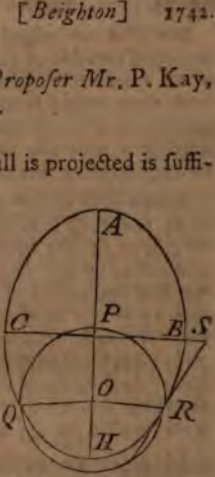
Given $Z\odot = 50^\circ\ 53'$, its coſ. $= b$; $ZO = 34^\circ\ 3'$, its coſ. $= a$; the $\angle\ \odot PZ = 54^\circ\ 0'$, its coſ. $= c$; $OPZ = 26^\circ\ 15'$, its coſ. $= d$; (ſee the laſt figure). Put x and y for the ſine and coſine of half the ſum of the arcs $\odot P$ and ZP, z and v for ſine and coſine of half their difference; then will $xv + yz$ and $yv - xz$ be the ſine and coſine of the greater arc $\odot P$; $xv - yz$ and $yv + xz$ thoſe of the leſſer ZP. And (by p. 176, Simpſon's Fluxions) $yyvv - xxzz + dxxvv - dzzyy = a$. And $yyvv - xxzz + cxxvv - czzyy = b$; and ſubſtituting $1 - zz = vv$, and redu. equ. we have $yy - zz + dxx - dzz = a$; and $yy - zz + cxx - czz = b$; and ſubſtituting $1 - xx$ for yy, we get $vv + zz = \frac{a - ca - b + db}{d - c}$ $= {\cdot}89449$ the coſine of the difference of the arcs ZP and $\odot P$ $(= 26^\circ\ 34')$ or the ſine of the ſun's meridian altitude, which anſwers to $63^\circ\ 26'$. Again, per equation above we have $\frac{b + db - a - ca}{d - c} = -{\cdot}38410, = yy - xx$, the coſine of the ſum of thoſe arcs $(112^\circ\ 36')$ or the ſine of the ſun's depreſſion at midnight, $= 22^\circ\ 36'$. Whence $90^\circ - \frac{63^\circ\ 26' + 22^\circ\ 36'}{2}$ $=$ latitude $46^\circ\ 59'$; and $\frac{63^\circ\ 26' - 22^\circ\ 36'}{2} = 20^\circ\ 25'$, the ſun's declination on the 11th day of May. *Q.E.I.*

THEOREM.

$$\frac{\text{s. } 90^\circ - \text{coſ. } 54^\circ \times \text{s. } 55^\circ\ 57' - \text{s. } 39^\circ\ 7'}{\text{coſ. } 26^\circ\ 15' - \text{coſ. } 54^\circ} + \text{s. } 39^\circ\ 7' = \text{s.}$$

ſun's alt. at noon. $\frac{\text{s. } 90^\circ + \text{coſ. } 54^\circ \times \text{s. } 55^\circ\ 57' - \text{s. } 39^\circ\ 7'}{\text{coſ. } 26^\circ\ 15' - \text{coſ. } 54^\circ}$ $-$ s. $39^\circ\ 7'$ $=$ s. ſun's depreſſion at midnight. Thence the meridian altitude is had $63^\circ\ 34'$; and the midnight depreſſion $22^\circ\ 34'$, half the difference of which is the ſun's declination; half of each ſubtracted from 90° is the latitude required.

V. QUESTION 223 *anſwered by the Propoſer Mr.* P. Kay, *and by* Merones.



Since the velocity with which the ball is projected is ſufficient to carry it over a ſpace of 6 miles per ſecond; and the velocity of the place of projection thro' the earth's rotation round its axis is 0·1802 miles per ſecond; the abſolute velocity compounded of theſe two will be 6·1115 miles per ſecond; and the angle which the true direction of the ball makes with the horizon 29° 24′, and the azimuth or bearing from the ſouth 46° 22′. Let *PQRO* repreſent the earth, *R* the place of projection, *ACHB* the required trajectory, *AOH* its tranſverſe axis, *CB* its conjugate, and *RS* a tangent to it at the point *R*. Putting $OR = d$; $6{\cdot}1115 = v$; nat. ſine of *ORS* (119° 24′) $= s$; radius $= 1$; and the diſtance deſcended in one ſecond, in parts of a mile, $= r$. Then, by p. 23 of Mr. Simpſon's Mathematical Eſſays, the tranſverſe *AH* will be $= \dfrac{d}{1 - \dfrac{vv}{4rd}} = 17165$ miles, the conjugate $CB = \dfrac{vs\sqrt{AH}}{\sqrt{r}} = 12633$ miles; and the time of one entire revolution = 4 h. 27 m. 13 s. from whence the time that the ball is in motion will be found 4 h. 6 m. 24 s. and the arch *RPQ* deſcribed, in the plane of a great circle round the center of the earth, in that time will be 209° 14′: but the angle, which that circle makes with the meridian, is found above to be 46° 22′; from which angle, and the two given ſides including it, the third ſide of the triangle, or the latitude of the place where the ball deſcends, may be found, and comes out 28° 16′ ſouth: Alſo the angle at the pole, from the ſolution of the ſame ſpherical triangle, will be 156° 30′, which being added to 61° 42′ the arch deſcribed by the earth about its axis during the time the ball is in motion, gives 218° 12′ weſterly, for the difference of longitude required.

Merones's *Anſwer*.

To anſwer every part of this problem at large, would require too much room: I ſhall therefore only explain the method of calculation:

1. Compounding the earth's motion, with the projectile's motion, I find its true velocity to be 6·11228, at an angle of elevation (above the horizon ſuppoſed at reſt) 29° 23½′, making an angle with the meridian 46° 22½′.

2. By propoſition 15 and 17 Princip. I. the latus of the orbit is = 9301·53; the tranſverſe axis of the ellipſis = 17252·42; and the periodic time = 4 h. 29 m.

3. By meaſuring the area of the ellipſis, and part cut off by the earth's radius; the time of the flight above the earth will be found 4 h. 8 m. 32·4 s. in which time it comprehends an arch of the earth (between its riſing and falling) = 208° 37′.

4. Therefore, by ſpherical trigonometry, the body falls in ſouth latitude 28° 48′; and the difference of longitude eaſt is 226° 29′, from the point of projection ſuppoſed at reſt.

5. But ſince the earth's motion in the time of the flight, transfers the place of projection 62° 23′ eaſtwards; therefore the place the body falls in will be in 164° 6′ eaſt longitude from the place of projection.

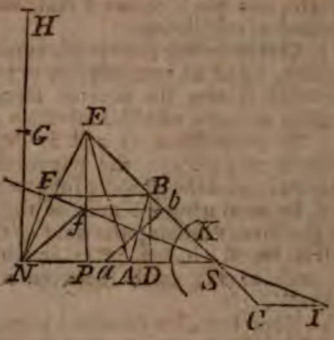
Mr. *Dunthorne*, Mr. *Ramſhay*, and others have attempted the ſolution, but not with the ſame ſucceſs. In a queſtion ſo curious and intricate, it is very ſurprizing, that two perſons, at ſo great a diſtance as above 200 miles, ſhould ſo preciſely agree in the numbers: This ſhews, that, in theſe things, Mr. *Simpſon*'s Eſſays, &c. come up to the ſame perfection of the wonderful Sir *Iſaac Newton*'s Principia.

VI. QUES-

I. QUESTION 219 *Conſtructed*. [Which by miſtake was omitted in page 135]

CONSTRUCTION. Upon PE deſcribe the ſegment of a circle capable of containing an $\angle$ of 60°; and having completed the circle, from F, the middle of the lower ſegment, through R draw a line to cut the circumference in B; join the points P, B and E, B, and $\perp$ FB draw DL, cutting EB and PB in C and A, and ABC will be the triangle whoſe area is required.——For the $\angle ABC$ being, by conſtruction, = 60°, the $\angle FBE$ = the half of it or 30°, and the $\angle DLB$ a right one, the $\angle BCL$ will alſo be = 60°, and of courſe the $\triangle ABC$ equilateral.

* VI. QUESTION 224 *anſwered by Mr.* W. Daniel.



From an analytic proceſs (which ſhall be publiſhed elſewhere) is deduced this theorem. The ſq. root of $\frac{796^2}{25^2 - 5^2} + 25^2$; that is, $\sqrt{1681{\cdot}026} = 41$ nearly, made more by (5) half the compared diſtances, make 46 leagues, the diſtance between the ſouth and middle port; and leſs by (5) half the compared diſtance $= 36$, between the middle and north port: therefore 82 leag. is the diſtance run. The area 796 ÷ by 25 gives $31{\cdot}84 = PE$ the perpendicular. Which ſquared and taken from

* VI. QUESTION 224.

CONSTRUCTION. Since the perpendicular of the $\triangle\ NES$ is given, being a 3d proportional to $\frac{1}{2}\ NS$ and the ſide of the ſquare expreſſing the given area, the triangle itſelf may be conſtructed by prob. 76 of SIMPSON's *Algebra*, 2d ed. Or, perhaps, more elegantly thus: At $N \perp$ to NS erect $NG =$ the given perpendicular, and continue it to H ſo that $GH = NG$; with center S and radius $SK = SE - NE$ deſcribe a circle; then by prob. XII of Mr. LAWSON's *Tangencies*, find the center of a circle which ſhall touch that already deſcribed, and likewiſe paſs through the points N and H, and that center will be the vertex of the triangle.—— By either of theſe methods the $\triangle$ being deſcribed; on ES produced and parallel to NS take SC and CI proportional to the uniform velocities with which the ſhips move from S and N; then on an indefinite line drawn through the points D, S, let fall the $\perp\ NF$, and complete the parallelogram $NFBA$, ſo will BA be the *minimum* ſought.——For conſtituting any other parallelogram $Nfba$; then ſince, by ſim. $\triangle$s, $Sb : bf$ or (*Euc.* I. 34) $Na :: SC : CI$, and likewiſe $SB : BF = NA :: SC : CI$, the points b and a, B and A will be contemporaneous poſitions of the two ſhips; and by *Euc.* I. 18, Nf or ba is greater than NF or BA.

Had BA been required of a given length, NF need only have *been taken equal* to that length, and, it is manifeſt, the remainder *of the conſtruction* would have been performed as above.

from the ſquare of SE, leaves $1102{\cdot}2144$, whoſe root is $33{\cdot}2 = SP$. Let B and A repreſent two cotemporary poſitions of the ſhips; and their motion at B to that at P; as 4 to $5\frac{1}{2}$, or as 1 to $1{\cdot}375$, and we have this theorem: $SB = \frac{SN \times \overline{1{\cdot}375\,SE + SE}}{1{\cdot}375^2\,SE + 2 \times 1{\cdot}375\,SA + SE} = 21{\cdot}95$. And as $1 : 1{\cdot}375 :: 21{\cdot}95 : 30{\cdot}18 = SP$, which taken from SN, leaves $19{\cdot}82 = PN$. Alſo $SP - SA$ leaves $13{\cdot}38 = PA$. And by 47 Euc. 1, $AE = 34{\cdot}11$ leagues the diſtance of the firſt ſhip from the middle port, when ſhe brings her ſtarboard tacks on board. Then by trigonometry, $\angle NSE = 43^\circ\ 47'$. Therefore the courſe of the ſhip S to E was N. E. $1^\circ\ 13'$ northerly; and the $\angle PEN = 27^\circ\ 51'$: $\therefore$ the ſhip bears from E to N, W.N.W. $5^\circ\ 21'$ northerly. Laſtly, the $\angle AEP = 21^\circ\ 53'$. Hence the port A bears from E, W.S.W. $0^\circ\ 37'$ weſterly.

The Propoſer, Mr. Farrer's *Anſwer.*

Let $2m = SN = 50$ leagues; $2n = SE - EN = 10$; $2y = SE + EN$ ſought. Then $m + y = \frac{1}{2}$ ſum of the 3 ſides, $n + y = SE$; $y - n = EN$; thence having the 3 ſides the ſquare area will be $m + y \times y - m \times m - n \times m + n$; that is, $y^2 - m^2 = \frac{A^2}{m^2 - n^2}$; ergo, $y = \sqrt{\frac{A^2}{m^2 - n^2}} = 41$ leagues; and $SE = 46$ leagues, and $EN = 36$. Hence the courſe on the larboard tacks is N. $43^\circ\ 48'$ E. and the diſtance 46 leag. and her courſe on the ſtarboard N. $62^\circ\ 11'$ W. and the diſtance 36 leagues.

Now, let A and B repreſent the two ſhips at their neareſt approach, and let fall the perpendicular BD; and put $q =$ ſine $\angle S$; $p =$ its coſine; $s = 5{\cdot}5$; $r = 4$ miles; $d = SN$; and $z = AN$ ſought. Then $d - z = AS$; and (per queſt.) $s : r :: z : \frac{rz}{s} = SB$; and (by trig.) $1 : \frac{rz}{s} :: p : \frac{prz}{s} = SD$; and $1 : \frac{rz}{s} :: q : \frac{qrz}{s} = DB$; then $SA - SD = AD = \frac{sd - sz - prz}{s}$; ſubſtitute $h = 1 + \frac{pr}{s}$; then $AD = d - hz$: And (by 47 Euc. 1.) $AD^2 + DB^2 = AB^2 = \frac{qqrrzz}{ss} + dd - 2dhz + hhzz$, which muſt be a minimum, and its fluxion $\frac{2qqrrz\dot{z}}{ss} - 2dh\dot{z} + 2hhz\dot{z} = 0$.

This reduced gives $z = \frac{dhzz}{qqrrhhss} = 29{\cdot}5698$ leagues $= AN$; and $SA = 20{\cdot}4320$ leagues; $SB = 21{\cdot}5053$ leagues. And the middle port bears from the firſt ſhip, when ſhe brings her ſtarboard tacks on board, ſouth 66° 52′ weſterly, and diſtant 34·62 leagues. *Q. E. I.*

This queſtion was anſwered by Mr. *R. Heath*, Mr. *Robert Robinſon*, Mr. *John Jackſon*, Mr. *J. Terey*, Mr. *Edw. Croſs*, Mr. *T. Ramſhay*, Mr. *J. Powle*, Mr. *J. Turner*, Mr. *W. Spicer*, Mr. *T. Bird*, Mr. *Walter Trott*, Mr. *Ri. Gibbons*, *Ens Rationalis*, Mr. *J. May*, Mr. *J. Hemmingway*, Mr. *Rich. Piercy*, Mr. *Proctor*, and others.

* VII. QUESTION 225 *anſwered by Mr.* John Watts.

The ſum of the three ſides (per queſt.) $= 100$,

$$\text{The angles} \left\{ \begin{array}{l} CAB = 50^\circ \\ ABC = 60 \\ ACB = 70 \end{array} \right\} \begin{array}{l} \text{and from} \\ \text{the point} \\ \odot \text{ within} \end{array} \left\{ \begin{array}{l} C\odot B = 110^\circ \\ C\odot A = 120 \\ A\odot B = 130 \end{array} \right.$$

Call CB unity (1); then per trig. as $s.\ \angle A : 1 :: s.\ \angle C : AB = 1{\cdot}2266$, and $s.\ \angle A : 1 :: s.\ \angle B : AC = 1{\cdot}1305$. And the ſum of theſe three (proportional) ſides is $= 3{\cdot}3671$. Whence by ratio, or proportion,

$$\left.\begin{array}{l} \text{As } 3{\cdot}3671 : 1 :: 100 : 29{\cdot}786 = BC \\ \quad 3{\cdot}3671 : 1{\cdot}2266 :: 100 : 36{\cdot}538 = BA \\ \quad 3{\cdot}3671 : 1{\cdot}1305 :: 100 : 33{\cdot}674 = AC \end{array}\right\}$$

Let x and m expreſs the number of degrees in the angles $AC\odot$ and $BC\odot$ reſpectively; n and y, thoſe of $BA\odot$ and $CA\odot$; v and s, the angles $AB\odot$ and $CB\odot$. Then $x + m = 70^\circ$, per queſt. And $m + 110 + s = 180$ (by 17 Euc. 1.) $\therefore m + s = 70^\circ$; conſequently x is $= s$. Again, $y + n = 50$; and $n + 130 + v = 180$; $\therefore n + v = 50$; conſequently $v = y$. Whence if x and y denote the ſine and coſine of the $\angle CB\odot$; c and m that of ABC; then will $cy - xm$ expreſs the ſine of the angle $AB\odot$ or $CA\odot$.

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* VII. QUESTION 225.

The $\triangle ABC$ itſelf may be determined by the conſtruction given to prob. 44 in the *Mathematician*, or by firſt deſcribing a triangle ſimilar to it; after which, if on AB and AC two ſegments of circles *capable of containing* $\angle$s = to 130° and 120° be deſcribed, their *point of interſection* will, it is plain, give the place of the ſpring.

And (per trig.) s. $\angle C\odot B\ (n) : CB\ (k) :: $ s. $\angle CB\odot\ (x) : \frac{xk}{n} = \odot C$; s. $\angle A\odot C\ (c) : CA\ (p) ::$ s. $CA\odot\ (cy - xm) : \frac{pcy - pxm}{c} = \odot C$. Conſequently $\frac{pcy - pxm}{c} = \frac{xk}{n}$; which, out of fractions, is $pcny - pxmn = xkc$; $\therefore pcny = xkc + pxmn$; and $\therefore \frac{y}{x} = \frac{kc + pmn}{pcn} = \frac{k}{pn} + \frac{m}{c}$, the cotangent of $\angle CB\odot$. And $\frac{y}{x} = 1{\cdot}51866$ &c. anſwering in the tables to 33° 21′. Therefore the angle $AB\odot = 26^\circ\ 41'$; whence (per trig.) is found,

	Chains	Poles			a.	r.	p.
$\odot A =$	21·383	= 85·5283	The area of	$A\odot B =$	15	1	38·96
$\odot B =$	18·917	= 75·6709		$B\odot C =$	15	1	39·19
$\odot C =$	17·431	= 69·7242		$C\odot A =$	16	0	22·31

And the content of the whole field $ABC = 47$ 0 20·46

Mr. John Jackſon's *Anſwer.*

Calling $AC\ (a)$; $AB\ (b)$; $BC\ (c)$. And finding the ſides as above. Putting $m =$ s. $A\odot C = CBA$, and $d =$ coſine; $n =$ s. $\angle C\odot B$; and $s =$ s. $A\odot B$. Then $y =$ s. $\angle B\odot A = CA\odot$; $\sqrt{1 - y^2} = z =$ coſine; and tang. $= \sqrt{\frac{1 - y^2}{y^2}} = q$; $x =$ s. $CB\odot = AC\odot$. Then $n : c :: x : \frac{cx}{n} = C\odot$; and $m : a :: y : \frac{ay}{m} = C\odot$; conſequently $\frac{cx}{n} = \frac{ay}{m}$, $\therefore x = \frac{any}{cm}$, and $mz - dy = \frac{any}{cm}$; tranſpoſed, $mz = \frac{any}{cm} + dy = \frac{any + cmdy}{cm}$, and $z = \frac{any + dcmy}{cm^2}$, $\frac{z}{y} = \frac{an + dcm}{cm^2} = \sqrt{\frac{1 - y^2}{y^2}}$; conſequently $q = \frac{an + dcm}{cm^2} = 1{\cdot}999 = 63^\circ\ 25'$, $\therefore B\odot A = 26^\circ\ 35'$. Whence the lines and areas as above.

This queſtion is very elegantly ſolved by Mr. *Turner* and Mr. *Farrer*, who ſay there's one of the ſame ſort in *Ronayne*'s Algebra, ſolved there two ways, but in a tedious and clumſey manner. Mr. *R. Robinſon*, Mr. *T. Robinſon*, Mr. *Bamfield*, Mr. *Williams*, Mr. *Chapple*, Mr. *Ramſay*, Mr. *Powle*, Mr. *Heath*, Mr. *May*, Mr. *Bird*, Mr. *Haſtings*, Mr. *Kingſton*, Mr. *Trott*, Mr. *Pilgrim*, Mr. *Gibbons*, and ſome others, have in various methods given ſolutions to this queſtion.

The PRIZE QUESTION *ſolved by Mr.* Ant. Thacker.

Putting $x = FV$ the focal diſtance, and proceeding with the nature of the parabola, we get $x = 25 = FV$, $\therefore VB = 50$, and $DB = 70{\cdot}71$.

For any variable abſciſſa put x; and y for its correſponding ſemi-ordinate; p for the latus rectum; then will $px = yy$ expreſs the nature of the curve DV; $\therefore \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{\dot{y}}{p}\sqrt{pp + 4yy} =$ the fluxion of the arc DV; whoſe fluent is $\frac{y}{2p}\sqrt{pp + 4yy} + \frac{p}{4} \times \mathrm{L}.\ \frac{2y + \sqrt{pp + 4yy}}{p} =$ the length of the variable arc, which when $y = 70{\cdot}71$ is $89{\cdot}8914 = DV$, the half of which $44{\cdot}9457$ is $=$ the arc VA; which call z, and put $\frac{2y + \sqrt{pp + 4yy}}{p} = h$; then will $\frac{h\ p - p}{hh} + 4p\ \mathrm{L}.\ h = 16z$; and $\frac{\overline{hh - 1}\,|^{3}}{h^{3}} \times pp = 64yx$; or if for p be put a^3, then (1ſt) $\frac{h^4a^3 - a^3}{h^2} + 4a^3\ \mathrm{L}.\ h = 16z$; and (2d) $\frac{hhaa - aa}{h} = 4\sqrt[3]{yx}$; whence per (1) equation $\dot{a} = \frac{2\dot{h}a + 4a\dot{h}h^2 + 2\dot{h}h^4a}{3h - 3h^5 - 12h^3\ \mathrm{L}.\ h}$; and (2d) $\dot{a} = \frac{\dot{h}a + \dot{h}h^2a}{2h - 2h^3}$; conſequently $\frac{2 + 4h\ + 2h^{4}}{3 - 3h^4 - 12h^2\ \mathrm{L}.h} = \frac{1 + hh}{2 - 2hh}$; hence $h^6 + h^4 - 12h^4\ \mathrm{L}.\ h - 12h^2\ \mathrm{L}.\ h - 1 = 0$; which in fluxions, and each term divided by $\dot{h}$, gives $6h^5 - 8h^3 - 4h^3\ \mathrm{L}.\ h - 24h\ \mathrm{L}.\ h - 14h = A$. And proceeding according to caſe 1ſt, p. 81 of Simpſon's Eſſays, we get (if no miſtake be made) $h = 4{\cdot}133926$; and from the equation $\frac{h^4p - p}{hh} + 4p\ \mathrm{L}.\ h = 16z$, is had $p = \frac{16hhz}{h^4 - 1 + 4hh\ \mathrm{L}.\ h} = 31{\cdot}6$ &c. And the area $VSA = 616$ nearly.

Merones *anſwers it thus*:

In the parabola DV we have $2VB^2 = DB^2$, and $\frac{2}{3}VB \times DB = 1473\frac{1}{8}$; whence $DB^2 = 4999{\cdot}92$; $VB^2 = 2499{\cdot}96$.

In any parabola, where $x =$ abſciſſa, $y =$ ordinate, $a =$ latus rectum; the flux. curve $= \dot{y}\sqrt{1 + \frac{4yy}{aa}}$; and the curve $= \sqrt{\frac{1}{4}yy + \frac{y^4}{aa}} + \frac{1}{4}a \times \text{H.L.}\ \frac{2y}{a} + \sqrt{1 + \frac{4yy}{aa}}$; therefore $VD = 89{\cdot}8918$, and $VA = 44{\cdot}9459$. Let a parabola be ſuch that $xy = 1$, and the curve a minimum; expunge a out of the foregoing expreſſion of the curve, and put $z = \text{H.L. of}\ \frac{2y}{a} + \sqrt{1 + \frac{4yy}{aa}}$; and then $\sqrt{\frac{1}{4}yy + \frac{1}{yy}} + \frac{y^3z}{4}$ is a minimum; the fluxion of this made $= 0$, there comes out $z = \frac{2}{3yy}\sqrt{1 + \frac{4}{yy}}$; or $\frac{2xx}{3}\sqrt{1 + 4x^4} = \text{H. L.}\ 2xx + \sqrt{1 + 4x^4}$; whence by reverſion of ſeries, $x = {\cdot}986408$; $y = 1{\cdot}01378$; the curve $= 1{\cdot}4787$; and in the parabola AV, where the curve is $44{\cdot}9459$; we ſhall have $VS = 29{\cdot}9823$; $SA = 30{\cdot}8142$; and the area $VAS = 615{\cdot}921$.

This queſtion has exerciſed the faculties of a great number of perſons verſed in the moſt abſtruſe and higher parts of the mathematics, has occaſioned a good deal of ſpeculation and controverſy; and the beſt of artiſts have been doubtful, whether it is poſſible to be ſolved by any ſcientific method; nor can I apprehend I have received any ſuch ſolutions, unleſs theſe two above.

The latter by *Merones*, a perſon ſo profound in theſe ſciences, that he is equal to the moſt arduous taſk; by his difference between the abſciſs and ordinate ſeems to be right, yet do not readily enough comprehend his proceſs: And as I have long wiſh'd I could diſcover who he is, or how to direct to him; I would now heartily beg that favour, and that he would pleaſe a little further to exemplify how the expreſſion above, $\sqrt{\frac{1}{4}yy + \frac{1}{yy}} + \frac{y^3z}{4}$ put into fluxions makes $z = \frac{2xx}{3}\sqrt{1 + 4x^4}$.

The propoſer Mr. *Turner*, Mr. *Heath*, Mr. *Farrer*, Mr. *Dunthorne*, and ſeveral others (in their ſolutions) make the abſciſſa and ordinate equal, which though it does not much differ from the truth, yet is not really ſo.*

Mr.

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*PRIZE QUESTION.

The Diary Author having as above expreſſed a deſire of having ſome parts of the ſolution by *Merones* (Mr. *Emerſon*) explained to him, and it no where appearing, that I know of, that he received any farther account of it; I think it my *duty*, as an *Editor*, to ſupply a particular explanation of the ſaid ſolution, notwithſtanding the Editor of the Repoſitory has thought fit to paſs this Queſtion by in ſilence; though his declared intention at the beginning was to 'ſupply defects by rendering the Queſtions perfectly clear, and their ſubſequent anſwers as eaſy to be underſtood as the nature of the ſubject will admit.'

The firſt principle of the ſolution is to find the curve a minimum under a given area, inſtead of the area a maximum and the curve given, as in the queſtion, which is the ſame thing. Mr. EMERSON aſſumes another curve ſimilar to the curve required, whoſe area is $\frac{2}{3}$, or $xy = 1$; and if $a =$ the parameter, then $ax = y^2$; from theſe two equations we have $x = \frac{1}{y}$, and $a = \frac{y^2}{x} = y^3$; alſo $\frac{y}{a} = \frac{y}{y^3} = \frac{1}{y^2} = xx$: Now the general expreſſion for the length of the curve being $\sqrt{\frac{1}{4}y^2 + \frac{y^4}{a^2}} + \frac{1}{4}a \times \text{H. L.}\ \frac{2y}{a} + \sqrt{1 + \frac{4yy}{aa}}$, by writing y^3 for a in this expreſſion, it becomes $\sqrt{\frac{1}{4}y^2 + \frac{1}{y^2}} + \frac{y^3}{4} \times \text{H. L.}\ \frac{2}{y^2} + \sqrt{1 + \frac{4}{y^4}} =$ a minimum; the fluxion of which being found in the common way, and made $= 0$, the equation gives z or $\text{H. L.}\ \frac{2}{y^2} + \sqrt{1 + \frac{4}{y^4}} = \frac{2}{yy}\sqrt{1 + \frac{4}{y^4}}$; that is (putting x inſtead of $\frac{1}{y}$) $\frac{2xx}{3}\sqrt{1 + 4x^4} = \text{H. L.}\ 2x^2 + \sqrt{1 + 4x^4}$, which thrown into a ſeries, and reverted, x is found; thence y, and the curve. Then, by ſim. figures, he proportions *thus, as the length of the curve thus found is to the length given in the queſtion, ſo is* x and y here found, to the abſciſſa and ordi*nate required.*

	DV	*VA*	S*A* *SV*	*SVA*
Mr. *Turner*	89·89	44·945	30·39	615·7
Mr. *Heath*	89·8914	44·9457	30·3905	615·721
Mr. *Dunthorne*	———	44·94	30·39	615·7
Merones	———	44·9459	*SA* 30·814 *VS* 29·982	615·92

If any ratio could be found between the ordinates and abſciſſas, then this problem might be ſolved by a ſimple equation; but as the nature of ſuch curves do not admit of that, it ſeems impracticable.

Of the Eclipſes in 1742.

Within the ſphere of the earth's orbit will happen four eclipſes; two of the ſun, and two of the moon; but all inviſible in our iſland. The firſt of the moon, May the 8th, near noon; the ſecond of the ſun, May the 22d, at 12 h. 30′ p. m. the third of the moon, November the 1ſt, 27′ p. m. and the fourth of the ſun, November the 15th, at 18 h. 16′ p. m.

New Queſtions.

I. Question 226, *by Mr.* Tho. Ramſay.

I'm in love with a damſel who has beauty in ſtore,
Beſides a large portion:—Her charms I adore;
Her eſteem and affections at length I've obtain'd,
But her father's conſent, as yet, is not gain'd,
Unleſs the conditions below I fulfil,
Which is a taſk far ſurmounting my ſkill.
He inſiſts that a method I to him make known,
How to find the content of a field of his own,
Without any more data than what he deſcribes,
I muſt find the ſaid * area, as alſo the ſides.
A triangular form is the ſhape of the ground;
The hedges as ſtraight as moſt are to be found:
Without the ſaid field grows an oak and a pine,
Betwixt which ſaid trees, if one ſuppoſe a ſtreight line,
To one of the ſides it would parallel lie,
As he ſays himſelf did accurately try.
If the field's longeſt ſide extended ſhould be,
Juſt eighteen chains farther, it would touch the oak tree.

---

* *The area to be a maximum.*

The pine in a ſtreight line lies with the third ſide,
Diſtant exactly twelve chains, when 'twas try'd:
Moreover the diſtance betwixt the ſaid trees
Is juſt forty chains.——And now, ladies, pleaſe
Some aſſiſtance to lend, my content it will prove:
Delays being dangerous in matters of love.

II. QUESTION 227, *by Mr.* Peter Kay.

Suppoſing a pendulum, whoſe length is 29·2 inches, and its bob or weight 27 pounds; to find at what part of the rod of that pendulum, a weight of one pound muſt be fixed, ſo as to have the greateſt effect in accelerating the pendulum; or, ſo that the time of the vibration may be the ſhorteſt poſſible? The rod itſelf being ſuppoſed void of gravity.

III. QUESTION 228, *by Mr.* J. May.

The great inundations we have had here lately in Holland, has laid above ſix hundred thouſand acres of land under water; and hath ruined and waſhed away the boundaries, that it is almoſt impoſſible again to determine each man's poſſeſſions; but to help a friend, and prevent diſputes, your aſſiſtance is deſired.

He had a piece of land, *AFPVD*, which was divided into two equal parts by the right line *VF*; of which the part *ADVF* was a geometrical ſquare; and the other part *VPF* an apollonian parabola, *V* the vertex, and *F* its focus: but all was defaced except the ſide *AD*; which we with ſome difficulty meaſured fifty-two chains. So that to fix the boundaries again there is required the lengths of the ſides *VP* and *FP*: It is alſo expected that the points *V*, *F*, *P*, &c. be determined by a geometrical conſtruction.

IV. QUESTION 229, *by* Hurlothrumbo.

Suppoſing an homogenous fluid, equal in denſity and magnitude to the earth, to revolve uniformly about an axis; ſo that the greateſt diameter thereof may be juſt double the axis from pole to pole; to find the time of one entire revolution; with a general theorem for the ſolution of other queſtions of this nature.

V. QUESTION 230, *by Mr.* John Turner.

Suppoſe the bung diameter of a ſpheroidal caſk were 40 inches, and its diagonal 48 inches; it is required to find the head diameter of the leaſt ſpheroidal caſk poſſible, having the aboveſaid dimenſion; and its content in ale gallons?

VI. QUESTION 231, *by Mr.* N. Farrer.

On the 20th of December, 1740, at ten minutes paſt eight at night, I obſerved two noted fixed ſtars on the meridian; the difference of their altitudes thirty-ſeven degrees; and at ten o'clock I found their azimuths 31° 40′ and 61° 45′, both weſt, and their difference of altitudes 32°. Quere the latitude of the place of obſervation?

VII. QUESTION 232, *by Mr.* Ant. Thacker.

If $x^3 + y^3 - 945yx = 0$ expreſs the nature of the curve AM, and 150400 be the area of the ſpace AMP; it is required to find the area of the greateſt parallelogram that can be inſcribed in the ſaid figure AMP?

VIII. QUESTION 233, *by Mr.* Rich. Piercy.

If $x^n \times \overline{n-x}|^x = y$; when n is $= 1000$, required the value of x, y being the greateſt number that poſſibly can be.

IX. QUESTION 234.

A father at his death bequeathed to his daughters theſe portions, viz.

To the eldeſt he gave $\cdot 2^2$ of 1000 pounds.
To the ſecond $\cdot 3^3$ of 1000 pounds.
To the youngeſt $\cdot 4^4$ of 1000 pounds.
How much was each daughters's portion?

The PRIZE QUESTION, *by Mr.* Heath.

Vincit amor patriæ.——

Where, with his fleet, our noble patriot ſails,
Succeſs o'er all his conduct ſtill prevails;
Intent on public wrongs, he ſtems the tide
Of Spain's oppreſſion, inſolence, and pride:

Brave, generous-minded, not to be withſtood,
A hero conqu'ring for his country's good.
Nor could vain boaſts their Porto Bella ſave,
He took it, as he was reſolv'd to have:
His thund'ring cannon rend the liquid ſky,
Hot iron balls the iron caſtle try,
And ſtorms of ruin round the harbour fly.
Pour'd on the land, behold each honeſt tar,
With ſword in hand, aſcending to the war,
Their country's wrongs are boiling in each breaſt,
Bold in revenge, dauntleſs, all forward preſs'd.
The Spaniards, aw'd, and in a deadly fright,
Shrink back for refuge to the woods in flight,
And leave the Engliſh maſters of the fight,
While loud huzzas from deck and ſhore reſound,
And the glad victor with applauſe is crown'd!
Th' obſcuring ſmoak uncurls itſelf in air,
And Britiſh heroes bright in arms appear!
The conqu'ror now, as wiſdom does approve,
Divides his treaſure, and withal his love:
With love and joy each Britiſh boſom burns,
And in new conqueſts all expreſs returns.
Chagre is conquer'd——America does ſhake,
Spain for her wrongs muſt reſtitution make;
From ſhore to ſhore the victor bears command,
And Vernon! Vernon! rings through all the land!
Whoſe actions an immortal record claim,
Amongſt his ſhining anceſtors of fame.
  Late * news I've heard, ladies, I wiſh it true,
  And as your hearts are there, ſo muſt all you.

The QUESTION.

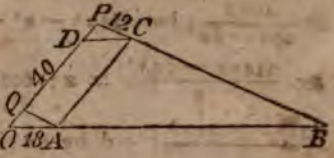
* Admiral Vernon ſailing on a ſouth courſe, from Jamaica to Carthagena, ſees *Don Blaſs* right before him, ſteering due weſt, along the ſhore. He now continually bears directly upon him, in a right line; when coming up with him, it appears that the *Don* had ſailed 8 leagues during the chace, and that the ſaid admiral was 7 leagues diſtant from him when the chace began: Now, ſuppoſing each ſhip's motion to be uniform during the whole chace, to find from thence the diſtance ſailed by Admiral Vernon.

1743.

Queſtions anſwered.

I. QUESTION 226 *anſwered by* Hurlothrumbo.

LET AQ be parallel to CP; $OP = a = 40$; $OA = b = 18$; $AQ = (PC) = c = 12$, and $OQ = x$. Then the area of the $\triangle AOQ$ is $\sqrt{\frac{2b^2c^2 + 2b^2x^2 + 2c^2x^2 - b^4 - c^4 - x^4}{16}}$; therefore, as $xx\ (OQ^2) : \overline{a-x}|^2\ (AC^2) ::$ the ſaid area : the area of the $\triangle ABC$, which, by the queſtion, is a maximum; $\therefore \frac{\overline{a-x}|^4}{x^4} \times : 2b^2c^2 + 2b^2x^2 + 2c^2x^2 - b^4 - c^4 - x^4$ is alſo a maximum: Whence $a \times \overline{bb-cc}|^2 - \overline{bb+cc} \times ax^2 - \overline{bb+cc} \times x^3 + x^5 = 0$: from which the value of x is found $= 7{\cdot}6986$; $AC = 32{\cdot}3014$; $BC = 50{\cdot}3473$; $AB = 75{\cdot}52$; and the area 61 a. 2 r. 9 p.



Merones's *Anſwer.*

Draw DC parallel to AB; let $PC = b$, $OA = c$, $PO = a$, $PD = x$. Then $BC = b \times \frac{a-x}{x}$, $AB = c \times \frac{a-x}{x}$, $AC = x \times \frac{a-x}{x} = a - x$; and let $s = cc - bb$, $t = cc + bb$: Then, by the rule for finding the area from the three ſides, $\frac{1}{4}\sqrt{\frac{-ss + 2txx - x^4}{x^4}} \times \overline{a-x}|^2 =$ area a max. this in fluxions, and reduced, gives $x^5 - tx^3 - atx^2 + ass = 0$, and $x = 7{\cdot}698$, and the area $= 615{\cdot}57$.

This queſtion is truly ſolved by the propoſer Mr. *T. Ramſay*, Mr. *Ant. Thacker*, Mr. *Hemmingway*, Mr. *Farrer*, and *Tho. Cowper*, by a proceſs much alike. But as I was at ſome trouble in ſolving it in a method different from any of them, before the receipt of theirs, I will, for variety's ſake, here give it, though I don't think it at all better.

$AQ=PC=12$; $AO=18$; $OQ=2x$. The ſum of the three ſides is $30+2x$, half ſum $=15+x$, the difference between half ſum and three ſides are $3+x$, $x-3$, and $15-x$; then $\sqrt{\overline{15+x}\times\overline{15-x}\times\overline{x+3}\times\overline{x-3}}$, i. e. $\sqrt{234x^2-x^4-2025}=z$, the area of OAQ. But as $xx:z::\overline{20-x}|^2:z\times\dfrac{\overline{20-x}|^2}{xx}=$ area of the $\triangle\, ABC=$ a maximum; which in fluxions is $\dot{z}\times\overline{20-x}|^2-2\dot{x}z\times\overline{20-x}\times xx-2\dot{x}z\times\overline{20-x}|^2=0$; which divided by $x\times\overline{20-x}$, gives $\dot{z}\times\overline{20-x}-2\dot{x}z\times x-2\dot{x}z\times\overline{20-x}=0$; or $20\dot{z}x-\dot{z}xx-40\dot{x}z=0$; $\therefore 20\dot{z}x-\dot{z}xx=40\dot{x}z$; $\therefore \dot{z}=\dfrac{40\dot{x}z}{20x-xx}$; but $234x^2-x^4-2025=z^2$; in fluxions, &c. $\dfrac{234\dot{x}x-2\dot{x}x^3}{z}=\dot{z}$; conſequently $=\dfrac{40\dot{x}z}{20x-xx}=\dfrac{234\dot{x}x-2\dot{x}x^3}{z}$; ÷d by $2\dot{x}$ gives $\dfrac{20z}{20x-xx}=\dfrac{117x-x^3}{z}$; $\therefore 20zz=\overline{117x-x^3}\times\overline{20x-x^2}$, or $20\times\overline{234x^2-x^4-2025}=\overline{117x-x^3}\times\overline{20x-x^2}$; in numbers is $4680x^2-20x^4-40500=2340x-20x^4-117x^3+x^5$; ordered, is $x^5-117x^3-2340x^2+40500=0$; from whence x is found $=3{\cdot}8493$, and $2x=7{\cdot}6986=OQ$.

II. QUESTION 227 *anſwered by Mr.* Kay.

Put $a=$ the length of the pendulum $=29{\cdot}2$ inches; $w=$ weight of the bob 27 pounds; $v=$ the weight to be fixed to the rod $=1$ pound; and $x=$ the required diſtance thereof from the point of ſuſpenſion; then the diſtance of the center of oſcillation from that point will be $\dfrac{aaw+xxv}{aw+xv}$, which by the queſtion ought to be a minimum; and therefore $2x\dot{x}v\times\overline{aw+xv}-v\dot{x}\times\overline{aaw+xxv}=0$, whence x is $=a\times\dfrac{\sqrt{ww+wv}-w}{v}=14{\cdot}4686$ inches.

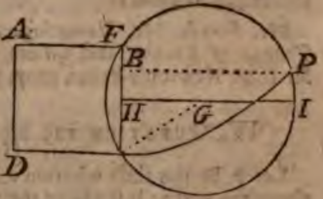
Mr. J. Watts.

The momenta of all moving bodies are as a rectangle of their celerity and maſs, and the celerity of a pendulum as its length, &c. Then the center of oſcillation of any compound pendulum (by Colin Mac Laurin's Fluxions, lib. 2, p. 453) is equal to the ſquares of the diſtances multiplied into their reſpective weights, and their aggregate divided by the ſum of the momenta; therefore [the anſwer as above].

This queſtion was anſwered by Mr. *Ramſay*, Mr. *Farrer*, Mr. *May*, Mr. *Sharp*, and *Hurlothrumbo*.

III. QUESTION 228 *anſwered by* Hurlothrumbo

Upon the given line AD deſcribe the ſquare $ADVF$, and from the focus F, and vertex V, the parabola VP; biſect FV with the perpendicular HG, and take $HG = \frac{3}{4} AD$, and from the center G with the radius GV deſcribe the circle VPF, and from the point P, where it cuts the parabola, draw FP, then will $ADVPFA$ (by book 1 prop. 30 Newton's Princip.) be the figure required.



[*In this fig. a line drawn from* F *to* P *is omitted, which the reader may ſupply.*]

Anſwered by Mr. Farrer.

Let $m = VF = 52$ chains, $x = VB$; then $m - x = BF$, and, per conics, $PB = 2\sqrt{mx}$, and the area of $VPF = \frac{4x}{3}\sqrt{mx} + \overline{m-x}\sqrt{mx} = mm$; this equation reduced is $x^3 + 6mx^2 + 9m^2x - 9m^3 = 0$; hence $x = 34{\cdot}77163$, $BP = 85{\cdot}044$, $FP = 86{\cdot}77157$, and the arch $VP = 93{\cdot}7442$ chains. And the points V, P, and F, will be determined by the following conſtruction.

Let $HG = \frac{3}{4}$ of VF biſect at right angles the ſide of the ſquare, and on the center G, with GF deſcribe the circle, which will cut the curve in P.

Mr. J. May *the Propoſer*,

After an analytic anſwer, gives this geometrical conſtruction: Having $xx = 4ay$ from the property of the parabola, and $yx = 6aa - 3ax$; then $yxx = 4ayy$, and $yxx = 6aax - 3axx$, whence $4ayy = 6aax - 3axx$, divided by $4a$ is $yy = \frac{3}{2}ax - \frac{3}{4}xx$, for $\frac{3}{4}xx$ put its value $3ay$, we have yy $\frac{3}{2}ax - 3ay$, to which add $xx = 4ay$, is $yy + xx = \frac{3}{2}ax + ay$ an equation to a circle, and $y = \frac{1}{2}a \pm \sqrt{\frac{1}{4}aa + \frac{3}{2}ax}$; put the ſurd $= 0$, then is $y = \frac{1}{2}a$; biſect FV in H, and draw HGI perpendicular to VF, in which the center of the circle will be; then $x = \frac{3}{4}a \mp \sqrt{\frac{13}{16}aa}$; make $HG = \frac{3}{4}a$; draw VG, this is $\sqrt{\frac{13}{16}aa}$ and the center is G; put $x = 0$, then $yy = ay + 0$, and $y = \frac{1}{2}a \mp \frac{1}{2}a$, or $y = 0$, or $= a$, then V, F are two points in the periphery. On the center G with GV, draw the circle which cuts the given parabola in the required points.

Mr. *Powle*, Mr. *Hemmingway*, Mr. *Ramſay*, Mr. *Watts*, *Teague of Exeter*, and others anſwered this, (which is in Sir Iſaac Newton's Princ. prop. 30.)

IV. QUESTION 229 *anſwered by the Propoſer.*

Let r be the time wherein a body would deſcribe a circle about the earth, juſt above the ſurface, by means of its own gravity $=$ 1h. 24′ 45″, and let a be the arch of a circle whoſe radius is 1, and ſecant n, ſuppoſing the given ratio of the equatorial diameter to the axis be as n to 1. Then by Simpſon's Mathematical Diſſertations, we ſhall have $r \times \sqrt{\dfrac{\overline{2n^2-2} \times \sqrt{nn-1}}{\overline{nn+2} \times \overline{3a - 9\sqrt{nn-1}}}}$ for the exact time of one entire revolution; which therefore, when $n = 2$, or the equatorial diameter is juſt double the axis, will be 2h. 31′ 20″.

Merones's *Anſwer.*

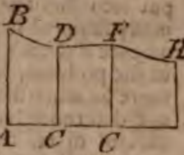
Let $r =$ earth's radius, $f = 16\frac{1}{12}$ feet. That the equinoctial diameter may be double the polar one, the centrifugal force muſt take away half the gravity; to do which, any point in the equinoctial muſt in 1″ deſcribe the arch whoſe verſed ſine is $\frac{1}{2}f$; or, which is the ſame thing, the arch itſelf will be $\sqrt{rf}$; and the periodic time $3{\cdot}1416\sqrt{\frac{4r}{f}}$ $=$

$= 7147'' = 2$ h. ferè; and therefore the body will revolve 12 times as faſt as our earth. But to give an accurate ſolution to this problem, the decreaſe of gravity ariſing from the earth's figure, ought to be taken into conſideration; but this would render the calculus very intricate, and too long for this place.

V. QUESTION 230 *anſwered by* Hurlothrumbo.

Put the diagonal $= 48 = a$, half the bung diameter $= 20 = b$, and half the head diameter $= x$; then the content will be $\frac{2bb+xx}{1077} \times 8\sqrt{aa-\overline{b+x}^2}$ ale gallons, which by the queſtion is to be a minimum. Now by making the fluxion of $\frac{2bb+xx}{1077} \times 8\sqrt{aa-\overline{b+x}^2}$, or that of $\overline{2bb+xx}^2 \times \overline{aa-\overline{b+x}^2} = 0$; we get $2a^2x - 2b^3 - 4b^2x - 5bx^2 - 3x^3 = 0$; that is, in numbers $-16000 + 3008x - 100x^2 - 3x^3 = 0$; whence $x = 7{\cdot}85$, or $x = 12{\cdot}63$, or $x = -53{\cdot}8$, but none of their roots is the required value of x.

For let $BDFH$ be a curve, whoſe abſciſſa AC is x, and ordinate $\overline{2b^2+x^2}^2 \times \overline{aa-\overline{b+x}^2}$, and it is evident that when the ordinate of this curve is a minimum, the caſk will be ſo too; but it appears, from what has been found above, that the ordinate CD grows leſs and leſs, till x or AC becomes $7{\cdot}85$ (becauſe till then its fluxion, or $-16000 - 3008x - 100x^2 - 3x^3$, is a negative quantity) after which it increaſes till x becomes $= 12{\cdot}63$ (the fluxion being affirmative) and then decreaſes again continually, till x arrives at 20, its greateſt value; in which circumſtance it will be leſs than in any former poſition, as will eaſily appear upon trial; therefore the head diameter is equal to the bung diameter indefinitely near, and the required content $236{\cdot}468$ ale gallons.



Mr. N. Farrer's *Anſwer.*

If $m = 40$, $n = 48$, $y =$ head diameter; then $\frac{m+y}{2}$ is the baſe of the right-angled triangle, and (per 47 Euc. 1) $\sqrt{\frac{4nn-mm-2my-yy}{4}} =$ ſemi-length. Let $4nn - mm$

$= d$; then its length is $\sqrt{\overline{d-2my-yy}}$, and its ſolidity $= \overline{2mm+yy} \times \sqrt{\overline{d-2my-yy}} \times {}^{\cdot}2618$; whoſe fluxion

$$2y\dot{y} \times \sqrt{\overline{d-2my-yy}} + \overline{2mm+yy} \times \frac{-m\dot{y}-y\dot{y}}{\sqrt{\overline{d-2my-yy}}}$$

$= 0$. Reduced, $3y^3 + 5my^2 + 2mmy - 2dy + 2m^3 = 0$. Hence $y = 15{\cdot}701465$ = head diameter, 78·187894 = length; 250·174 ale gall. the content. And farther obſerves,

1. The caſk will be greateſt when it becomes a ſpheroid, (whoſe length is then 87·266 inches, and content 259·28 a. g.) and leaſt when a cylinder (length 53·066, content 236·46 a. g.); therefore the queſtion, properly ſpeaking, does not admit of either maximum or minimum, for the leaſt ſpheroidal caſk will be infinitely near the cylinder, and greateſt near the ſpheroid. 2. If the length be 76, the content is 252·18 a. g. If 84 inches long, the content is 252·85, between theſe two the leaſt is that found above. 3. Between this leaſt and the cylinder, there is another, whoſe capacity is a maximum.

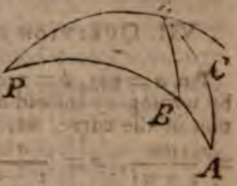
Mr. Hemingway,

After his anſwer, remarks, That when any expreſſion is put into fluxions, the roots of the equation determine ſo many limits of increaſe and decreaſe of the flowing quantity; whence if there's but one root which anſwers the conditions of the problem, we obtain a maximum or minimum; but if there be more than one, the limits thoſe roots exhibit may not determine either. In the preſent caſe, if the ſemi-diff. of diam. $= x$ increaſes gradually from its firſt value 7·3749, till it arrives at its ſecond value 12·149, the caſk continually decreaſes from 250·87 to 250·174; but x ſtill increaſing, the caſk will increaſe again, till it becomes a whole ſpheroid, whoſe content is 259·259 a. g. Again, let x gradually decreaſe from its firſt value, and the caſk will decreaſe likewiſe, till it becomes a cylinder, whoſe length is 53·066 inches, and content 236·47, which is the minimum, and may be conſidered as the middle fruſtum of a ſpheroid of an infinite length.

This queſtion was anſwered by Mr. *Watts*, Mr. *Bamfield*, Mr. *Richardſon*, Mr. *Ramſay*, Mr. *Wright*, Mr. *Robinſon*, Mr. *Oats*, Mr. *Ant. Thacker*, and others.

VI. QUESTION 231 *ſolved.*

This queſtion happened to be over-limited, the latter given time of obſervation being ſuperfluous, for it will not agree with the reſt of the data; (the hour angle may be only 26° 30′, and the times of obſervation 14m. after 8 and 10 o'clock).



Given $\begin{cases} AB = 37° = \text{1ſt diff. alt.} \\ AZ - BZ = 32° = \text{2d diff. alt.} \\ AZB = 30° \; 5' = \text{diff. azim.} \\ BZP = 118° \; 15' = \text{zenith} \angle. \\ ZPB = 26° \; 30' = \text{hour} \angle. \end{cases}$

To find $AZ + BZ$ and PZ.

Put x and y for the ſine and coſine of half the difference of AZ and BZ, s and q for thoſe of half their ſum. Then, by our new theorem, $sy + qx$ and $qy - sx$ is the ſine and coſine of ZA, and $sy - qx$ and $qy + sx$ thoſe of ZB; alſo let z repreſent the coſine of AZB, and d the coſine of AB. Then, by 1ſt theorem, $\overline{qy - sx} \times \overline{qy + sx} + \overline{sy + qx} \times \overline{sy - qx} \times z = d$; which when multiplied is $qqyy - ssxx + zssyy - zqqxx = d$, or $dqq + dss$; being tranſpoſed, $zssyy - ssxx - dss = dqq + zqqxx - qqyy$, $\therefore \frac{zyy - xx - d}{d + xxz - yy} = \frac{qq}{ss}$; to which add 1, its $\frac{zyy - xx - d}{d + zxx - yy} + 1 = \frac{qq}{ss} + 1 = \frac{qq + ss}{ss}$, or $\frac{z - 1}{d + zxx - yy} = \frac{1}{ss}$; $\therefore ss = \frac{d + zxx - yy}{z - 1}$, or $\frac{yy - d - xxz}{1 - z}$; $\therefore 2ss = \frac{2yy - 2d - 2xxz}{1 - z}$ = the verſed ſine of $AZ + BZ$.

Operation.

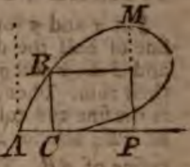
74°	16
74	16
148	32 Sum
Whoſe ver. s. is 1·8480481 $= 2yy$	Whoſe verſ. s. is ·1519519 $= 2xx$
Sub. $2d + 2xxz =$ 1·7287543	multip. by $z =$ ·8652972
$2yy - 2d - 2xxz =$ ·1192934 rem.	gives $2xxz =$ ·13148337
this ÷d by $1 - z =$ ·1337028 gives	add $2d =$ 1·5972710
the quotient = ·8856064.	Sum $2d + 2xxz =$ 1·7287543

which gives = 83° 25′ 53″ = $AZ + BZ$, the half of which 41° 42′ 56″ added to half the diff. 16°, gives 57° 42′ 56″ = AZ the greater ſide, and ſubtracted, leaves 25° 42′ 56″ = BZ.

BZ the leſſer ſide; then as s. 37° : s. 30° 5′ :: s. 57° 42′ 56″ : s. 44° 44′ $= PBZ$; again, as s. 26° 30′ : s. 25° 42′ 56″ :: s. 44° 44′ : s. 43° 13′ $= PZ$ the complement of the latitude, ∴ 36° 47′ is the latitude required.

This queſtion has been truly anſwered by Mr. *W. Daniel*, Mr. *Joſ. Spilſbury* of *Birmingham*, Mr. *John Worth*, and Mr. *Francis Parrot.*

VII. QUESTION 225 *anſwered by* Hurlothrumbo.



Put $a = 945$, $b = 150400$, $PM = y$, $AP = x = vy$; then, by writing vy inſtead of x in the equation of the curve, &c. we ſhall have $y = \frac{av}{1+v^3}$, $x = \frac{av^2}{1+v^3}$, and $y\dot{x} = \frac{2a^2v^2\dot{v} - a^2v^5\dot{v}}{\overline{1+v^3}|^3}$; whoſe fluent is $\frac{a^2}{6} - \frac{a^2}{2\times\overline{1+v^3}|^2} + \frac{a^3}{3\times\overline{1+v^3}}$, which, by reſtoring x and y, will be $\frac{aa}{6} + \frac{xy}{2} - \frac{ayy}{6x}$ or $\frac{xy}{2} + \frac{axx}{6y}$. Now by putting $z = 1 + v^3$, $\frac{aa}{6} - \frac{aa}{2\times\overline{1+v^3}|^2} + \frac{aa}{3\times\overline{1+v^3}} = b$, and $c = \frac{6b}{aa} - 1$, we ſhall get $z = \frac{1-\sqrt{1-3c}}{c} = 1{\cdot}512$; whence $v = 0{\cdot}8$, $y = 500 = PM$, and $x = 400 = AP$. Hence the area of the greateſt inſcribed parallelogram PB, will be found $= 92133$.

Note, This curve, having its equation affected alike by x and y, returns into itſelf, and has its convex and concave parts exactly ſimilar to each other.

VIII. QUESTION 233 *anſwered by Mr.* J. Watts.

Since $x^n \times \overline{n-x}|^x$ is to be a maximum, its h. l. $n \times$ l. x + $x \times$ l. $\overline{n-x}$ = max. in fluxions $\frac{n\dot{x}}{x} + \dot{x} \times$ l. $\overline{n-x} - \frac{x\dot{x}}{n-x}$ $= 0$; which divided by $\dot{x}$ is $\frac{n}{x}$ + l. $\overline{n-x} - \frac{x}{n-x} = 0$, i. e. $\frac{1000}{x}$ + l. $\overline{1000-x} - \frac{x}{1000-x} = 0$. By trials it is found x

muſt be greater than 800, ∴ put $800+z=x$, which ſubſtituted above, gives $\frac{1000}{800+z}+\mathrm{l.}\,\overline{1000-800-z}-\frac{800+z}{1000-800-z}=0$; contracted, it is $\frac{1000}{800+z}+\mathrm{l.}\,\overline{200-z}-\frac{800+z}{200-z}=0$; out of fractions, is $200000-1000z+\overline{160000-600z-zz}\times\mathrm{l.}\,\overline{200-z}-640000-1600z-zz=0$; or $-440000-2600z-zz+\overline{160000-600z-zz}\times\mathrm{l.}\,\overline{200-z}=0$; ſigns changed, $440000+2600z+zz-\overline{160000-600z-zz}\times\mathrm{l.}\,\overline{200-z}=0$; ordered by infinite ſeries, is $440000+2600z+zz-\overline{160000+600z+zz}\times:\mathrm{l.}\,200,-\frac{z}{200}-\frac{z^2}{80000}-\frac{z^3}{24000000}-\frac{z^4}{6400000000}$, &c. $=0$; or finding the hyp. log. of 200, viz. $5{\cdot}2983$, &c. its $440000+2600z+zz-\overline{160000-600z-zz}\times:5{\cdot}2983-\frac{z}{200}-\frac{z^2}{8000}-\frac{z^3}{24000000}$, &c. $=0$; that is, $-407728+6578{\cdot}98z+5{\cdot}2983z^2-{\cdot}005833z^3-{\cdot}0000125z^4=0$; or $6578{\cdot}98z+5{\cdot}2983z^2-{\cdot}005833z^3-{\cdot}0000125z^4=407728$; which divided by the coefficient of z, (per Simpſon's Flux. p. 101) gives $z+{\cdot}0008054z^2-0000008z^3-{\cdot}000000001z^4=61{\cdot}9742$, compared with the ſeries, in the ſame page, we have $b={\cdot}0008054$, $c={\cdot}0000008$, $d={\cdot}000000001$, &c. and $z=61{\cdot}9742$ which call y, then will $z=y-by^2+\overline{2bb-c}\times y^3$ &c. $=59{\cdot}37973$, which added to 800, gives $x=859{\cdot}37973$. *Q.E.I.*

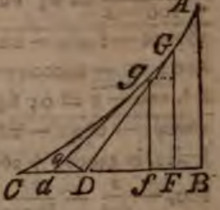
IX. QUESTION 234 *anſwered by Mr.* J. Watts.

Subtract the log. of $2=0{\cdot}3010300$ from the log. of $10=1{\cdot}0000000$, the remainder is $0{\cdot}69897000$, which multiplied by 2, is ${\cdot}1397940=$ the log. of ${\cdot}2^{\cdot 2}$, which ſubtract from unity, and the difference is ${\cdot}8602060$, which ſought in the logarithm tables, gives the abſolute number anſwering ${\cdot}72478$. And proceeding in like manner with ${\cdot}3^{\cdot 3}$ and ${\cdot}4^{\cdot 4}$, we ſhall have the eldeſt daughter's portion $724{\cdot}78=724$ l. 16 s. 4 d. the 2d $=696{\cdot}84=696$ l. 16 s. 11 d. and the youngeſt $=693{\cdot}14=693$ l. 2 s. 8 d.

This queſtion was alſo anſwered by *Hurlothrumbo*, Mr. *Powle*, Mr. *Seyth*, Mr. *Ramſay*, Mr. *Farrer*, Mr. *Bamfield*, Mr. *May*, Mr. *E. Ruſton*, Mr. *W. Miles*, and ſeveral others.

The PRIZE QUESTION *anſwered by* Me.ones.

Let $CB = b = 8$, $AB = c = 7$; let G, g be two points in the curve infinitely near each other, draw GF and gf parallel to AB, and to the points G, g draw the tangents GD, gd, and Do perpendicular to gd. Let $BD = x$, $BF = v$, $GD = y$, $AG = z$, and $z = ax$, and conſequently $\dot{z} = a\dot{x}$, where a is a conſtant, but unknown, quantity. By ſim. triangles $\dot{z} : \dot{v} :: \dot{x} \, (= \frac{\dot{z}}{a}) : do = \frac{\dot{v}}{a}$; then y is increaſed by do, and decreaſed by Gg, that is, $\dot{y} = \frac{\dot{v}}{a} - \dot{z}$; or $\dot{z} = -\dot{y} + \frac{\dot{v}}{a}$; and the fluent is $z = c - y + \frac{v}{a} = ax$; and when they arrive at C, $y = 0$, and $v = x = b$; whence $c + \frac{b}{a} = ab$; and $a = c + \frac{\sqrt{cc + 4bb}}{2b} = 1{\cdot}529$; and therefore the whole curve $z = c + \frac{b}{a} = 12{\cdot}2321$ leagues the diſtance ſailed by Admiral Vernon.



Mr. Powle

Takes notice that this queſtion is a particular caſe of prob. 8 p. 170 Simpſon's Fluxions. And if $\frac{1}{x}$ be the ratio of the velocities of the two ſhips, and $AB = a$, and $BC = b$; then from the nature of the curve we have $\frac{ax}{1 - xx} = BC$, and $\frac{a}{1 - xx} = AC$ ſought: Now the equation $\frac{ax}{1 - xx} = b$ will give $x = \frac{\sqrt{4bb + aa}}{2b} - \frac{a}{2b}$, $\therefore \frac{ax}{1 - xx} = \frac{2bb}{\sqrt{4bb + aa} - a} = 12{\cdot}2324 = AC$ required.

Mr. *Ant. Thacker*, Mr. *Scyth*, Mr. *Croſs*, Mr. *Ramſay*, Mr. *Farrer*, Mr. *Moore*, Mr. *Watts*, Mr. *May*, Mr. *Robinſon*, Mr. *W. Hanbury*, jun. Mr. *Jo. Spilſbury*, and ſeveral *others have* anſwered this queſtion.

Of

Of the Eclipſes in 1743.

Within the ſphere of the earth's orbit will happen ſix eclipſes this year; three times will the moon, in her wandering courſe, interpoſe and hide the ſplendour of the ſun from falling upon the earth, or its atmoſphere; and three times will the earth in its annual elliptic motion, be ſo full in a line between the ſun and moon, as to hinder her receiving the light ſhe borrows from the ſun, to enlighten the earth by reflection.

1. Sun ecliped April 13, at 10 morning, inviſible.

2. Moon eclipſed April 27, inviſible.

3. Sun eclipſed May 12, at 5 morning, inviſible.

4. Moon eclipſed on thurſday the 22d day of October, at a quarter paſt 3 in the morning, total and viſible.

Calculated by	Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig.
Aſtron. Carolina, Coventry	1 31	3 17	5 3	3 32	22 c
An. Amanuenſis, London	1 48	3 54	5 29	3 30	
Jo. Taylor, Snaith, Yorkſhire	0 34	3 25	5 15	3 42	22 19
J. Williams, Whitford, Flint.	1 10	3 0	4 52	3 42	22 c
P. Timkin, London	1 29	3 17	5 6	3 36	
Ben. Claridge, Warwick	1 29	3 16	5 2	3 34	21 44
Will. Wollams, Oxford	1 24	3 10	4 57	3 33	20 14
John Thornet, Glouceſter	1 30	3 16	5 3	3 33	20 15
T. Cowper, Wellingborough	1 18	3 5	4 52	3 34	21 25
S. Bamfield, Honiton, Devon.	1 16	3 2	4 48	3 32	21 42
Ni. Farrer, Sunderland	1 18	3 6	4 53		
Alexandria	3 23	5 11	6 5	3 35	21 51
Cape Farewell	10 33	12 21	2 8		
W. Honor, Burcheſter	1 19	3 8	4 54	3 34	21 5c
Ruſhden	1 23	3 10	4 58		
John Brown, Bp. Auckland	1 30	3 18	5 5	3 35	21 5c
C. Facer, Benſon	1 26	3 13	4 59		22 [illegible]
Tho. Wade, London	1 23	3 11	4 58	3 35	21 5c
Ens Rationalis, Newcaſtle	1 14	3 1	4 48	3 34	21 34
M. Rafton, Hartlepool	1 20	3 7	4 55	3 35	21 5
T. Robinſon, London	1 23	3 11	4 58		
Morton Hill	1 18	3 6	4 53	3 35	22 [illegible]
Nic. Oats, Topſham	1 14	3 4	4 52	3 38	21 54
Teague of Oxford, Greenwich	1 23		4 58		

The beginning of total darkneſs 2. 28. End 4. 6.

The other two eclipſes are very inconſiderable.

The paſſage of Mercury over the Sun, October 25.*

	Beg.	Mid.	End	Dur.
Joſeph Taylor	9 25	11 37	1 43	4 18
Peter Timkin, for London	11 6	0 21	1 16	2 9
T. Cowper, Wellingborough	8 36	10 48	1 0¼	4 23
Annus Amanuenſis, London	8 54	11 40	2 30	4 23
T. Wade, London	8 35	10 51	1 7	4 32
Ens Rationalis, Newcaſtle	8 23	10 40	0 56	4 33

New

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* *This* TRANSIT *of* MERCURY

Was obſerved at *London* by Dr. BEVIS thus:

Temp. Ap.	h	m.	s.	
Oct. 24.	20	28	57	The 1ſt exterior contact.
		29	48½	The ingreſs of the center.
		30	40	The 1ſt inter. contact.
25.	1	0	33	The laſt inter. contact.
		1	24½	The egreſs of the center.
		2	16	The laſt ext. contact.

Alſo at *London* by Mr. GEO. GRAHAM.

h	m.	s.	
1	0	42	The laſt inter. contact.
	2	16	The laſt ext. contact.

And at *Gieſen* by Mr. CHRIST. LEWES GERSTEN.

h.	m.	True time.
9	4	Central ingreſs.
1	37	Central egreſs.

A COMET

Was obſerved, of a remarkable ſize, in the month of December this year, and in the months of Jan. and Feb. the year following. From the obſervations Mr. JOS. BETTS determined that the place of the aſcending node was in ♉ 15° 45′ 20″; the log. of the perihelion diſt. 9·346472; the log. of the diurnal motion 0·940920; the place of the perihelion ♎ 17° 12′ 55″; the diſtance of the perihelion from the node 151° 27′ 35″; the log ſine and coſine of the inclination of the orbit to the ecliptic 9·865138 and 9·832616: and thence the time the comet was in the vertex of the parabola, or the time of the perihelion, Feb. 19d 8h. 12m. the motion of the comet in its orbit thus ſituated, was direct, or according to the *order of the* ſigns. This amazing comet was not leſs than the

earth;

New Queſtions.

I. QUESTION 235, *by Mr.* Thomas Cowper.

When that immenſe amazing orb of day
Gilded the northern tropic with his ray,
From eaſtern ſkies emitting lucid ſtreams,
And ſpread his radiance in prolific beams;
In pleaſing ſolitude I did repair
To view the fields, and breathe in purer air;
Where joyous birds ſtretch'd forth their tuneful throats,
And pierc'd the yielding air with thrilling notes:
On verdant ſprays the thruſh and blackbird ſings,
While warbling larks diſplay'd aërial wings;

Artful

---

earth; and travelled with the wonderful velocity of 13 millions of miles in a day, or 9 thouſand miles in a minute; but when in its perihelion the rapidity of its motion exceeded that of lightning itſelf; and drew after it a tail of fire of above 16 degrees or 23 millions of miles in length.

Another COMET

Alſo appeared in the beginning of this year, but I do not meet with any determination of its elements.

A COMET

Was obſerved in the ſpring of the year 1742 by ſeveral perſons.

From the obſervations made at *Pekin* in *China*, Mr. JAMES HODGSON computed theſe following articles: It is manifeſt, he ſays, that the comet came to the equator March 3d about 6 h. *ant. merid.* and that it paſſed in r. aſc. 282° 30′, with inclination of its path to the equator 84° 30′ very nearly; and therefore that its longitude was 13° 35′ in ♑ with N. lat. 22° 54′. Hence we may collect, that the path of the comet, which did not ſeem to deviate from a great circle, met the ecliptic in ♑ and ♋ 9° 19′ with incl. of 80°: and the colure of the equinoxes in the diſtance of 50° $37\frac{1}{2}'$ from the poles of the world toward the equinoctial points with the angle of incl. 77° $33\frac{1}{2}'$: and the colure of the ſolſtices in the diſt. of 23° $57\frac{1}{3}'$ from the poles of the world toward the ſolſticial points with ang. of incl. 13° 38′ equal to the greateſt elongation of its orbit from the ſame colure in the adverſe part, and to the diſt. of the poles of the orbit from the equinoctial points.

[*This Note was by miſtake omitted in its proper place laſt year.*]

Artful aſcending through the elaſtic way,
And hail'd triumphant the ſolſtitial day.
Each object round a grateful ſcene did yield,
While teeming plenty crown'd the blooming field,
And beauteous verdure deck'd the enamell'd plain,
And pearly dews hung on the rip'ning grain;
The fragrant flowers with diff'rent colours dy'd,
On ſmiling ground diſplay'd their gaudy pride:
The air with light effluvia did abound,
Which ſpread their aromatic odours round:
The winds were huſh'd, and zephyr's gentle breeze
Scarce heard to murmur through the ſhady trees.
To one of them a bough did appertain,
Whoſe ſhadow I obſerv'd upon the plain;
And found its diſtance from the bough to be,
One hundred inches wanting only three.
As towards the ſouth bright Phœbus journied on,
And on the earth with greater luſtre ſhone,
While up the ſkies progreſſive time he led,
Seventy-ſix minutes and one-fifth were fled.
It happen'd that another branch I found,
Whoſe altitude above the level ground
Was thirteen inches and juſt eight-tenths more
Than that which I obſerv'd the time before;
Yet, as the firſt, its diſtance from the ſhade,
No more than ninety-ſeven inches made;
And th' baſe was ſhorter at this ſecond view,
By one foot and ſix-tenths of one inch too.
Now, from this data, 'tis requir'd to ſhow
Their height*, the time, and latitude alſo?

* i. e. The perpendicular height of the end of each bough, from which the ſhadows were projected and meaſured.

N. B. That 17 minutes muſt be deducted from the ſun's altitude at the time of the firſt obſervation, and 16½ at the ſecond, as an allowance for his ſemidiameter and refraction.

II. QUESTION 236, *by* Hurlothrumbo.

If two bodies, *L* and *T*, whoſe maſſes are reſpectively equal to thoſe of the moon and earth, were projected at the ſame time, and in the ſame plane, from two places, *A* and *B*, at the diſtance of a hundred thouſand miles from each other; the former, *L*, with a velocity of 5 miles per ſecond, making an angle with *AB* of 100 degrees, and the latter with a velocity of 2 miles per ſecond, an angle (on the ſame ſide *AB*) of 60 degrees; it is required to find the diſtance *and poſition* of the two bodies with reſpect to each other, alſo with reſpect to the points *A* and *B*, after they have been 48 hours in motion, ſuppoſing them, when in motion, to be only acted upon by each other.

III. QUESTION 237, *by Mr.* Nich. Oats.

—— *Hic validum Vernon cano magnâ ſorte juvanti.*

Free from domeſtic broils and homeward jars,
The hero ruſhes in the din of wars;
On proud Iberia's coaſt, in dread alarms,
Appears the fury of Britannia's arms.
Each honeſt heart with indignation burns,
And long-call'd vengeance on the inſulter turns.
The elements are mix'd, with fire the main;
Deſtruction's hurl'd through the indignant plain:
The watchful ſquadrons triumph o'er the deep,
Whilſt thunder-ſtruck the daſtard tribes can't peep,
Nor ſcarce a petty-auger from her moorings creep.
Him conqueſt ſtill attend, and let your ſongs
Blazon his triumphs, num'rous as our wrongs:
So ſhall our ſhips in diſtant climates roam,
And bring the wealth of Peru's Indies home.
Then let each heart with gratitude be fraught,
Each ſailor love, who for his country fought.
Ladies, ere long (believe the muſes true)
Diſplay your wit, the conq'rors you'll ſubdue.

The QUESTION.

A fleet of ſhips at Portſmouth is bound with military ſtores, &c. for our brave admiral in the Weſt Indies; and being informed, by experienced navigators, that a ſhip, in ſailing upon a wind, having her larboard tacks on board, which makes her way good ſix miles an hour, will, when got into a trade wind (which blows in the latitude of thirty) make her way nine miles an hour; now admit the fleet can ſail at the rate above, I demand the courſe and diſtance, before and after their arrival in the trade wind, to be performed in the ſhorteſt time poſſible, from the Lizard to Jamaica, and the minimum, according to Wright's projection. Lizard in lat. 49° 56′, long. 5° 14′ weſt, Jamaica 18° and 76°.

IV. QUESTION 238, *by Mr.* Peter Kay.

To find in what arc of a circle a pendulum muſt vibrate, ſo that the time of one whole deſcent ſhall be equal to the time in which a heavy body would fall along the chord of the ſame arc.

V. QUESTION 239, *by Mr.* Robert Heath.

Four maids, wiſe and fair, and as Aſtrea rare,
(Impatient to tie Hymen s nooſe)
Burn with amorous flame for an artiſt of fame:
Then which for her age would you chooſe?

Suppoſing their ſeveral ages to be repreſented by the following equations.

$$a \times : ee + uu + yy = 20850.$$
$$e \times : aa + uu + yy = 23238.$$
$$u \times : aa + ee + yy = 24654.$$
$$y \times : aa + ee + uu = 24750.$$

What was each of their ages, and the analytical inveſtigation?

VI. QUESTION 240, *by* F. R. S.

Let there be the fruſtum of a cone, whoſe leſs diameter is 20 inches, its greater 40, and length 90; which being cut by a plane diagonally through the contrary extremities of its two diameters, will divide it into two parts (called hoofs) a greater and a leſs. There is required a ſcientific theorem for finding the ſolid content of each part; there being none yet given by any author (except perhaps thoſe got from a tedious ſeries) which will give the contents preciſely true, when at the ſame time we have found, by an analytical method, the true ſolidity of each. All which may be made fully appear, and ſhall be demonſtrated in the next year's Diary.

VII. QUESTION 241, *by Mr.* William Daniel.

It is univerſally agreed, that the heat at any moment of time, on any day, is proportionable to the rectangle made of the ſine of the ſun's altitude, and the arc of time expreſſing his continuance above the horizon: which being allowed, it is required to find what time of the day will be the hotteſt at Coventry (lat. 52° 30′) on Auguſt 14, 1743.

N. B. This is one of the two problems which Mr. Stone (in his tranſlation of L'Hoſpital's Fluxions) challenges the mathematicians of Europe to anſwer.

VIII. QUESTION 242, *by Mr.* J. May, *jun.*

Laſt ſpring, 1742, being at ſea in north latitude, we had great ſtorms for ſeveral days together, ſucceeded with cloudy weather, which hindered our making obſervations; at laſt, when it cleared up, with Mr. Hadley's octant, upon deck, I endeavoured to take the ſun's meridian altitude, but unfortunately thick clouds prevented it. Now being at a great loſs to know where we were, we endeavoured to contrive ſome other way; and accordingly waiting a few minutes, it cleared up again, and we took the ſun's altitude (after allowance for refraction and dip of horizon, &c.) 57° 24′ 52″; tarrying 26 minutes by a good watch, we obſerved the ſun's height 55° 35′ 19″; and 25 min. after this, the ſun's altitude was 53° 16′ 15″. From whence is required the latitude we were in, the time of each obſervation, and the ſun's declination, by a general theorem for all problems of this nature.

IX. Question 243, *by Mr.* John Powle.

Let there be three ſpherical and perfectly elaſtic bodies, *A*, *B*, *C*: the weight of *A* = 3 pounds, and of *C* = 27. Now it is required to find the weight of the intermediate body *B*, ſo that *A* ſtriking *B* at reſt, and *B*, with the motion acquired by the ſtroke, ſtriking *C* at reſt, the motion produced in *C*, ſhall be a maximum?

Prize Question, *by the late illuſtrious Sir* I. Newton.

Three ſtaves being erected, or ſet up on end, in ſome certain place of the earth, perpendicular to the plane of the horizon, in the points *A*, *B*, and *C*; whereof that which is at *A*, is 6 feet long; that in *B*, 18; that in *C*, 8; the line *AB* being 33 feet long: It happens on a certain day in the year, that the end of the ſhadow of the ſtaff *A* paſſes through the points *B* and *C*; and of the ſtaff *B*, through *A* and *C*; and of the ſtaff *C*, through the point *A*.

To find the ſun's declination, and the elevation of the pole, or day and place where this ſhall happen.

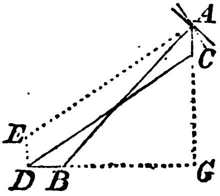
Note, this is the 42d problem in Sir Iſaac Newton's Univerſal Arithmetic; and it may ſeem a piece of vanity in attempting to give a ſolution after the greateſt of men; but having in the winter 1740, taken a great deal of pains to bring out a ſolution, and never being able to get his numbers for the declination and latitude preciſely the ſame, I was fond to think his were exact, and wrought it over and over again; at firſt it came out an adfected equation, then a quadratic, and at laſt happily by a ſimple equation; and having taken the pains to prove all the numbers (not depending on the logarithms) found them agree in every particular, and by conſtruction to form a true conic ſection. We therefore humbly preſume, that in a calculus ſo prolix and difficult (in Sir Iſaac's method) there might happen a ſmall error, or at leaſt ſome preſs fault of the editions, or in the tranſlation; which we hope to make more fully appear in the next year's Diary.

1744.

Queſtions anſwered.

* I. QUESTION 235 *anſwered by Mr.* J. Turner.

THERE is given $AB = CD = b = 97$ inches, $AC = 13{\cdot}8 = c$, and $DB = 12{\cdot}6 = d$; to find $CG = x$. Put $y = BG$.



By 47 E. I. $\begin{cases} cc + 2cx + xx + yy = bb, \\ dd + 2dy + yy + xx = bb; \end{cases}$ conſequently they are equal to one another, or $cc + 2cx = dd + 2dy$; and by the firſt equation, $y = \sqrt{bb - cc - 2cx - xx}$; which ſubſtituted for y, in $cc + 2cx - dd = 2dy$, is $cc + 2cx - dd = 2d\sqrt{bb - cc - 2cx - xx}$; and ſquaring both ſides of the equation, makes $c^4 + 4c^3x - 2ccdd + 4ccxx - 4ddcx + d^4 = 4ddbb - 4ddcc - 8ddcx - 4ddxx$; tranſpoſed, $4ccxx + 4ddxx + 4c^3x + 4ddcx = 4ddbb - 2ddcc - c^4 - d^4$; and dividing all by the coefficient of the higheſt power, it will ſtand, $xx + cx = \dfrac{4ddbb - 2ddcc - c^4 - d^4}{4cc + 4dd}$. The ſquare compleated, and properly reduced, gives this theorem,

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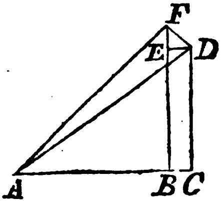
* I. QUESTION 235 *Conſtructed.*

Make DB = the given diff. of the ſhadows [Fig. 1], and perpendicular thereto DE = the given diff. of the heights of the boughs; then from E and B as centers, with the ſame given diſtance from each bough as a radius, deſcribe two circular arcs, and having drawn EA and BA to the point of interſection, A, draw DC ∥ and = EA, and AC drawn to meet DB, produced, in G, will determine CG and AG for the two required heights of the boughs.—— DC being, by conſtruction, $= EA = BA$, AC (by *Euc.* I. 34) = *and* ∥ DE, and conſequently the angle at G = the angle at D = a *right one.*

theorem, $x = \sqrt{\frac{ddbb}{dd+cc} - \frac{dd}{4}} - \frac{c}{2} = 58\cdot2 = GC$. Thence is found $y = 65$, $GA = 72$, and $DG = 77\cdot6$; whence, by common trigonometry, may be found the angles $GAB = 42°$ 4′ 30″, $ABG = 47°$ 55′ 30″, and $GDG = 36°$ 52′ 12″. Now having the two altitudes of the ſun found, and the differences of time given per queſtion $= 76\frac{1}{5}$ minutes, the azimuth is eaſily had; then by the theorem in the anſwer to queſtion 220, the latitude will come out 52° 19′ 37″, and the times of the day 8h. 0m. 11s. and 9h. 16m. 23s. the anſwer required.

The ſame anſwered by Mr. Betts.

Put x and y for the ſine and coſine of the angle EFD, radius $= 1$. Then by trigonometry, as s. $\angle EFD : ED :: $ s. $\angle EDF : EF$, i.e. $x : d :: y : c$; therefore $cx = dy$ and $\frac{x}{y} = \frac{d}{c} =$ tangent of 42° 23′ 51″ the angle EFD, or half the ſum of the angles BAD and $BAF =$ half the ſum of the two altitudes of the ſun. Again, ſine $EDF : EF ::$ rad. $: DF$, i.e. $y : c :: 1 : \frac{c}{y}$; and as AD : radius $:: \frac{1}{2}FD$: ſine of half the angle DAF, viz. $b : 1 :: \frac{c}{2y} : \frac{c}{2by}$; or as $2b : c :: \frac{1}{y} : \frac{c}{2by}$, viz. $2b : c ::$ ſecant 42° 23′ 51″ : ſine 5° 31′ 39″ equal to half the angle DAF, which is half the difference of the required altitudes; whence the altitude $CAF = 47°$ 55′ 30″ and $CAD = 36°$ 52′ 12″, and the times of the day 8h. and 9h. 16m. the latitude 52° 19′ 37″, the anſwer required.

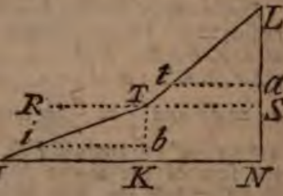


This queſtion was anſwered by *Bironnos*, Mr. *S. Bamfield*, Mr. *J. Aſh*, Mr. *R. Sowerby*, Mr. *Ramſhay*, Mr. *Watts*, Mr. *Scyth*, Mr. *Tho. Cowper* the propoſer, Mr. *W. Kingſtone*, and Mr. *Daniel*, in the former method, and by Mr. *Dan. Howard* in the latter.

II. QUESTION 236 *anſwered.*

After the two bodies have been 48 hours in motion, T is diſtant from the point of projection (B) 393270 miles, and makes an angle with it of 59° 55′. Alſo L makes with A an angle of 98° 23′, and is diſtant from it 861570 miles. Through the induſtrious labours of ſome of our correſpondents we have got an anſwer to this abſtruſe and curious philoſophical problem; but as it is doubted whether ſome errors are not in it, and believing no one ſo equal to the taſk as the author, whoſe knowledge and penetrability in ſuch difficult and uncommon problems, is ſcarce to be rivalled in an age; we have given only the numbers above (beſides the ſcheme and anſwer we have inſerted in our 1ſt vol. of Diary Queſtions, p. 193) * till ſuch time as the propoſer favours us with his.

III. QUESTION 237 *anſwered by* Bironnos.



Let LN repreſent the enlarged diff. lat. between the Lizard and Jamaica, LS that beween the Lizard and the trade wind. Then we have $LS = a = 1579{\cdot}9$; $TK = 790{\cdot}2 = b$; $Tb = 720 = q$; $JN = 4246 = n$. Let $y = TS$; then $n - y = IK$; and per ſim. triangles, $a : y :: m : \frac{my}{a} = ta$; $b : n-y :: q : q \times \frac{n-y}{b} = ib$; and per 47 Eu. 1, $\sqrt{m^2 + \frac{m^2y^2}{a^2}} = Lt$; alſo $\sqrt{qq + qq \times \frac{\overline{n-y}|^2}{b^2}} = Ti$; hence the required minimum is $\frac{m}{6a}\sqrt{aa+yy} + \frac{q}{9b}\sqrt{bb + \overline{n-y}|^2}$; which being fluxed, there comes out $\overline{qn - qy} \times 2a\sqrt{aa+yy} = mq \times 3b\sqrt{bb + \overline{n-y}|^2}$; which ſolved, $y = 1857{\cdot}9215 = TS$. Hence the courſe from the Lizard to the trade wind is ſouth 49° 37′ 25″ weſterly, diſtance = 1846·23 miles; and thence to Jamaica S. 71° 41′ 27″ W. diſtance 2291·95 miles.

Anſwered by Mr. J. Powle.

RS is the parallel of 30°, L the Lizard, J Jamaica, T the point where the fleet muſt come to in their way, the $\angle TLS$ the firſt courſe ſteered, &c. (per Wright's projection) LS

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* *The 1ſt vol. of queſtions here referred to, is Thacker's* Miſcel. *where the calculation of this queſtion is given.*

$LS = 1597 \cdot 8979$ miles, which call b; $SN = TK = 790 \cdot 1607 = c$; $JN = 4246 = d$. Put $x = TS$; $m = 6$; $n = 9$: then (per 47 E. 1) $TL = \sqrt{bb + xx}$, $TJ = \sqrt{cc + dd - 2dx + xx}$; and, by a uniform velocity, the times of deſcription will be $\frac{\sqrt{bb + xx}}{m}$, and $\frac{\sqrt{cc + dd \mp 2dx + xx}}{n}$, the ſum of which two muſt be a minimum; being fluxed, made $= 0$, and ordered, there will ariſe this equation:

$$\left.\begin{matrix} +n^2 \\ -m^2 \end{matrix}\right\} x^4 \left.\begin{matrix} -2dn^2 \\ +2dm^2 \end{matrix}\right\} x^3 \left.\begin{matrix} +ddnn \\ +ccnn \\ -bbm^2 \\ -ddm^2 \end{matrix}\right\} xx + 2dbbm^2 x = bbddm^2.$$

In numbers, $52x^4 - 441584x^3 + 771997102 \cdot 9985x^2 + 763080806290 \cdot 03x = 1620020551753144$.

Solved, $x = 1458 \cdot 016$, or $2891 \cdot 56$, or $5488 \cdot 45$, or $1346 \cdot 03$; but it is the firſt of theſe values ($1458 \cdot 016$) which ſerves our preſent purpoſe. Now having found TS, by plain trigonometry $TL = 2149 \cdot 86$ is had) being the diſtance ſailed before the fleet's arrival in the trade wind) and the courſe ſteered S. W. by S. 8° 27′ weſterly: Thence to Jamaica TJ is $2897 \cdot 79$; courſe W. S. W. 6° 40′ weſterly.

The reſult of ſome other anſwers are theſe following:

	1 courſe		Diſt. ſailed.	2 courſe		Diſt. ſailed.	x's val.	Time	
	°	′	miles	°	′	miles	miles	d.	h.
Mr. N. Oats	S. 48	44	1813·56	72	5	2343·41	1801	23	10½
Mr. Bamfield	49	43	1849·6	71	39	2287·6	1864		
Mr. J. Aſh	49	43	1849·7						
Mr. Ramſay	39	50	1555	73	33	2542		22	6
Mr. J. Watts	148	44	1813	172	5	2343		32	10½
Mr. Rr. Scyth	48	40	1853	72	8	2346	1796	23	17
Mr. Cowper	39	44	1555·4	73	32	2542		22	13

IV. QUESTION 238 *anſwered by* Merones.

Put $c =$ chord of the arch required; $x =$ any variable part of it; $r =$ the length of the pendulum; $t =$ time of the deſcent in the chord; and $z =$ time of the deſcent in the arch. Then (by ex. 10 prop. 13 Mr. Emerſon's Doctrine of Flux. p. 114) $\dot{z} = \frac{tr}{2} \times \frac{\dot{x}}{\sqrt{cx - xx}\sqrt{4rr - cx}} = \frac{t\dot{x}}{4\sqrt{cx - xx}} \times : 1 + \frac{cx}{2.4r^2} + \frac{3.c^2x^2}{2.4.16r^4} + \frac{3.5.c^3x^3}{2.4.6.64r^6}$ &c. And by forms

ſo

10 and 17 (pages 62, 68, ibid) of the table, $z = \frac{tt}{2} \times$ arch whoſe ſine is $\sqrt{\frac{x}{c}} \times : 1 + \frac{cc}{16r^2} + \frac{3^2c^4}{4.16^2r^4}$ &c. = (when $x = c$) $\frac{3{\cdot}1416t}{4} \times : 1 + \frac{cc}{16r^2} + \frac{3^2c^4}{2^216^2r^4} +$ &c. $= t$, per queſt. ∴ (putting $r = 1$) $cc + \frac{3^2c^4}{2^216} + \frac{3^25^2c^6}{2^23^216^2} + \frac{3^25^27^2c^8}{2^23^24^216^3}$ &c. $= 4{\cdot}3744$: whence, by reverſion, $cc = 2{\cdot}5107$; and $c = 1{\cdot}5846$, the chord of 104° 48′ the arch through which the pendulum muſt deſcend.

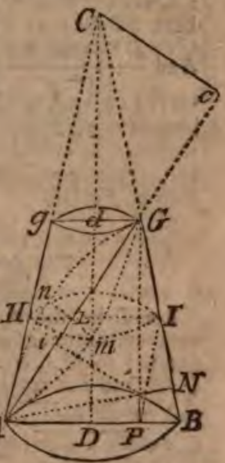
The ſame anſwered by the Propoſer Mr. Peter Kay.

Let $P = 3{\cdot}14159$; $a =$ length of the pendulum; $c =$ the verſed ſine of the arch deſcribed in the deſcent; then the time of deſcent, or half the time of vibration, will be $\frac{a^{\frac{1}{2}}P}{\sqrt{2}} \times : 1 + \frac{c}{2.2.2a} + \frac{c^2}{2.2.4.4.4a^2}$ &c. and the time along chord by $2\sqrt{2a}$; (as demon. p. 140, 141. of Simpſon's Flux.) which two expreſſions muſt by the queſtion be equal to each other; divide both by $\frac{a^{\frac{1}{2}}p}{\sqrt{2}}$, and put $x = \frac{c}{2a}$, and the equation will become $1 + \frac{x}{2.2} + \frac{3.3x^2}{2.2.4.4}$ &c. $= \frac{4}{p}$, and therefore $x + \frac{3.3x^2}{4.4} + \frac{3.3.5.5x^3}{4.4.6.6} + \frac{3.3.5.5.7.7x^4}{4.4.6.6.8.8}$ &c. $(\frac{16}{p} - 4) = 1{\cdot}093$; whence $x = \frac{c}{2a}$, will be found $= {\cdot}642$, and therefore $\frac{c}{a} = 1{\cdot}284$; and conſequently the required arch = 106° 32′.

Mr. *Ramſhay*, by an eaſy proceſs, finds the arch 104° 5′, Mr. *Watts*, = 104° 48′. *Gamſton Retford*, = 106° 32′.

V. QUESTION 239 was anſwered by the propoſer, by Mr. *John Landen*, Mr. *Samuel Bamfield*, Mr. *Ramſhay*, Mr. *Watts*, and *Gamſton Retford*. The ages 15, 18, 21, 25.*

VI. QUESTION 240 *anſwered by* F. R. S.



In the given fruſtum of the cone let $AB=D=40$; $gG=d=20$; $GP=h=90$. And put $AG=t$ the tranſverſe diameter; $nm=c$ the conjugate; $GB=z$; and $Cc=x$. Extend the tranſverſe to c, and draw the two perpendiculars to it cC and iB, alſo $f=GC$, and $x=Cc$.

Then by ſim. triang. $$\begin{cases} t:h::D:\dfrac{Dh}{t}=Bi, \\ z:\dfrac{Dh}{t}::f:\dfrac{Dhf}{zt}=Cc, \\ \dfrac{D-d}{2}:z::\dfrac{d}{2}:\dfrac{dz}{D-d}=f. \end{cases}$$

Subſtitute $\frac{dz}{D-d}$ for f, then Cc (x) $=\frac{Ddh}{Dt-dt}$, and x is nearly 38; which multiplied by $\frac{1}{3}$ of the area of the ellipſis, $AnGm$, viz. $\frac{.7854tc}{3}$, or ntc, gives $\frac{Dhdnc}{D-d}$, the ſolidity of $ACGmA$; from which take the ſolid gCG, i. e. $\frac{ddnhd}{D-d}$, leaves $\frac{Dhdnc-d^3nh}{D-d}$ the content of $AgGA$. But nm $(=c)$

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* V. QUESTION 239.

The ſolution of this queſtion, as given by Mr. *Aſh*, is omitted, as being ſo very erroneous. It is not perhaps worth one's while to reduce the equations by a regular algebraic proceſs, which would be a very tedious operation: but they might be much eaſier ſolved by the method of Trial-and-Error, or ſome other approximating method. Or by a few trials the numbers are found to be as above.

is known to be a mean proportional between AB and gG; i. e. $\sqrt{Dd} = c$; whence $\frac{Dhdn\sqrt{Dd} - d^3nh}{D-d} =$ leſſer hoof $AgGA$. Alſo $\frac{D^3hn - Ddn\sqrt{Dd}}{D-d} =$ greater hoof $ABGA$.

But in this example, as $2d = D$, the theorems are reducible, $\frac{D\sqrt{Dd} - dd}{D-d} \times {\cdot}2618db$ is $\frac{2d\sqrt{dd} - dd}{2d-d} \times {\cdot}2618dh$, i. e. $\overline{2dd\sqrt{2} - dd} \times {\cdot}2618b$, or $\overline{2\sqrt{2} - 1} \times {\cdot}2618hdd$. In numbers, $1{\cdot}8284 \times 90 \times 400 \times {\cdot}2618 = 17232{\cdot}304$ the leſſer hoof. And $\frac{D^2 - d\sqrt{Dd}}{D-d} \times {\cdot}2618Dh$ becomes $\frac{4dd - dd\sqrt{2}}{2d-d} \times {\cdot}2618hhd$, that is, $\overline{4dd - dd\sqrt{2}} \times {\cdot}2618 \times 2h$, or $\overline{4 - \sqrt{2}} \times {\cdot}2618 \times 2hdd$: but $\sqrt{2} = 1{\cdot}4142$, and $4 - 1{\cdot}4142 = 2{\cdot}5858$; therefore in numbers $2{\cdot}5858 \times 2618 \times 180 \times 400 = 48741{\cdot}2956$ the greater ungula. Which ungulas together make the whole fruſtum $65973{\cdot}6$.

The ſame anſwered by Merones.

Let L be the center of the ellipſis $AnGm$, mLn the common ſection of it and the circle $HnIm$, and draw IP parallel to HA, &c. Since GA is biſected in L, $\therefore LI = \frac{1}{2}AB$, and $LH = \frac{1}{2}gG$; whence $nL^2 = HLI = \frac{1}{4}gG \times AB$, and $nm = \sqrt{Gg \times AB}$; and the area of the ellipſis $c \times GA\sqrt{Gg \times AB}$ (c being $= {\cdot}7854$). By ſimilar triangles, $AB - Gg : GP :: AB : CD = \frac{AB \times GP}{AB - Gg} :: Gg : Cd = \frac{Gg \times GP}{AB - Gg}$; and, by the ſame, $GA : Gg :: CD : Cc = \frac{CD \times Gg}{GA}$; $\therefore$ the area ellipſis $\times Cc = \frac{c \times CD}{3}\sqrt{Gg^3 \times AB} =$ ſolidity of GCA; but $c \times Ggg \times Cd =$ ſolidity GCg; and $\frac{c \times AB^2 \times CD}{3} =$ ſolid BCA; whence $\frac{c \times CD}{3} \times \overline{AB^2 - Gg\sqrt{Gg \times AB}} =$ ungula GAB; and $\frac{c \times CD}{3} \times \sqrt{Gg^3 AB} - \frac{c \times CD}{3}Ggg =$ ungula GgA. Hence, putting the height $GP = H$, $AB = B$, $Gg = b$, $c = {\cdot}7854$; then $\frac{cHB}{B-b}$ $\times$

$\times \frac{B^2 - b\sqrt{Bb}}{3}$ = the greater ungula $GAB = 48740{\cdot}55$; and $\frac{cHB}{B-b} \times \frac{b\sqrt{Bb} - bb}{3}$ = the leſſer ungula $GgA = 17232{\cdot}055$. *Q.E.I.*

Bironnos *has anſwered this Queſtion in a Method ſomething different from the two laſt.*

Let fall the ⊥ AN upon GB; put $2a$ = greater diameter AB, $2q$ = leſſer gG. Their half ſum $AP = m$; half diff. $PB = n$; the fruſtum's height $GP = d$; and ·785398, &c. $= s$; then $\sqrt{d^2 + m^2} = AG$ the tranſverſe, and $2\sqrt{aq}$ = the conjugate: Then, per ſim. △s, $PB : GP :: gd : Cd = \frac{dq}{n}$, and $:: BD : DC = \frac{da}{n}$; and, per 47 Euc. 1, $CG = \frac{q\sqrt{nn + dd}}{n}$; again, $GB : GP :: AB : AN = \frac{2da}{\sqrt{dd + nn}}$, and $AG : AN :: CG : cC = \frac{2daq}{n\sqrt{d^2 + m^2}}$; hence the ſolidities, of the cone $ACB = \frac{4a^3sd}{3n}$, of $CgG = \frac{4q^3sd}{3n}$, and the ſcalenous CAG, is $= \frac{4sdaq\sqrt{aq}}{3n}$; whence the ſolidity of the greater hoof $AGB = \frac{4asd}{3n} \times \overline{aa - q\sqrt{aq}}$, and leſſer $\frac{4sdq}{3n} \times \overline{a\sqrt{aq} - qq}$. Or putting the fruſtum's height $= A$, greater diameter $= D$, leſſer $= d$, diff. $= x$, mean proportional between B and ·78539 $= s$; then

Greater hoof $= \frac{DAs}{3x} \times \overline{DD - Bd} = 48741{\cdot}05$.

Leſſer hoof $= \frac{dAs}{3x} \times \overline{DB - dd} = 17232{\cdot}55$, &c.

Q.E.I.

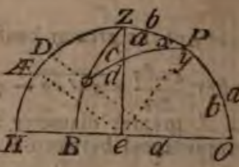
Mr. *Tho. Atkinſon*, and Mr. *Tho. Ramſhay*, have each brought out theorems and true ſolutions by the common method of algebra. Solutions were alſo given by Mr. *Turner* and Mr. *Arch. Scyth.*

VII. QUESTION 241 *anſwered by Mr.* J. Turner *of York.*

Let b = length of the ſemidiurnal arch $= 1{\cdot}8233$; x = arch of the hour from noon, its coſine $= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. d = rectangle of the ſines of the lat. and declination $= {\cdot}1493$; $c = {\cdot}5979$ the rectangle of their coſines; then $c \times : 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. $+ d$ = ſine ſun's altitude; and $\overline{cb + cx} \times : 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. $+ db + dx$ by the queſtion muſt be a maximum; which put into fluxions and reduced, gives $cbx + \frac{3cx^2}{2} - \frac{cbx^3}{6} - \frac{5cx^4}{24} + \frac{cbx^5}{120}$ &c. $= c + d$; or $1{\cdot}09015x + {\cdot}8968x^2 - {\cdot}1817x^3 - {\cdot}1246x^4 + {\cdot}0991x^5$ &c. $= {\cdot}7472$. Here $x = {\cdot}5049$ = an arch of 28° 56′, ſo the time is 1h. 55′ 44″ poſt meridiem.

An Anſwer by Mr. Thacker.

Put radius = 1; d and c for the ſine and coſine of the ſun's declination; b and a = ſine and coſine of the latitude given; x and y the ſine and coſine of the hour from noon; z the arc itſelf: then by ſpherics $db + cay$ = ſine of Bo the ſine of the ſun's altitude; which being multiplied by $n + z$ (n = ſemid. arc) the arc of time paſſed over from ſun-riſing, gives $\overline{db + cay} \times \overline{n + z}$; which by the queſtion is proportionable to the heat, and muſt therefore be a maximum; put into fluxions, is $ca\dot{y} \times \overline{n + z} + \dot{z} \times \overline{db + cay} = 0$; but by the nature of the circle $x : 1 :: -\dot{y} : \dot{z}$, $\therefore -\frac{\dot{y}}{x} = \dot{z}$; which put for $\dot{z}$ is $ca\dot{y} \times \overline{n + z} - \frac{\dot{y}}{x} \times \overline{db + cay} = 0$; and therefore $ca\dot{y} \times \overline{n + z} = \frac{\dot{y}}{x} \times \overline{db + cay}$, which divided



divided by $\dot{y}$ gives $ca \times \overline{n+z} = \frac{1}{x} \times \overline{db + cay}$; this divided by ca will make $n + z = \frac{1}{x} \times \overline{\frac{db}{ca} + y}$.

From which equation it appears that the hotteſt time of the day is, when the arc of time paſſed over from ſun-riſing is equal to the rectangle of the tangents of the ſun's declination and latitude, plus the coſine of the hour or arc from noon, drawn into the co-ſecant of the arc paſſed over ſince noon. In numbers, the tangent of ſun's declin. 10° 53′ = 9·2839070 multiplied by tangent lat. 52° 30′ = log. 10·1150195 gives 9·3989265 = coſine 75° 29′ 40″ the arc of time from midnight, which ſubtracted from 180 leaves 104° 30′ 20″ the ſemi-diurnal arc = 12; to which 28° 56′ time after noon makes 133° 26′ 20″. This in decimals of degrees, multiplied by ·0174532, &c. (i. e. 360) 6·28318 (·01745) gives the arch of the circle 2·3263535. And tang. decl. × tang. lat. = ·2505642, plus coſ. arc 28° 56′ = ·8751832 from noon (= $n + z$) is 1·1257474 multiplied by the co-ſecant 28° 56′ = 2·0670056 gives 2·326 &c. equal to the other ſide of the equation, which is proved right. But the arch from noon is found thus: If v be put for the number of degrees paſſed over ſince noon; then will $·017453 \times \overline{104^\circ\, 30' + v} =$ co-ſecant $v \times \overline{·2505642 + \text{coſ.}\, v}$; whence by a few trials, v may be found = 28° 57′, which in time is 1h. 55′ 48″ afternoon.

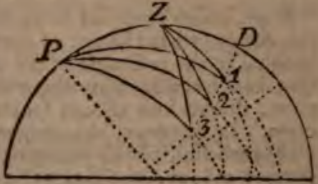
If any one ſhould diſlike this gueſſing method, this following may be thought more valuable, viz.

Put t = tangent of half the time from noon; then will $\frac{2t}{1+tt} = x$; and $\frac{1-tt}{1+tt} = y$; alſo, by the nature of the circle, $z = 2t - \frac{2t^3}{3} + \frac{2t^5}{5} - \frac{2t^7}{7}$, &c. whence $n + z = \frac{1}{x} \times \overline{\frac{db}{ca} + y}$ is $n + 2t - \frac{2t^3}{3} + \frac{2t^5}{5}$ &c. equal to $\frac{1+tt}{2t} \times \frac{db}{ca} \times \frac{1-tt}{1+tt} + 1 - tt$; and the equation ordered

$$\left.\begin{matrix}+2n\\-2\end{matrix}\right\}t \left.\begin{matrix}+4\\-\frac{bd}{ca}\end{matrix}\right\}t^2 + 2t^3 - \frac{4t^4}{3} + \frac{4t^6}{5} + \frac{4t^8}{7} + \frac{4t^{10}}{9}, \text{ \&c.}$$

$= \frac{db}{ca}$; which ſeries reverted will give the value of t = tangent 14° 28′.

VIII. QUESTION 242 *anſwered by Mr.* T. Cowper.



Let c = coſine of $Z1 = 57° 24' 52''$; t = coſine of $Z2 = 55° 35' 19''$; d = coſine of $Z3 = 53° 19' 15''$; b = ſine of $1P2 = 6° 3'$, its coſine $= p$; m = ſine of $1P3 = 12° 45'$, n = its coſine; x = ſine $DP1$, y = its coſ. then $py - bx$ = coſ. $ZP2$; $ny - mx$ = coſine $ZP3$. Then by theorem 1 in the ſolution to queſt. 222, we have $\frac{c + ty - t - cpy + c^{bx}}{y - py + bx}$ = coſine ZD, and $\frac{c + dy - d - cny + cmx}{y - ny + mx}$ = coſine ZD; which tranſpoſed and reduced, is $cpyy + tnyy + dyy - dpyy - cnyy - tyy + cbx + tmx - dbx - cmx = cpy + tny + dy - dpy - cny - ty + cbxy + tmxy - dbxy - cmxy$. Put $r = cp + tn + d - dp - cn - t$, or $= {}^{\cdot}0001692$; and $s = cb + tm - db - cm$, or $= {}^{\cdot}00077322$; then the equation above, after ſubſtitution, will ſtand, $ryy + sx = ry + syx$, i.e. $sx - sxy = ry - ryy$, and dividing by $1 - y$, we have $sx = ry$; $\therefore \frac{x}{y} = \frac{r}{s}$ the tangent of 12° 20′ 36″: conſequently the 1ſt obſervation was 12 h. 49′ 22″, the 2d at 1 h. 15′ 22″, the 3d at 1 h. 40′ 20″ *P. M.* And by the 1ſt and 2d theorem in the ſame 222d queſt. is found the ſun's meridian altitude = 58° 15′ 57″, his ſeptentrional depreſſion = 17° 20′ 35″; hence the latitude is 51° 58′ 49″, and ☉'s declination = 20° 45′ 36″ north.

The ſame anſwered by F. R. S.

Let the quantities be repreſented as above; alſo let r and s = ſine and coſine of the latitude; and a and c the ſine and coſine of the ſun's diſtance from the pole.

Then in the 3 triangles, by the ſpheric theo. we have
$$\begin{cases} (1)\ er + asy = c, \\ (2)\ er + aspy - asbx = t, \\ (3)\ er + asny - asmx = d. \end{cases}$$
In the (1) $er = c - asy$, which ſubſtituted in the other two, make (4) $c - asy + aspy - asbx = t$, (5) $c - asy + asny - asmx = d$; per (4) $-asy + aspy - asbx = t - c = g$, $\therefore as = \frac{g}{-y + py - bx} = \frac{g}{-y \times \overline{1 - p} - bx}$: call $1 - p =$

$v =$

$v =$ verſed ſine 6° 30′, then will $as = \frac{g}{-yv-bx}$; by the (5) $-asy+asny-asmx=d-c=k$, $as = \frac{k}{-y+ny-mx} = \frac{k}{-y\times\overline{1-n}-mx}$; and if $1-n=w=$ verſ. ſine 12° 45′, then will $as = \frac{k}{-yw-mx}$; conſequently $\frac{g}{-yv-bx} = \frac{k}{-yw-mx}$, or $\frac{g}{yv+bx} = \frac{k}{yw+mx}$, which out of fractions $gyw+gmx=kyv+kpx$, $\therefore \frac{x}{y} = \frac{kv-gw}{gm-kb} =$ tangent of the $\angle DP1 = 12^\circ\ 20'\ 36'' =$ time 49′ 22′. Whence the anſwer will come out as above.

I cannot but be perſuaded that this curious problem will point out a way to be very uſeful in navigation, for determining the longitudes as well as latitudes: for, ſuppoſing at ſea we know neither, or had not the time of the day, but were furniſhed with a quadrant to take altitudes of the ſun, and could find the difference in time between each obſervation, which a common pocket watch with a minute hand would give us very well, or with a ſecond hand better. For though that watch or a clock was incapable of keeping true time at ſea, yet it might very well meaſure a few minutes between one obſervation and another, in which ſpace the error muſt be very inconſiderable. Now when this was done, by ſolving this problem, we get the latitude and the true times of the day, and then it would be no very difficult taſk to rectify the longitude pretty near. In order to this we ſhall deduce a theorem in words, by which any one that is but ſkilled in the common caſes in trigonometry may put it in practice.

A Theorem for the Hour of the Day.

1. The difference between the firſt and ſecond altitude, drawn into (*i.e.* multiplied by) the verſed ſine of the arch of time between the firſt and ſecond obſervation; made leſs by the difference between the firſt and ſecond altitude, multiplied into the verſed ſine of the arch of time between the firſt and third obſervations.

2. The difference between the firſt and ſecond altitude, drawn into the right ſine of the arch of time, between the firſt and third obſervations; minus the difference between

the firſt and third altitude, drawn into the right ſine of the arch of time between the firſt and ſecond obſervations.

Laſtly, Divide the former difference by the latter, and the quotient will be the tangent of the arch of the time from noon.

The ſame anſwered by Mr. J. May *the Propoſer.*

Put $a =$ ſine $57^\circ\ 24'\ 52''$, $b =$ that of $55^\circ\ 35'\ 19''$, $c =$ that of $53^\circ\ 15'\ 16''$, the ſine of 26 min. or $6^\circ\ 30' = m$, coſ. $= n$, $f =$ ſine $12^\circ\ 42'$ ($=$ 51 min.) the time between the firſt and laſt obſerv. its coſ. $= g$, radius $= 1$. Then put $a - b = h$, $a - c = i$, $r - n = s$, $r - g = t$; then the tangent of the hour angle from noon when the greateſt altit. was taken will be $= \dfrac{ht - is}{mi - hf}\,r = {}^{\cdot}2190328 = 12^\circ\ 21'\ 16''$, which in time is $49'\ 25''$, or after 12 o'clock; the ſecond 1 h. $15'\ 25''$; and the laſt at 1 h. $40'\ 25''$, according to the altitude's decreaſe.

Now put the coſine of $12^\circ\ 21'\ 16'' = d$; of $25^\circ\ 6'\ 16'$ (the arc of 1 h. $40'\ 25''$ time of the laſt obſervation) $= e$; put likewiſe $d - e = q$, $r - e = l$, and $r + d = p$.

Then the ſine of the ſun's ſouthern altitude will be $= b + \dfrac{li}{q}$ (ſee his anſwer to queſt. 222 Diary 1742) the degrees of which put $= w$. Likewiſe the ſine of the ſun's depreſſion in the north will be $\dfrac{pi}{q} - a$; the degrees of it put $= u$; then the ſun's declination will be $\dfrac{w + u}{2} = 20^\circ\ 47'$ nearly; and the coſine of the latitude $\dfrac{w - u}{2} = 38^\circ\ 4'\ 45''$, and $51^\circ\ 55'\ 15''$ the latitude required.

IX. QUESTION 243 *anſwered by Mr.* J. Landen.

$\left.\begin{array}{l}\text{The bodies are } A,\ B,\ C.\\ \text{Their weights } \ 3,\ x,\ 27.\end{array}\right\}$ Then according to Mr. Keil's Introduct. Phyſ. we have $\dfrac{2a}{x + a} =$ the celerity wherewith the body B will approach C, and $\dfrac{4xa}{xx + xa + xc + ac} =$ the velocity of C after the impulſe; the fluxion of which being made $= 0$, and reduced, we have $x = 9$, a mean proportional between A and C.

Mr. Tho. Cowper's *Anſwer.*

The bodies and weights denoted as above, and putting 1 to expreſs the velocity of *A*; from Dr. Keil's demonſtration about the motions of elaſtic bodies is deduced this analogy: As the ſum of the bodies : twice the weight of the moving or ſtriking body : : the velocity of the ſtriking body before percuſſion : the velocity of the quieſcent body after it.

That is, $x+a : 2a :: 1 : \frac{2a}{x+a} =$ velocity of *B* after the ſtroke. Again, $x+c : 2x :: \frac{2a}{x+a} : \frac{4ax}{xx+cx+ax+ca}$ = the celerity of *C* after the ſtroke; which, per queſtion, is a maximum, and the fluxion thereof $4ax^2\dot{x}+4acx\dot{x}+4aax\dot{x}-8ax^2\dot{x}-4acx\dot{x}-4aax\dot{x}=0$. Reduced, gives $x=\sqrt{ac}=9$ pounds.

Mr. *J. Watts*'s anſwer is in the ſame method. Mr. *William Honnor*, from Mac Laurin's Fluxions, p. 429, Mr. *Powle* the propoſer, Mr. *J. Turner*, *Bironnos*, Mr. *S. Bamfield*, Mr. *Aſh*, Mr. *Ric. Sowerby*, and *Philotechnus*, have anſwered this queſtion.

The PRIZE QUESTION *anſwered by Mr.* A. Thacker.

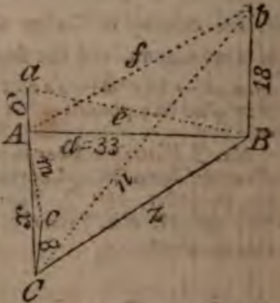
Put $\left.\begin{array}{l} a=6 \text{ feet} \\ b=18 \\ c=8 \end{array}\right\}$ the height of the ſtaves at $\left\{\begin{array}{c} A \\ B \\ C \end{array}\right\}$; the line $AB = 33$ feet $= d$; the diſtance between the top of the ſtaff *A* and bottom of the ſtaff *B* $(= aB)$ call *e*; the diſtance between the top of the ſtaff *B* and bottom of ſtaff *A* $(= bA)$ call *f*; then will $\frac{\overline{c+a}\times\overline{e+f}}{2b+2a}+\frac{\overline{b+a}\times\overline{c-a}}{2e+2f}=m=$ 21·0789, the diſtance from the top of *C* to the bottom of *A*. Then (by 47 Euc. 1.) *AC* is found = 19·501, which call *x*; alſo $\frac{\overline{c+b}\times\overline{e+f}}{2b+2a}+\frac{\overline{b-c}\times\overline{b+a}}{2e+2f}=n=40\text{·}216$, the length between the top of *B* and bottom of *C*; and hence *GB* is found $= 35\text{·}962 = z$. Now putting $\frac{be-fa}{df+de}=h=$

16111;

16111; $\sqrt{xx+aa} = s = 20{\cdot}404$; $\frac{cs - am}{xm + xs} = k = {\cdot}0454$; and $\frac{dd + xx - zz}{2dx} = v = {\cdot}13501$; then will $\frac{2 - 2vv}{1 + hh + kk - 2hkv - vv} = 1{\cdot}948315$, the verſed ſine of an arc which is double the latitude 161° 30′; whoſe half is = 80° 45′, the true latitude required.

Now calling the ſine of the latitude p, then will $\frac{pc + pa}{s + m} = {\cdot}33309$ = the ſine of 19° 27′, the ſun's declination north.

Sir *I. Newton*, the author of this problem, finds the lat. 80° 45′ 20″, and declination 19° 27′ 20″, as may be ſeen in the 42d prob. of his Univerſal Arithmetic, in the Engliſh edit. 1720, p. 151, where the tranſlator Mr. Raphſon, Mr. Cunn, or the printer, committed a blunder, making the line *AB* = 30, inſtead of 33. By reaſon of which diſappointment, the ſolution is here ſhorter than was deſigned; but the inveſtigation of the theorem above we have printed in the firſt volume of Diary Queſtions [*Thacker's Miſcel.*] If the line *AB* was = 30, then the latitude is 80° 4′ 7″, and declination 21° 7′ 4″. And Mr. *Ed. Croſs* has found it 80° 4′ 11″, and declination 21° 7′ 48″.



The Prize of 10 *Diaries fell to Mr.* Ed. Croſs.

Of the Eclipſes in 1744.

Within the ſphere of the earth's orbit will happen four eclipſes this year; twice will the moon, in her wandering courſe, interpoſe and hide the ſplendour of the ſun from falling on the earth, or its atmoſphere; and twice will the earth, in its annual elliptic orbit, be ſo full in a line between the ſun and moon, as to hinder her from receiving the light ſhe borrows from the ſun, to enlighten the earth by re*flection.*

1. Sun eclipſed April 1, at 10 at night, inviſible.

2. Moon eclipſed April 15, at half an hour after 8 at night, viſible.

Calculated by		Beg. h.m.	Mid. h.m.	End h. m.	Dig.
From Aſtron. Carol. at Coventry		7 4	8 28	9 52	8 0
J. Randles of Wemm	London	7 5	8 30	9 58	8 9
	Rome	7 57	9 22	10 50	
	Wemm	6 55	7 53	9 48	
William Leighton,	London	7 8	8 32	9 57	
	Arlington	7 15	8 39	10 4	
S. Bamfield, Leadbet. Tab. Lond.		7 3	8 16	9 29	6 53
Will. Brown, Brent's Comp. A.		7 3	8 29	9 55	8 1
Car. Cleobury		7 1	8 25	9 49	8 0
W. Honnor,	Burcheſter	6 54	8 23	9 52	8 23
	Ruſhden	6 58	8 27	9 56	
Mr. Ral. Hulſe, London		7 8	8 28	9 56	
Tho. Cowper, Wellingborough		7 9	8 35	10 1	8 2
Mr. Poole, Hereford		7 5	8 30	9 50	7 50
Mr. Betts, Oxford		7 7	8 35	10 3	8 35

3. Sun eclipſed September 25, inviſible.

4. Moon eclipſed October 10, inviſible. Mr. Hulſe has given the calculation of this eclipſe at Moſcow: The beginning 11. 30. Middle 12. 46. End 1. 8.

New Queſtions.

I. QUESTION 244, *by Mr.* J. Turner.

If a flexible chain, eighteen inches long,
On two pins horizontal was hung,
Whoſe diſtance aſunder exactly ſhall be
A foot; its loweſt deſcent then let's ſee.
A theorem that's general give, for to find
The areas of all ſuch curves of that kind.

II. QUESTION 245, *by Mr.* John Landen.

I have one hundred pieces of gold; ſome of which are piſtoles, ſome guineas, and the reſt moidores. Now if a piſtole was worth 18s. 6d. a guinea worth 1l. 3s. and a moidore 1l. 10s. my hundred pieces would be worth juſt one hundred pounds. Quere, How many I have of each ſort?

III. QUES-

III. QUESTION 246, *by Mr.* Peter Kay.

To find the center of oſcillation of a pendulum, whoſe bob is compoſed of two equal and ſimilar parabolical conoids, joined together at their baſes; the thickneſs of the bob being three inches, the diameter of its greateſt circle ſeven inches, and the diſtance of its center from the point of ſuſpenſion 39·5 inches?

IV. QUESTION 247, *by Mr.* J. Betts.

A ſet of men and women were drinking together, and their reckoning came to juſt ſix guineas; towards the diſcharging of which, each man agreed to pay a certain ſum, and each woman the ſquare root of the ſame: Now it was found, if the number of men and women were mutually changed the one for the other, the reckoning would have come to half a Portugal piece leſs, or only to 4l. 10s. Again it was found, that each man paid as many ſhillings more than each woman, as there were women in company. It is required, what number were of each, and what each paid?

V. QUESTION 248, *by Mr.* William Daniel.

In an oblique-angled triangle (*EGF*) there is given the difference of the two ſides, which compoſe the oblique angle (ED) $= 2$; the difference of the ſegments of the baſe (EB) $= 2{\cdot}4$; and the oblique angle (EFG) $= 112° \; 37'$: It is required to find all the other parts of the triangle.

VI. QUESTION 249, *by Mr.* William Brown.

In the latitude 52° 30′, on the 10th of June (ſuppoſing it the longeſt apparent day) I aſked a mathematical friend, what o'clock it was? who made me this puzzling anſwer: Count (ſays he) the hours from the viſible time of the riſing of the ſun's center, and add their cube root to the ſquare root of the hours to the apparent time of its ſetting; and it will give you the hour of the day. Quere, What o'clock was it?

VII. QUESTION 250, *by Mr.* John Hill.

There is a river, whoſe ſtream is divided into two parts; and after running ſome ſpace, the waters are united; between which it has incloſed an iſland in the form of a geometrical ellipſis, whoſe tranſverſe diameter is forty chains (according to Gunter) and conjugate = thirty chains. Upon the tranſverſe diameter is built a farm or cottage houſe, 132 yards from the center; and as this piece of land is to be divided by ſtraight hedges from the houſe to the water, one of them, which ſhould be the ſhorteſt that can be made, is to convey the water from the river to fill a ciſtern by the cellar. It is required to find the ſhorteſt diſtance, and the poſition it will make with the tranſverſe.

VIII. QUESTION 251, *by Mr.* J. Powle.

To determine the law of the weights, which preſs each particle of a perfect flexible line, in ſuch manner, as that it ſhall form a curve, whoſe equation is $ax = y^4$?

IX. QUESTION 252, *by Mr.* T. Sandalls.

In an oblique-angled plain triangle, there is given the difference of the ſides which include the angle of 112° = 20, and the perpendicular let fall from the angle on the baſe = 60: Required a theorem to determine the baſe and ſides of the triangle?

X. QUESTION 253, *by Mr.* J. Powle.

Granting the reſiſtance, as the ſquare of the celerity; in what law of denſity will a heavy body moving deſcribe a curve, whoſe equation is $ax = y^3$?

XI. QUESTION 254, *by* Diophantus.

Since the doctrine of triangles has an unbounded uſe and application in moſt parts of the mathematics, and the ſimilarity of them generally had recourſe to; let it be required to find eight right-angled plain triangles, whoſe hypothenuſes are all equal; and ſhew a general method for determining the ſame.

XII. QUESTION 255.

The various contrivances for meaſuring time have employed the curious in all ages; the true determining of which is a matter of no ſmall importance in civil life; and perhaps I may ſurprize ſome, if I ſay, algebra is uſeful to know the time of the day by a clock, when it cannot be done otherwiſe; which is the reaſon for putting in this eaſy queſtion, in order to convince others, the facetious Hudibras did not joke, when he ſays,

——And wiſely tell what hour o'th' day
The clock does ſtrike, by algebra.

THE QUESTION. Being at ſo large a diſtance from the dial-plate of a great clock, that I could not diſtinguiſh the figures; but as the hour and minute hands were very bright and glaring, I could perceive, that the minute hand pointed upwards to the right hand, at the ſame time the hour index pointed downwards to the left, ſo as both were in a right line, or diametrically oppoſite, and in ſuch poſition, as that the elevation (I gueſſed) was ſome few degrees more than 50 above the horizon. Quere, The hour and minute of the day.

PRIZE QUESTION *by Mr.* J. May, *jun. of Amſterdam.*

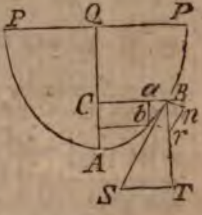
An architect, or maſter carpenter, in Holland, had (from that ſlender knowledge which uſually attends mechanics) conceit enough to fancy, he could find the dimenſion of any piece of timber in a building, of which a deſign ſhould be given: A burgo-maſter of the city of Amſterdam, intending to build a handſome houſe fronting the ſtreet, where his length was limited, becauſe he would ſave the charges of a double roof and gutter, and at the ſame time put his beſt ſide outward, gives the ſaid architect theſe dimenſions, viz. That the building ſhould be forty feet wide, and the front wall twelve feet higher than the back wall: Alſo, becauſe too much of a large roof ſhould not appear in view of the ſtreet, he will have the length of the rafters, from wall to ridge, on the back ſide of his houſe, juſt 37 feet; but the rafters on the front ſide to be of ſuch a length, as may form the pitch, or ſteepneſs of the roof, the ſame on each ſide. The owner being frugal (not to ſay wiſe) orders the builder to ſit down and count up the coſt: But although he was ſkilful in numbers, and pretty well verſed in ſome parts of geometry, yet he found the firſt would be ſo much adfected, and the latter only an approximation, that he was not able to know how high the roof would riſe, nor the length of the rafters in front, and therefore was incapable of computing the timber and roofing. The burgo-maſter ſurprized, probably thinking ſo famed an architect muſt be little leſs than a conjuror (when himſelf was none) reſolves not to have his houſe begun until he can have the meaſures exact, and leaves him bare-breach'd, riding on the ſtrange roof, although he is furniſhed with mathematical inſtruments, to deſcribe curves and conic ſections organically. But having heard of ſuch things being effected by geometrical conſtruction, he has, through the mediation of a friend, applied to the artiſts of Great Britain; and thinking the author of the Ladies' Diary deals in quibbles and quaint queſtions, hopes to ſee both methods in the next year's production.

Queſtions

1745.

Queſtions anſwered.

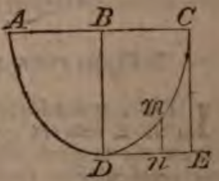
I. QUESTION 244 *anſwered by Mr.* N. Farrer.



LET $a =$ the force which keeps the chain in its poſition at A, $x = AC$, $y = BC$, $z = AB$; draw the tangent BS, and $BT = BA$ perpendicular to the horizon, and TS parallel thereto. Then will the line AB be ſuſtained by three forces; for its gravity acts in the direction BT, it is drawn at A in an horizontal direction, by the force a, and it is ſuſtained in B by the tenſion of the line SB; which three forces are conſequently as BT, TS, and BS; as $BT = z : TS = a :: \dot{x} : \dot{y}$. Take $Br = Bb$, and draw rn parallel to BS, and Bn perpendicular thereto; then BT is the perpendicular force or weight of the line at B, $\dot{x}$ is the fluxion of the tenſion at B, whoſe fluent (x) is the tenſion or force in the direction nr; but in A where $x = 0$, this tenſion $= a$, ergo by correction the whole tenſion drawing in the direction of the curve is $a + x$; then $BS = a + x : z :: \dot{z} : \dot{x}$, $\therefore a\dot{x} + x\dot{x} = z\dot{z}$, its fluent is $2ax + xx = zz$, therefore $\dot{y} = \frac{a\dot{x}}{\sqrt{aa + xx}}$, and $y =$ hyperb. log. of $\frac{a + x + \sqrt{2ax + xx}}{a}$; which equation when $y = PQ$ will give $x = AQ = 6{\cdot}0314$ the loweſt deſcent; hence $a = \frac{zz - xx}{2x} = 3{\cdot}6992$; the ſupplemental fluxion $x\dot{y} = \frac{ax\dot{z}}{\sqrt{aa + zz}}$ gives $az - aa \times$ hyp. log. $\frac{z + \sqrt{aa + zz}}{a}$, and the area $PAQ = xy - az + aa \times$ hyp. log. $\overline{z + \sqrt{aa + zz}} = 25{\cdot}0916$.

The ſame anſwered by Mr. Cockſon.

Put $DC = c$, $BC = b$, $Dn = x$, $mn = y$, and $Dm = z$. Then will $\dot{x} = \frac{a\dot{y}}{z}$, $z = \sqrt{aa + yy}$; whence, the curve being the catenaria, $y = \sqrt{aa + zz} - a$, $\dot{y} = \frac{z\dot{z}}{\sqrt{aa + zz}}$; this put for $\dot{y}$ in the above value of $\dot{x}$, makes $\dot{x} = \frac{a\dot{z}}{\sqrt{aa + zz}}$; hence $y\dot{x}$ the flux. of the variable area Dmn is $\frac{a\dot{z}}{\sqrt{aa + zz}} \times \overline{\sqrt{aa + zz} - a} = a\dot{z} - \frac{aa\dot{z}}{\sqrt{aa + zz}}$, whoſe fluent is $za - ax =$ the area, and when $z = c$ (x being then $= b$) becomes $\overline{c - b} \times a$ the expreſſion for the area CDE, ∴ the area of the curve is 50·1888, and DB the loweſt deſcent of the chain = 6·0317 inches.



II. QUESTION 245 *anſwered by Mr.* Heath.

Let $x =$ the number of moidores, $y =$ guineas; then $b - y - x =$ the piſtoles (putting b for 100 pieces); hence $60x + 37b - 37y - 37x + 46y = 4000$ the number of ſix-pences; whence $x = \frac{300 - 9y}{23}$ or $y = \frac{300 - 23x}{9}$; from whence x and y are determined in whole numbers 6 and 18, alſo $b - y - x = 76$.

This queſtion was alſo truly anſwered by Mr. *Brown*, Mr. *Farrer*, Mr. *Terey*, Mr. *Croſs*, Mr. *Adams*, Mr. *Landen* the propoſer, and ſeveral others.*

III. QUES-

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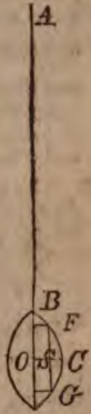
* *This queſtion* may alſo be ſolved after the manner of ſeveral *others of* the ſame kind already given in this work.

* III. QUESTION 246 *anſwered by Mr.* N. Farrer.

Let a repreſent the point of ſuſpenſion. Now here is given $AO = 39·5 = d$, $BO = 3·5 = m$, and $CO = 1·5 = n$: Let $y = SF = SG$, $x = CS$; then $x : yy :: n : mm$, $\therefore x = \frac{nyy}{mm}$; and then the fluent of $\overline{n-x}|^2 \times \overline{d-y}|^2 \times \dot{y}$ divided by the fluent of $\overline{n-x}|^2 \times \overline{d+y} \times \dot{y}$ gives $\frac{112dd - 70dm + 16mm}{112d + 35m}$, which let $= a$; again the fluent of $\overline{n-x}|^2 \times \overline{d+y}|^2 \times \dot{y}$ divided by the fluent of $\overline{n-x}|^2 \times \overline{d+y} \times \dot{y}$ gives $\frac{112d^2 + 70dm + 16m^2}{112d + 35m} = b$; hence the diſtance of the center of oſcillation from A the point of ſuſpenſion is $\frac{aa + bb}{a + b} = 37·35728$ inches.



Mr. *Cockſon* and Mr. *Powle* have made the diſtance of the point of ſuſpenſion from the center of oſcillation $= 40·0125$ inches; and Mr. *Wm. Hanbury* the ſame with the former.

IV. QUES-

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* III. QUESTION 246.

The original ſolution above given to this queſtion is wrong, for the center of oſcillation ought to be farther below the point of ſuſpenſion than the center of gravity is; but it is here made leſs than it.

Putting the quantities as in the notation above, and c for the diſtance of the center of oſcillation from the point of ſuſpenſion, then will c be $= d + \frac{\text{the flu. of } y^4\dot{x}}{2d \times \text{the flu. of } y^2\dot{x}}$ (by the nature of the center) $= d + \frac{\text{the flu. of } m^2x^2\dot{x}}{2d \times \text{flu. of } nx\dot{x}} = d + \frac{m^2x}{3dn} = d + \frac{y^2}{3d} =$ (when $y = m$ or $x = n$) $d + \frac{m^2}{3d} = 39·5 + \frac{3·5^2}{3 \times 39·5} = 39·5 + ·103376 = 39·603376$ the diſtance required.

IV. QUESTION 247 *anſwered by Mr.* R. Gibbons.

Put $m =$ the number of men, $n =$ the number of women, and $x =$ what each woman paid; then per queſtion each man paid xx. Hence $mxx + nx = 126$, $nxx + mx = 90$, and $xx - x = n$. Now by taking the value of m in the two firſt equations, we have $\frac{126 - nx}{x} = 90 - nxx$. From which and the third equation we get $x^5 - x^4 - x^3 + x^2 = 90x - 126$; whence $x = 3$ the ſhillings each woman paid, and $x^2 = 9$ what each man paid; conſequently there were 12 men and 6 women.

This queſtion was anſwered in the ſame manner by Mr. *Landen*, Mr. *Croſs*, Mr. *Rubins*, Mr. *Brown*, Mr. *Adams*, Mr. *Dent*, Mr. *Cockſon*, Mr. *Holliday*, and others.

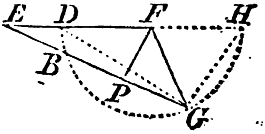
V. QUESTION 248 *anſwered by Mr.* Heath.

Put $m =$ s. $\angle EFG = 112° 37'$, [ſee the following fig.] $x = BP = PG$, and $n = EB$ the diff. of the baſe's ſegments; $y = DF = FG$, and $ED = d$ the diff. of the ſides containing the oblique angle. Now, per 47 Euc. 1, $EF^2 - EP^2 = PF^2 = FG^2 - PG^2$, i. e. $dd + 2dy + yy - nn - 2nx - xx = yy - xx$; whence $\frac{dd + 2dy - nn}{2n} = x$ by reduction. Again $EG :$ s. $\angle EFG :: FG :$ s. $\angle FEG$, i. e. $2x + n : m :: y : \frac{my}{2x + n} =$ s. $\angle FEG$; and rad. $: EF ::$ s. $\angle FEG : PF$, i. e. $1 : d + y :: \frac{my}{2x + n} : \overline{yy - xx}|^{\frac{1}{2}}$; whence $yy - xx = \frac{mmyy}{4xx + 4xn + nn} \times \overline{dd + 2dy + yy}$, or $\overline{yy - xx} \times \overline{4xx + 4xn + nn} = mmyy \times \overline{dd + 2dy + yy}$. In which if the value of x before found be ſubſtituted, an equation will come out in which there will be but one unknown quantity, viz. y.

Anſwered by Mr. James Rubins.

As $EF - GF : EP - PG ::$ coſ. $\frac{\angle E + \angle G}{2}$: coſ. $\frac{\angle G - \angle E}{2}$ (per Thacker's Miſcellany, theor. 18 p. 28.) Hence the ſides are eaſily found by plain trigonometry, and are $EF = 12{\cdot}9$, $FG = 10{\cdot}9$, and $EG = 19{\cdot}874$.

The ſame anſwered by Mr. Arch. Seyth.



In the triangle EFG draw the line DG, then the angle $FDG = \frac{\angle G + \angle E}{2}$, and $BGD = \frac{\angle G - \angle E}{2}$; but in the triangle EGH, as $EG : EH :: \text{s.} \angle H : \text{s.} \angle HGE$ $= \text{coſ.} \frac{\angle G - \angle E}{2}$), and in the $\triangle EGD$, as $\text{s.} EG : ED :: \text{s.} EDG : \text{s.} EGD$ $(= \text{s.} \frac{\angle G - \angle E}{2})$; that is, putting $ED = a$, $EB = b$, $FG = \frac{1}{2}x$, $BG = y$, and c and $d =$ ſine and coſine of half the ſum of the angles at the baſe, $y + b : x + a :: d : d \times \frac{x+a}{y+b}$ = coſine of half the difference; and $y + b : a :: c : \frac{ac}{y+b}$ = s. of half the difference; $\therefore dd \times \overline{a+x}|^2 + aacc = \overline{y+b}|^2$, or $\sqrt{ddxx + 2addx + aa}$ $= y + b$: but $\overline{x+a} \times a = \overline{y+b} \times b$, $\therefore y + b = \frac{xa + aa}{b} =$ $\sqrt{ddxx + 2addx + aa}$, $\therefore xx + 2ax = \frac{aabb - a^4}{aa - bbdd}$ and $x = -a \pm \sqrt{\frac{aabb - aabbdd}{aa - bbdd}} = \frac{abc}{\sqrt{aa - bb}} - a =$ 21'96; and therefore $\frac{abc}{2\sqrt{aa - bbad}}$ = the greater ſide $\frac{abc}{2\sqrt{aa - bbdd}} - a$ = the leſs, and $\frac{ac}{y+b} =$ '056033 = s. 3° 12½', ſo that the greater angle is 36° 54', and the leſs 30° 29'.*

VI. QUES-

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* A conſtruction of this problem may be ſeen at prob. 8 of *Simpſon's* Algebra.

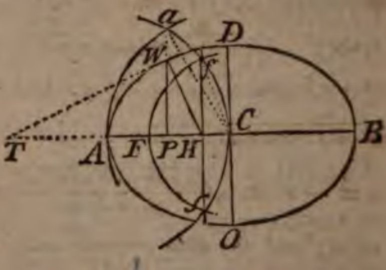
VI. QUESTION 249 *anſwered by Mr.* Farrer.

Let $y=$ the hours from ſun-riſing, n his true time of riſing, and m the length of the day; then the time to his ſetting is $m-y$, and per queſtion $\sqrt[3]{y}+\sqrt{m-y}=\overline{n+y}$ the hour of the day: Put $y=x^3$, and we have $x+\sqrt{m-x^3}=n+x^3$, $\therefore m-x^3=n^2+x^6+x^2+2nx^3-2nx-2x^4$, which in numbers is $x^6-2x^4+8{\cdot}4002x^3+x^2-7{\cdot}4022x=m^2-n^2=2{\cdot}8997$; hence $x=1{\cdot}0919$, and $x^3=y=1{\cdot}3083$, and $n+y=5{\cdot}0029=5$h. 0′ 10·44″ the time required.

Mr. *Brown* the propoſer, Mr. *Heath*, Mr. *Adams*, Mr. *F. Holliday*, Mr. *Gibſon*, Mr. *Cockſon*, and ſeveral others, have anſwered this queſtion in the ſame method.

VII. QUESTION 250 *anſwered by Mr.* Wm. Lax.

Put $AC=t=20$, $DC=c=15$, $CH=d=6$, $BH=a=26$, $AH=b=14$; and let $PH=x$. Then, per property of the curve, we have $tt:cc::\overline{a+x}\times\overline{b-x}:cc\times\frac{ab+bx-ax-xx}{tt}=PW^2$, $\therefore cc\times\frac{ab+bx-ax-xx}{tt}+xx=HW^2$, which by the queſtion is to be a minimum, $\therefore$ its fluxion $cc\times\overline{\dot{x}b-a\dot{x}-2\dot{x}x}+2tt x\dot{x}=0$; whence $x=\frac{cc}{2}\times\frac{a-b}{tt-cc}$; or (by writing $2d$ for $a-b$) $x=\frac{ccd}{tt-cc}=169{\cdot}7$ yards; conſequently $HW=13{\cdot}36$ chains, making an angle with the tranſverſe of 54° 45′.



Mr. N. Farrer's *Anſwer.*

Here is given $AB=40$, $OD=30$, $CH=6$ chains: let $CB=m=20$, $DC=n=15$, $CH=d=6$, and $PC=x$;

then, per conics, $mm : nn :: mm - xx : \frac{nnmm - nnxx}{mm} = PW^2$, and $\frac{n}{m}\sqrt{mm - xx} = PW$; $x : m + x :: m - x : \frac{mm - xx}{x} =$ ſubtangent PT; then, per ſim. triangles, $x - d : \frac{n}{m}\sqrt{mm - xx} :: \frac{n}{m}\sqrt{mm - xx} : \frac{mm - xx}{x}$, therefore $\frac{mmx - x^3 - dmm + dxx}{x} = \frac{nnmm - nnxx}{mm}$, therefore $m^4x - m^2x^3 - dm^4 + dmmxx = nnmmx - nnx^3$, that is $m^2x^3 - n^2x^3 - dmmxx + nnmmx - m^4x + dm^4 = 0$, which divided by $xx - mm$ gives $mmx - nnx - mmd = 0$, $\therefore x = \frac{mmd}{mm - nn} = 13{\cdot}714286 = PC$, and $PH = 7{\cdot}714286$, $PW = 10{\cdot}91806$, and $HW = 13{\cdot}368406$, making an angle with the tranſverſe of 54° 45′ 23″ nearly.*

This queſtion was alſo truly anſwered in the former method by Mr. *Tho. Cowper*, Mr. *J. Terey*, Mr. *John Landen*, Mr. *W. Dent*, Mr. *Adams*, Mr. *Fr. Holliday*, and Mr. *Gibbons*.

VIII. QUES-

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* VII. QUESTION 250 *Conſtructed.*

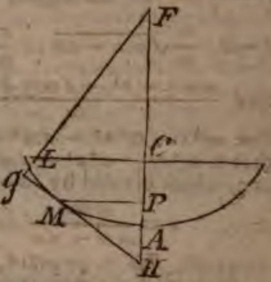
The point F being the focus, with the center C and radius CF deſcribe an arc cutting $fHf \parallel OD$ in f and f; through f, C, f deſcribe a circle; and with the center C and radius CA deſcribe another interſecting it in a; then aWP being drawn $\parallel CD$ will give the point W in the curve from whence the ſhorteſt line WH muſt be drawn.

For, having drawn the chords Cf, Ca, by the nature of the circle it will be $CP : CH ::$ (Ca^2 or) $CA^2 :$ (Cf^2 or) CF^2; hence, by diviſion, PH (or $CP - CH$) $: CH :: CD^2$ (or $CA^2 - CF^2$) $: CF^2$; $\therefore HW$ is $\perp$ the curve by prop. XV *Emerſon's* Conics, which it ought to be when it is ſhorteſt.

COROLLARY. Hence, when HW is $\perp$ the curve, CF^2 or $CA^2 - CD^2 : CA^2 :: CH : CP$. And from hence alſo the calculation is very eaſy; for $CP = \frac{CH \times CA^2}{CA^2 - CD^2} = \frac{6 \times 20^2}{20^2 - 15^2} = \frac{6 \times 400}{175} = \frac{6 \times 16}{7} = \frac{96}{7} = 13\frac{5}{7}$.

VIII. QUESTION 251 *anſwered by* Amicus.

Draw the lines FH, Fg, Hg, and LC will repreſent the weights upon every particle of any curve: let $PM = \dot{y}$, $PH = \dot{x}$, $CF = a$, $AM = \dot{z}$, and by ſimilar triangles, we ſhall have $\dot{y} : \dot{x} :: a : LC = \frac{a\dot{x}}{\dot{y}}$, whoſe flux. is $LC = \frac{a\dot{y}\ddot{x} - a\dot{x}\ddot{y}}{\dot{y}\dot{y}}$, a general expreſſion for all curves. In the preſent caſe we have $ax = y^4$, whoſe fluxion is $\dot{x} = \frac{4y^3\dot{y}}{a}$,



which ſubſtituting in the above equation $LC = \frac{a\dot{x}}{\dot{y}}$, and taking its fluxion we ſhall have $L\dot{C} = 12yy\dot{y}$; and if w expreſſes the weight that preſſes the particle M, we ſhall have $w\dot{z} = 12yy\dot{y}$, or $\frac{w\dot{y}}{a}\sqrt{aa + 16y^6} = 12yy\dot{y}$, or as the cube of the ordinate directly and tangent inverſely. *Q. E. I.*

Anſwered by Mr. Farrer.

Draw the lines as per figure to the ſolution of queſt. 244; and let a repreſent the tenſion in A, $x = AC$, $y = CB$, and $z =$ curve AB. Then we have, by the nature of the curve in the ſolution to queſtion 244, $z\dot{y} = a\dot{x}$, $2ax + xx = zz$ and $a + x =$ the tenſion: Now per queſtion $ax = y^4$, its fluxion $a\dot{x} = 4y^3\dot{y}$, $\therefore 4y^3\dot{y} = z\dot{y}$, and $4y^3 = z$ the gravity of the line or required law of the weights preſſing every particle of the line; which is as the cubes of the ordinates.

IX. QUESTION 252 *anſwered by Mr.* T. Atkinſon.

Given $AC - CB = 20 = 2n$, [ſee the fig. to queſt. 248] $CE = 60 = m$, s. $\angle BCD$ or $68^\circ = s$, its coſine $= c$; put $2y = BC + AC$, then $y + n = AC$, and $y - n = BC$, and per trig. $y+n : 1 :: m : \frac{m}{y+n} =$ s. $\angle CAE$, and $\frac{m}{y+n} : y-n :: s : \frac{syy-snn}{m} = AB$; alſo $1 : y-n :: c : cy-cn = CD$; but $AB^2 = AC^2 + BC^2 + 2AC \times CD$, i.e. $\frac{s^2y^4 - 2s^2n^2y^2 + s^2n^4}{mm} = 2y^2 + 2n^2 + 2cy^2 - 2cn^2$, or $y^4 - \frac{2s^2n^2 + 2m^2 + 2cm^2}{ss} \times y^2 = \frac{2m^2n^2 - 2cm^2n^2 - s^2n^4}{ss}$; or if for the coefficients of yy and the quantities on the other ſide of the equation be wrote $2A$ and B reſpectively, it will be $y^4 - 2Ay = B$, $\therefore y = \sqrt{A + \sqrt{B + A^2}} = 108{\cdot}4271$; hence $AC = 118{\cdot}4271$, $BC = 98{\cdot}4271$, and $AB = 180{\cdot}1279$.

The ſame anſwered by Mr. W. Kingſton.

Let $AF = a = 20$, $EC = p = 60$, s and $c =$ ſine and coſine of half the ſum of the angles at the baſe, x and y the ſine and coſine of half their difference; then will $sy + cx$ be the ſine of the greater angle at the baſe, and $sy - cx$ the ſine of the leſs; alſo $cy - sx$ and $cy + sx$ their reſpective coſines; then as $sy - cx : p :: 1 : \frac{p}{sy-cx} = AC$, and as $sy + cx : p :: 1 : \frac{p}{sy+cx} = CB$; hence $\frac{p}{sy-cx} - \frac{p}{sy+cx} = a = \frac{2cpx}{ssyy-ccxx}$, $\therefore 2cpx = assyy - accxx$: But if for yy be put its value $1 - xx$, and for cc, $1 - ss$, it will become $ass - axx = 2cpx$, $\therefore axx + 2cpx = ass$, ſubſtitute $2n = \frac{2pc}{a}$ and it will be $xx + 2nx = ss$; whence, by compleating the ſquare, and extracting the root, we have $x = \sqrt{ss + nn} - n$, and the required ſides as above.

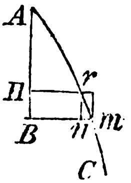
Anſwered by Mr. A. Scyth.

Put $p = 60$, $d = 20$, s and $c =$ the ſine and coſine of half the ſum of the angles at the baſe; $y =$ the baſe, and $x =$ the ſum of the ſides; then will $y : x :: c : \frac{cx}{y} =$ the coſine of half the difference of the angles at the baſe, and $y : d :: s : \frac{sd}{y} =$ its ſine, $\therefore ccxx + ssdd = yy$, and $xx = \frac{yy - ssdd}{cc}$;

but $\frac{csx+csd}{y}$ = the ſine of the greater, and $\frac{cxs-csd}{y}$ = the ſine of the leſs; therefore $\frac{csx+csd}{y} : p :: 1 : \frac{py}{cs\times\overline{x+d}}$ = the leſſer ſide, ∴ $\frac{py}{cs\times\overline{x-d}}$ = the greater, and their ſum $\frac{2pyx}{sc\times\overline{xx-dd}} = x$, ∴ $xx = \frac{2py}{sc} + dd = \frac{yy-ssdd}{cc}$, or $yy - \frac{2pcy}{s} = ccdd + ssdd = dd$; which reduced gives $y = \frac{pc \pm \sqrt{ppcc+ssdd}}{s} = 180{\cdot}122$; whence $\frac{x \pm d}{2} = \frac{\sqrt{dd + \frac{2p}{ssc} \times \overline{pc + \sqrt{ppcc+ssdd}}}}{2} \pm \frac{d}{2} = 118{\cdot}42$ the greater ſide, or $98{\cdot}42$ the leſs.

This queſtion was anſwered by Mr. *Brown*, Mr. *J. Boſton*, *E. Sugget*, *Brancepeth*, Mr. *John Corbett*, Mr. *Fr. Holliday*, Mr. *Heath*, Mr. *Gibbons*, Mr. *E. Croſs*, Mr. *Farrer*, and others.*

X. QUESTION 253 *anſwered by Mr.* N. Farrer.



Let *ArC* repreſent the curve deſcribed; draw the lines as in the figure, and let the velocity at *r* in the direction $rC = v$, $AH = x$, $rn = \dot{x}$, $Hr = y$, $mn = \dot{y}$, $rm = \dot{z}$, the required denſity as *D*, *c* the celerity, and law of reſiſtance as ac^n; then $\dot{y} : v :: \dot{x} : \frac{vx}{\dot{y}}$ = the velocity in the direction *nr*, its fluxion is $\frac{\dot{v}\dot{x}+v\ddot{x}}{\dot{y}}$ = the increaſe of velocity during the time of deſcribing *rm*; $\dot{y} : \dot{x} :: \dot{v} : \frac{\dot{v}\dot{x}}{\dot{y}}$ the part ariſing from the reſiſtance of the medium; therefore $\frac{v\ddot{x}}{\dot{y}}$ = the part ariſing from the force of gravity. The reſiſtance is to the force of gravity as $-\frac{v\ddot{x}}{\dot{y}}$ to $\frac{v\ddot{x}}{\dot{y}}$, or as $-\frac{v\ddot{x}}{v\dddot{x}}$ to 1; but $\frac{v\ddot{x}}{\dot{y}}$, the velocity ariſing from gravity, being proportional to the time $\frac{\dot{y}}{v}$ of deſcribing *rm*, may be expreſſed thereby; hence $\frac{v\ddot{x}}{\dot{y}} = \frac{\dot{y}}{v}$ or $vv\ddot{x} = \dot{y}\dot{y}$, in fluxions $2v\dot{v}\ddot{x} + vv\dddot{x} = 0$, or $-\frac{\dot{v}}{v} = \frac{\dddot{x}}{2\ddot{x}}$, which ſubſtituted in the foregoing proportion $-\frac{\dot{v}\dot{z}}{v\ddot{x}}$: 1, gives $\frac{\dot{z}\dddot{x}}{2\ddot{x}^2}$: 1 the ratio of the reſiſtance to the gravity.

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* To this prob. a Conſtruction is given in prob. 78 of *Simpſon's* Algebra.

Again, ſince the abſolute velocity is $\frac{v\dot{z}}{\dot{y}}$ the reſiſtance by by ſuppoſition will be $a \times \overline{\frac{v\dot{z}}{\dot{y}}}\Big|^{n}$; hence D as $\frac{\dddot{x}}{a\dot{z}^{n-1} \times \ddot{x}^{\frac{4-n}{2}}}$ Which when $n = 2$, or the reſiſtance as the ſquare of the velocity will be $\frac{\dddot{x}}{\dot{z}\ddot{x}}$: But, per queſtion, the equation of the curve is $ax = y^3$, ∴ $a\dot{x} = 3y^2\dot{y}$, $\ddot{x} = \frac{6y\dot{y}^2}{a}$, and $\dddot{x} = \frac{6\dot{y}^3}{a}$, ∴ the denſity $\frac{\dddot{x}}{\dot{z}\ddot{x}}$ will be as $\frac{1}{\frac{y}{a}\sqrt{9y^4+a^2}}$, or as the tangents reciprocally.

Anſwered by the Propoſer.

Becauſe the reſiſtance is as the ſquare of the celerity, $\frac{\dddot{x}}{\ddot{x}\sqrt{\dot{x}^2+\dot{y}^2}}$ (Simpſon's Eſſays, p. 60) will expreſs the required denſity: But by the equation of the curve ($ax=y^3$) we have $\dot{x}=\frac{3y^2\dot{y}}{a}$, $\ddot{x}=\frac{6y\dot{y}^2}{a}$, $\dddot{x}=\frac{6\dot{y}^3}{a}$, and $\sqrt{\dot{x}^2+\dot{y}^2}=\frac{\dot{y}}{a}\sqrt{a^2+9y^4}$; therefore, by ſubſtitution, the required denſity becomes $\frac{6\dot{y}^3}{\frac{6y\dot{y}^3}{a}\sqrt{a^2+9y^4}}=\frac{a}{y\sqrt{a^2+9y^4}}$; or by a farther reduction $=\frac{1}{\sqrt{yy+9xx}}$; i. e. it is always as the tangent of the diſtance from the vertex to unity.

This queſtion was anſwered in like manner by *Amicus*, Mr. *C. Cockſon*, Mr. *J. Landen*, and Mr. *Hanbury*.

XI. QUESTION 254 *anſwered.*

Let $bb=$ the ſquare of the common hypothenuſe, and xx the ſquare of one leg, then will $bb-xx$ be the ſquare of the other leg. Suppoſe $b-nx=$ that leg whoſe ſquare is $bb-xx$, then will $bb-xx=bb-2bnx+nnxx$, which equation reduced gives $x=\frac{2bn}{1+nn}=$ to one leg, and if inſtead of x in $b-nx$ we take its value laſt found, we ſhall have $\frac{1\backsim nn}{1+nn}=$ the other leg; and ſince n may be aſſumed at pleaſure, not only eight, but any number of right-angled plain triangles whatſoever may be found; which will alſo appear by making the common hypothenuſe the diameter of a circle.

XII. QUESTION 255 *anſwered by Mr.* J. Landen.

By conſidering the queſtion, I find that the firſt time the hands are diametrically oppoſite after 6 o'clock, is the time of the day ſought. Let x hours be the time from 6: now the minute hand going round once in an hour, the rounds it will be carried in the time x, may be repreſented by x; and the hour hand going round once in 12 hours, the part of the circumference it will be carried in the ſame time by $\frac{x}{12}$. It is evident that the index whoſe motion is ſwifteſt, will outgo the ſloweſt, one circumference in x hours; whence $x - \frac{x}{12} = 1$, and $x = \frac{12}{11}$; therefore the time required is 5 min. 27 ſec. paſt 7.

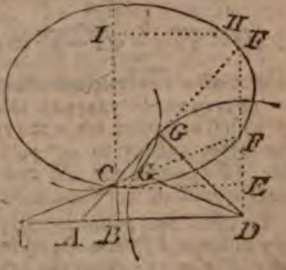
Anſwered by Mr. James Terey.

By the data it appears to be ſomewhat paſt 7. Let $x =$ the number of minutes paſt 7; then will $\frac{x}{12}$ be the diſtance of the hour hand from 7, and $25 - \frac{x}{12}$ the diſtance of the hour hand from 12; hence $25 + x - \frac{x}{12} = 30'$; $\therefore x = \frac{60}{11}$ $= 5\frac{5}{11}$ paſt 7, the time required.

This queſtion was truly anſwered by *Lincolnienſis*, Mr. *Jepſon*, Mr. *Holliday*, Mr. *Kingſton*, Mr. *Brown*, Mr. *Williams*, and others.

The PRIZE QUSTION *anſwered by Mr.* Ja. Terey.

Put BC, the height of the front wall above the back, $= 12 = d$, $BD = 40 = a$ the breadth of the houſe, $GD = GF = 37 = b$, and CG the required length of the front rafters $= x$; then $EF = \sqrt{xx + 2bx + bb - aa}$; and $AC : BC :: CF : FE$, i. e. $b - x : d :: b + x : \sqrt{xx + 2bx + bb - aa}$, therefore $db + dx = \overline{b - x} \times \sqrt{xx + 2bx + bb - aa}$; which equation, ordered,



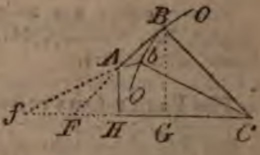
makes $x^4 \begin{matrix}-2bb\\-aa\\-dd\end{matrix} xx \begin{matrix}+2baa\\-2ddb\end{matrix} x = \begin{matrix}ddbb\\bbaa\\-b^4\end{matrix}$. In numbers, $x^4 - 4482xx + 107744x = 513375$; whence $x = 6\cdot51224$ &c. or $x = 20\cdot21768$ &c. either of which determines the length of the front rafters.

Geometrically thus: Let $AG = 37$ always paſs through the point C, the point A ſliding along the line AB; then will G the point at the other end of the line AG deſcribe the curve $CGGL$. On D as a center with the radius $= 37$, deſcribe a circle, and it will cut the ſaid curve in the points G, G, which determines the length and poſition of the front rafters, as is evident by inſpection.

Or let $AF = 2AG$ be moved as before, and it will cut the perpendicular DF in the points F and F, whence CG &c. is known as before.

N. B. If AF, CB, CI, and IH be called a, b, x, and y reſpectively; then will $\frac{xxaa}{xx+2bx+bb} - xx = yy$ expreſs the nature of all ſuch curves.

The ſame anſwered by Mr. John Corbett.

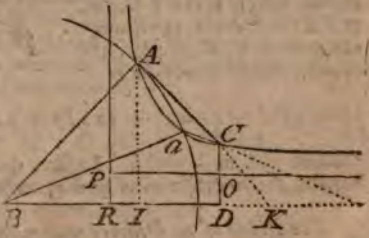


Put $x =$ ſine of the angle $AFH = GCB$, whoſe coſine is $\sqrt{1-xx}$, radius 1; $BC = 37 = b$, $HA = 12 = c$, and $HC = 40 = d$: Then as $x : c :: 1 : \frac{c}{x} = FA$, and as $1 : b :: \sqrt{1-xx} : b\sqrt{1-xx} = GC$, for the triangles BGC and AHF are alike. As $1 : \frac{c}{x} (= FA) :: \sqrt{1-xx} : \frac{c}{x}\sqrt{1-xx} = FH$; conſequently $\frac{c}{x}\sqrt{1-xx} + d = 2b\sqrt{1-xx}$; which, reduced, gives $x^4 - \cdot324324x^3 - \cdot681519x^2 + \cdot324324x = \cdot0262965$. Two of the roots of which equation are $x = \cdot71507$ and $x = \cdot39403$, which in the tables anſwer to 45° 39′ and 23° 12′; and thence the ſide $AB = 20\cdot22$ and $AB = 6\cdot54$ are found, either of which lengths will anſwer the conditions of the queſtion.

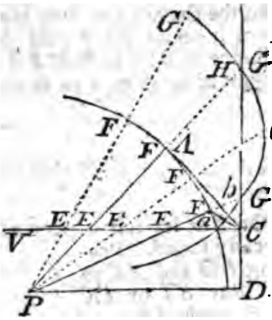
To effect this geometrically, Extend the line $HC = 40$ to any diſtance Ff, at one end of which H erect the perpendicular $HA = 12$; from the other end C with the extent of 37 deſcribe the arc *oo*; then lay a ruler upon the point *A*, and move it about till it cuts the arc *oo* and the line *fH* at a diſtance $= BC$; ſo ſhall *BA* or *bA* be the length of the roof ſought $= 20\cdot22$ or $6\cdot54$ as above.

Anſwered by the Propoſer.

On *BD* let fall a perpendicular from *A*, being ſuppoſed already found. Prolong *BD* and *AC* to *K*. Put *BI* or *IK* $= x$, and $AI = y$; then is $BK = 2x$, from which ſubtract $BD = a$, and there will remain $DK = 2x - a$. Now DK $(2x - a) : DC\ (b) :: IK\ (x) : AI\ (y)$, $\therefore$ $2xy - ay = bx$, an equation belonging to an hyperbola. Put x infinite, then is $y = \frac{1}{2}b$; and by putting y infinite, $x = \frac{1}{2}a$. Hence, if *BD* and *DC* are biſected, and the lines *PO*, *PR* drawn reſpectively parallel thereto, they will be two aſymptotes to the ſaid hyperbola. To find a point in the curve, put $x = a$ in the hyperbolic equation, then is $y = b$. Conſequently *C* will be a point in the curve, through which draw the hyperbola *AaC*.



Per 47 Euc. 1, $BI^2 + AI^2 = AB^2\ (= cc)$, or $xx = cc - yy$ (which is an equation belonging to a circle), therefore $x = \sqrt{cc - yy}$. Put the ſurd $= o$, then $x = o$, whence the center will be in *B*; whence alſo $y = c$ or $y = -c$, which determines the radius. Therefore, if on the center *B*, with the radius *AB*, be deſcribed the arc *Aa*, which cuts the above conſtructed hyperbola in the points *A*, *a*, and the lines *AB* and *AC*, or *aB* and *aC*, be drawn, they will be the ſides of the roof required.



This queſtion admits likewiſe of a very eaſy and elegant ſolution, by the help of another curve, and is thus performed: *BD* and *DC* being drawn as before, and likewiſe on the center *B* the arch *Aa*; lengthen the line *DC*, drawing *CV* parallel to *BD*, and ſuppoſe an infinite number of radii drawn from the center *B*, on which make *FG*, &c. = *FE*, &c. drawing through the points *G*, *G*, *G*, &c. the curve, whoſe equation is

$$y^4 + 2y^3b \begin{array}{l} +xx \\ -4aa \\ +bb \end{array} y^2 + 2xxby + xxbb = 0,$$

and it will cut the line *DC* in *H* and *b*. Laſtly, draw *BH* and *Bb*, which cut the above circle in *A* and *a*, as before.

The Prize of 10 *Diaries was won by Mr.* J. Corbett.

Of the Eclipſes in 1745.

There will happen two eclipſes this year; twice will the ſun loſe its light by the interpoſition of the moon's opaque body to ſome part of our terraqueous globe, in the following manner.

The firſt is on March 22, betwixt 2 and 3 of the clock in the morning, and therefore inviſible in England, but conſpicuous to our Antipodes, and much greater in the Eaſt Indies; eſpecially in ſome of the Phillippine Iſlands, the ſun will be vertically eclipſed

The ſecond happens Sept. 14, in the afternoon, but inviſible at London, by reaſon of the paralax in lat of the ☽ a ☉. Whence the moon is far depreſſed below the ſun's limb, which proves it inconſpicuous; but in ſome part of America it will be total and central; and near the meridian of port Royal, in Jamaica, it will happen in the following order:— Begins 14th day, 10h. 15m. 10s. in the morning, apparent time. Mid. 11h. 35m. 8s. Duration 2h. 34m. 4s. Ends 12h. 43m. 14s. Digits eclipſed 7°.

W. Leighton.

New

New Queſtions.

I. QUESTION 256, *by Mr.* J. Turner.

In a 'pothecar's ſhop an old mortar I found,
Which being deem'd uſeleſs, was thrown on the ground.
The inſide dimenſions are plac'd * here below;
From whence its content in wine gallons I'd know.

* Given the perpendicular height of the mortar = 9 inches, bottom diameter = 6, top diameter = 12, and the curvature of the mortar's ſides are ſuppoſed to be the apollonian parabola, whoſe vertex is a point on the uppermoſt edge of th mortar, or extremity of the top diameter.

II. QUESTION 257, *by Mr.* T. Cowper.

The late phænomenon conſpicuous here,
Each eve ſerene i'th' weſtern hemiſphere,
(When Sol withdrew his radiance from our ſight)
With blazing tail and tremulating light,
Amongſt thoſe orbs in the concave expanſe,
Which ſeem around this penſile world to dance;
Its nucleus firſt we in the æther ſaw,
'Twixt Pegaſus and fair Andromeda;
From whence, by motion retrograde, it run
With gentle pace towards th' approaching ſun,
Till courſe and declination ſo conſpire,
Both eve and morn preſents its ſanguine fire:
This diff'rence only, that its ſtreaming tail
Deſcends direct below th' horizon's vail,
But in the morn unfolds its orient light
In oblique glances, to the wond'ring ſight;
Whoſe length'ning train, glowing in azure ſkies,
Fills gazing mortals with immenſe ſurprize.
Whether they in elliptic orbits run,
By gravitation, round the central ſun?
Or but as tranſient fiery balls appear,
Thrown off in tangents from the ſolar ſphere?
That, the Newtonian ſyſtem doth regard;
This, the late the'ry of a modern bard.
As themes uncertain, leave we them behind,
As yet inſcrutable to human kind,
Perhaps reſerv'd for future years to find.
Soon as Aurora with refulgent beams,
Obſcur'd each leſſer conſtellation's gleams,

The cyprian ſtar her ſcintillating rays
Near the horizon ſplendidly diſplays;
* Fifty-three minutes paſt † apparent time,
I likewiſe ſaw the comet eaſtward ſhine;
Whoſe nucleus (by a common quadrant view'd)
Had five degrees one-third of altitude:
Its diſtance from bright Venus (taken true)
Was fifty-ſix degrees and ſix-tenths too.
Deduct refraction from its height before:
By ſpherics hence the comet's place explore.

* In lat. 52° 20′ N. Feb. 12, 1744. † Paſt 5.

III. QUESTION 258, *by Mr.* J. Powle.

Given $1 + 2 + 3 + 4$, &c. continued to x terms; to find $1 + 4 + 9 + 16$, &c. the ſum of the ſquares of thoſe numbers.

IV. QUESTION 259, *by Mr.* Landen.

A cannon ball projected from the ground, in a direction making an angle of 14° 29′ with the horizon, fell at the feet of a perſon ſome diſtance off the very moment he heard the report of the piece: Quere, how far he was from the place of projection?

V. QUESTION 260, *by Mr.* N. Farrer.

In an oblique-angled triangular grove, one of whoſe ſides is 20 chains, and the angle oppoſite thereto 78° 45′, if a perpendicular be let fall from each angle to its oppoſite ſide, they interſect at a fountain within the grove, whoſe neareſt diſtance to the given ſide is 8·19 chains: Quere its diſtance from each of the other ſides, by a ſimple equation?

VI. QUESTION 261, *by Mr.* C. Cockſon.

There are two ponds of water of the ſame quality and depth, under the ſame meridian, one in the lat. of $a - 16b$* north, and the other $8b = 5125$ miles due north from it. In the year 1743-4, the 16th of January, at three of the clock in the morning, the thickneſs of the ice in the ſouthermoſt was 6 inches. Quere the latitude, and thickneſs of the ice of the northermoſt pond at the ſame time?

* $\frac{\sqrt{aaa} + \sqrt[4]{aaa} - \sqrt[3]{aa}}{b} = a.$

VII. QUES-

VII. QUESTION 262, *by Mr.* Powle.

To determine the aſymptotes of a curve whoſe equation is $x^5 - y^5 + axy = 0$.

VIII. QUESTION 263.

Let *AFK* be the conchoid of Nicomedes, [*ſee the fig. to the prize queſt. for* 1738] and *BC* the aſymptote whoſe length is 60, and *P* the pole; a line drawn from *P* perpendicular to the aſymptote, to the curve at *A*, is 40; alſo from the pole to the aſymptote is 20: Required the length of the curve line *AFK*, with the analytical inveſtigation?

IX. QUESTION 264, *by* Amicus.

To determine the greateſt area that can be encloſed by a parabolic curve of the ſecond kind, whoſe (equation is $ax^2 = y^3$, and) length 100 feet, and an ordinate rightly applied to its greateſt axis?

PRIZE QUESTION *by Mr.* N. Farrer.

A bragging young gauger pretending to ſhew
The content of a caſk from what's given below,
Occaſion'd this wager—Five guineas to two:
He's try'd all his ſkill, but all will not do;
So begs the aſſiſtance, fair ladies, of you.

The length of the caſk is 31·907 inches, bung diameter 24 inches, and is the lower fruſtum of two equal conoids, generated by the rotation of a curve about its axis, whoſe equation is $y^y - 1000000x = 0$.

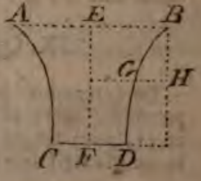
A PARADOX, *by* Amicus.

It has been aſſerted by a late celebrated mathematician, that if a veſſel formed by the rotation of an hyperbola round one of its aſymptotes be filled with water, and a hole made in the bottom of this veſſel, (let the hole be ever ſo large, and the depth of the veſſel ever ſo ſmall, it will take an infinite time to be exhauſted. Quere how this can be?

1746.

Queſtions anſwered.

I. QUESTION 256 *anſwered by Mr.* J. Landen.



LET *ABCD* repreſent the mortar, generated by the revolution of the parabolic curve *BD*, about the axis *EF*, parallel to *BH*. Call *EB*, b; *BH*, x; *GH*, y; and put $3{\cdot}1416 = p$. Then will $p\dot{x} \times \overline{bb - 2by + yy}$ be the fluxion of the required ſolidity; which by putting for y and yy, their values $x^{\frac{1}{2}}$ and x (found by the equation of the curve) will be $p\dot{x} \times \overline{bb + x - 2b\sqrt{x}}$, and its fluent $pbbx + \frac{1}{2}pxx - \frac{4}{3}pbx\sqrt{x}$; which, when $x = 9 = EF$, will be 466·5276 inches = 2·0108, &c. wine gallons. *Q. E. F.*

This queſtion was alſo ſolved by Mr. *Heath*, Mr. *Farrer*, Mr. *Powle*, *Bironnos*, Mr. *Aſh*, Mr. *R. Williams*, Mr. *Arch. Scyth*, and Mr. *Bamfield*.

II. QUESTION 257 *anſwered by* Bironnos.

The place of Venus at the given time is ♑ 20° 50′ 45″, her latitude 1° 22′ 7″ N. declination 20° 27′ ſouth, and is 44° 29′ 45″ ſhort of the meridian: Hence there is known, $Z\odot$, the comet's true zenith diſtance = 84° 48′ 35″, [*ſee the fig. to Q.* 220] ZP, the co-latitude of the place, = 37° 40′, $\angle ZPO$ = 44° 29′ 45″, OP the diſtance of Venus from the pole = 110° 27′, and $O\odot$ the comet's diſtance from Venus = 56° 36′. In $\triangle ZPO$ is known ZP, PO, and $\angle P$: Find the $\angle ZOP$ = 25° 35′ 49″, and ZO = 82° 25′ 41″. In the $\triangle ZO\odot$, are known the three ſides: Find the $\angle ZO\odot$ = 88° 44′ 54″: From which take the $\angle ZOP$, and there remains the $\angle PO\odot$ = 63° 9′ 5″. Then, in the $\triangle PO\odot$, is known PO, $\odot O$, and $\angle PO\odot$: Find the $\angle OP\odot$ = 48° 59′ 54″, and $P\odot$ = 80° 44′ 20″. Therefore the comet's declination is 9° 15′ 40″. N. and right aſcen. 341° 19′ 34″. Hence the comet's longitude is ♓ 16° 25′ 26″, and latitude 15° 54′ 54″ N.

III. QUESTION 258 *anſwered by Mr.* J. Powle.

Put $S = 1 + 4 + 9 + 16$, &c. $\overset{\prime}{S} =$ the next ſucceeding term in the ſeries to the *laſt*, or the increment of S when the number of terms are $x + 1$. Then, becauſe the firſt term and common difference are each unity, $x + 1 = \sqrt{\overset{\prime}{S}}$, which in the ſeries $\underset{\prime\prime}{x}\,\underset{\prime}{x}\,\overset{\prime}{x}\,\overset{\prime\prime}{x}$ &c. anſwers to x, $\therefore \overset{\prime}{S} = \underset{\prime}{x}\underset{\prime}{x}$, or $\underset{\prime}{x}x + \underset{\prime}{x}$. Whence $S = \frac{1}{3}\underset{\prime}{x}xx + \frac{1}{2}\underset{\prime}{x}x =$ (by ſubſtitution) $\frac{1}{3} \times x + 1 \times x - 1 \times x + \frac{1}{2} \times x + 1 = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$. *Q. E. F.*

Anſwered by Bironnos.

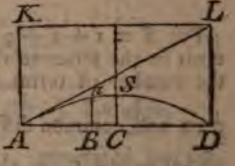
Let $Ax^{n+1} + Bx^n + Cx^{n-1} + Dx^{n-2}$ &c. .... $+ L = 1^n + 2^n + 3^n$ &c. to x^n; then will $\overline{Ax+1}|^{n+1} + \overline{Bx+1}|^{n} + \overline{Cx+1}|^{n-1}$ &c. .... $+ L = 1^n + 2^n + 3^n$ &c. to $x^n + \overline{x+n}|^{n}$; from which ſubtract the former, and there remains $A \times \overline{x+1}|^{n+1} - x^{n+1} + B \times \overline{x+1}|^{n} - x^n + C \times \overline{x+1}|^{n-1} - x^{n-1}$ &c. $= \overline{x+1}|^{n}$. By expanding the ſeveral powers of $x + 1$ we get $A = \frac{1}{n+1}$, $B = \frac{1}{2}$, $C = \frac{n}{3.4}$, $D = 0$, $E = \frac{n \times n - 1 \times n - 2}{2.3.4.5.6}$. Conſequently $1^n + 2^n + 3^n$ &c. $= \frac{x^{n+1}}{n+1} + \frac{x^n}{2} + \frac{nx^{n-1}}{3.4} - \frac{n \times n - 1 \times n - 2}{2.3.4.5.6}x^{n-3}$ &c. Which when $n = 2$ will become $\frac{2x^3 + 3x^2 + x}{6}$, the theorem required.

This queſtion was anſwered by Mr. *Farrer*, Mr. *Jepſon*, Mr. *Peter Wood*, Mr. *Arch. Scyth*, Mr. *Landen*, Mr. *J. Aſh*, Mr. *Williams*, and laſtly by Mr. *Heath* in an elegant and general manner.

IV. QUES-

IV. QUESTION 259 *anſwered by Mr.* N. Farrer.

Let ASD repreſent the path of the projectile, thrown from A, with a velocity that will carry it in a perpendicular aſcent to K; or in the direction AL, to L; and let this celerity carry it through the diſtance d in the time u, and the diſtance run through by a falling body in that time $= w$. Put any diſtance $AB = y$, s and e the ſine and coſine of the angle of direction LAD; then will $\frac{usy}{de}$ be the time in which the projectile runs through the curve Aa, and $w \times \overline{\frac{usy}{de}}|^2$ the diſtance deſcended by a heavy body in that time; $\therefore Ba = \frac{seddy - wuussyy}{ddee}$; and when this $= CS$, then $y = \frac{sedd}{w}$, and the time of deſcription $= \frac{sdes}{w}$. Put 1142 the feet ſound moves in 1 ſecond $= q$; then $q : 1 :: \frac{sedd}{w} : \frac{sdu}{w}$, therefore $d = \frac{qu}{e}$; conſequently $\frac{sqquu}{ew} =$ the required diſtance. Now let $u = 1''$, then $w = 16\frac{1}{12}$ feet; hence $AD = 20895{\cdot}106$ feet $=$ 3 m. 7 f. 21 p. and 2 yards nearly, and the time of deſcription $\frac{sdu}{w} = 18{\cdot}29$ ſeconds.



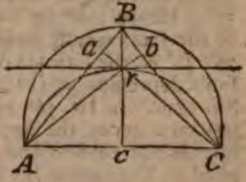
Mr. Tho. Cowper's *Anſwer.*

Let AL be the line of direction, and AD the diſtance of the perſon from the place of projection. Put $t =$ tangent of the $\angle DAL$; $d = 16\frac{1}{12}$ feet, the perpendicular deſcent of heavy bodies in 1 ſecond; $b = 1142$ feet, the velocity of ſound in the ſame time; and $x =$ the time the ball was in motion. Then $bx = AD$, and $dxx = DL$; but as $1 : t :: bx : tbx = DL$, $\therefore x = \frac{tb}{d}$; hence $\frac{tbb}{d} = AD = 20945{\cdot}59$ feet, the diſtance ſought.

This queſtion was ſolved by Mr. *Heath*, Mr. *Powle*, Mr. *Jepſon*, Mr. *Bamfield*, Mr. *Davies*, Mr. *Terey*, *Bironnos*, Mr. *Williams*, Mr. *T. Garrard*, Mr. *J. Aſh*, and Mr. *Landen* the propoſer.

* V. QUESTION 260 *anſwered by Mr.* Heath.

It is evident by the data that the triangle muſt be iſoſceles. In the $\triangle ABC$, the $\angle A = \angle C$, or the $\angle B$ muſt be the given angle; and either way the $\angle ArB = \angle CrB$, or $\angle ArC$ is given $= 101° 15'$ oppoſite to the ſide given $= 20$ chains, on which deſcribing a ſegment of a circle to contain it, and drawing the parallel diſtance 8·19 (or rather 8·2 as it ſhould be) chains $= cr$, and it will be found a maximum (as is alſo proved by trigonometry), therefore the data are rc, $Ac = cC$, and $\angle B$, $\angle A = \angle C$, and $\angle BAb$; whence follow by trigonometry $Ar = Cr = 12{\cdot}925$, $ar = rb = 2{\cdot}5215$ $(AB = BC = 15{\cdot}64)$ chains, required.



VI. QUES-

V. QUESTION 260.

The meaning of this prob. is thus: In a triangle we have given the baſe (AC), the vertical angle (ABC), and the diſtance (cr) of the baſe from the common point of interſection of three lines drawn from the three angles perpendicular to the oppoſite ſides; to determine the triangle.

We are not at liberty to ſuppoſe the triangle iſoſceles, for that would be introducing a condition too much into the problem; and whether it be iſoſceles or of any other form, can only appear from the conſtruction or calculation.

CONSTRUCTION. On the given baſe deſcribe two ſegments of circles, the one ABC to contain the given vertical angle, and the other ArC to contain its ſupplement; parallel to AC, and at the given diſtance of the point from it, draw a line to cut or touch ArC in r; then through r draw $crb \perp AC$, and B will be the vertex of ABC the triangle required.

For, through r drawing Arb and Crc, ſince by the conſtruction AC is the given baſe, ABC the given vertical angle, and rc the given diſtance, we have only to prove that the angles at a and b are right angles. Now by the conſtruction it appears that ArC, ABC are ſegments of equal circles, and that the two ſegments to-

gether

VI. QUESTION 261 *anſwered by Mr.* N. Farrer.

Given $\frac{\sqrt{aaa}+\sqrt[4]{aaa}-\sqrt[3]{aa}}{b}=a$. In which write $y^{12} = a$, and it is $y^{10}+y-640\cdot625y^{4}-1=0$, whence $y = 2\cdot936056$, and $y^{12}=a=410367$; from which ſubtract $16b = 10250$ and there remains 400117 miles = 6668° 37′, from which ſubtract $360 \times 18 = 6480°\ 0'$, and there remains 188° 37′. Hence $188°\ 37' - 180°\ 0' = 8°\ 37'$ S. the latitude of of the ſouthermoſt pond; which taken from 85° 25′, leaves 76° 48′ N. the latitude of the northermoſt. In lat. 8° 37′ S. ſun's depreſſion = 34° 35′, continuance under the horizon 8h. 48m. In lat. 76° 48′ N. ſun's depreſſion = 27·48; continuance under the horizon 91 days. And, ſuppoſing the intenſity of cold as the ſines of the ſun's depreſſion multiplied by the time of his continuance under the horizon, and the ſolidity or thickneſs of ice in the ſubtriplicate ratio thereof, we have $\sqrt[3]{\cdot 2081} : \sqrt[3]{42\cdot 441} :: (\cdot 5926 : 3\cdot 488 ::)$ 6 inches : 35·31 inches, the thickneſs of the ice of the northermoſt pond.

VII. QUES-

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gether make up a whole circle; then the $\angle ABr$ or $aBr = \angle ACr$ as ſtanding on equal ſegments Ar, and the oppoſite angles arB, crC are alſo equal; ∴ the third angles are equal, that is $\angle a = \angle c =$ a right angle. In the like manner b is proved to be a right angle.

SCHOLIUM. Another method of conſtruction might be by firſt finding the point r as before, through which draw the indefinite lines Arb, Cra, and perpendicular to them the lines CbB, AaB. ——And then we ſhould have to prove that theſe laſt two lines and cr, produced, meet in the ſame point B, and that ABC is = the given angle.

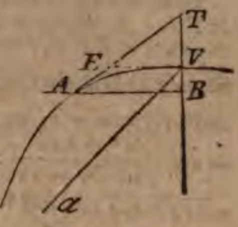
COROLLARY 1. The equal oppoſite angles Ara, Crb, being the ſupplements of the $\angle ArC$, are each = the $\angle ABC$, which is alſo the ſupplement of ArC by the conſtruction.

COROLLARY 2. Hence alſo the $\angle BAr = BCr$, and the four triangles BAb, aAr, BCa, bCr are all ſimilar.

THE METHOD OF CALCULATION will be, firſt to calculate the angles rAC, rCA, of the $\triangle ArC$, by prob. V. *Simpſon*'s Algebra, and then the ſegments Ac, Cc; then to each of theſe two angles adding the $\angle BAb$ or $BCa =$ the comp. of $\angle ABC$, there will be had the $\angle BAC$ and BCA; from which and the ſegments Ac, Cc, the two hypotheuuſes AB, BC are eaſily got, and will come out 15·386 and 16·127, the angles at A and C being 52° 16′ and 48° 59′.

* VII. QUESTION 262 *anſwered by Mr.* N. Farrer.

Here is given the equation of the curve $x^5 - y^5 + axy = 0$.



Let $zx = y$, then $x = \sqrt[3]{\frac{az}{z^5 - 1}}$. $y = \sqrt[3]{\frac{az^4}{z^5 - 1}}$, and $\frac{y\dot{x}}{\dot{y}} = BT = \frac{4z^5 + 1}{z^5 + 4} \times \sqrt[3]{\frac{az}{z^5 - 1}}$, $\therefore VT = \overline{\frac{4z^5 + 1}{z^5 + 4} - 1} \times \sqrt[3]{\frac{az}{z^5 - 1}}$; and when the flowing quantity becomes infinite, the tangent AT will become an aſymptote, in which caſe $VT = 4\sqrt[3]{\frac{a}{z^4}}$ and $\dot{x} : \dot{y} :: 1 + 4z^5 : 4 + z^5 :: VT : EV = \frac{16 + 4z^5}{1 + 4z^5} \times \sqrt[3]{\frac{a}{z^4}}$; hence the poſition of the aſymptote is determined.

VIII. QUES-

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* VII. QUESTION 262.

This queſtion may be much better performed from the original equation alone without any ſubſtitution. Thus in the given equation $x^5 - y^5 + axy = 0$, ſuppoſing x to be infinite, the term axy will vaniſh in compariſon of x^5 or y^5, and then $x^5 - y^5 = 0$, and $x = y$; that is, at an infinite diſtance the abſciſſa is $=$ the ordinate, and therefore the aſymptote, or tangent at the infinite diſtance, muſt make an angle of 45° with the abſciſſa. Again the given equation in fluxions gives $\dot{x} = \frac{5y^4 - ax}{5x^4 + ay} y$, hence the ſubtangent BT or $\frac{y\dot{x}}{\dot{y}}$ is $= \frac{5y^5 - axy}{5x^4 + ay} =$ (by expunging y^5) $\frac{5x^5 + 4axy}{5x^4 + ay}$, and conſequently $VT = BT - x = \frac{3axy}{5x^4 + ay} =$ (when $x = y =$ infinite) $\frac{3a}{5x^2} = 0$, and therefore the aſymptote paſſes through the vertex V and makes an angle of 45° with VB. And the form of the curve is as repreſented in the above figure, where Va is the aſymptote.

VIII. QUESTION 263 *anſwered by Mr.* Heath.

To rectify the conchoid of Nichomedes generally. Let $b = PB$, [*ſee the fig. to the Prize Q. for* 1738] $a = BA$ ($= b = 20$), $y = EF$ any ordinate, and $x = MD = BE$. Then becauſe DF is always equal to BA, $EF = \frac{b+x}{x}\sqrt{aa-xx}$ and $AE = a - x$, its correſponding abſciſſa. In fluxions $-\dot{x} \times \frac{x^3+baa}{xx\sqrt{aa-xx}} = \dot{y}$, whence $\sqrt{\dot{y}\dot{y}+\dot{x}\dot{x}} = ax^{-2}\dot{x} \times \sqrt{bbaa+2bx^3+x^4} \div \sqrt{aa-xx} = ax^{-2}\dot{x} \times : ba + \frac{x^3}{a} + \frac{x^4}{2ba} - \frac{x^6}{2ba^3}$ &c. $\times : \frac{1}{a} + \frac{x^2}{2a^3} + \frac{3x^4}{8a^5}$ &c. the fluent of which terms collected is $\frac{bx}{2a} + \frac{x^2}{2a} + \frac{x^3}{6ba} + \frac{bx^3}{8a^3} + \frac{x^4}{8a^3} + \frac{x^6}{16a^5}$ &c. $- \frac{ba}{x} - \frac{x^5}{20ba^3} - \frac{x^7}{112ba^5} - \frac{3x^9}{114ba^7}$ &c. but when $y = 60$, $x = 8{\cdot}6$ fere; conſequently when $-x$ in the correſponding abſciſſa $= 8{\cdot}6$, or $x = -8{\cdot}6$, then the fluent or curve $= 43{\cdot}4$; but when $x = a$, and $x = -a$ in the abſciſſa, the fluent or curve $= 19{\cdot}6$, which added to the former is 63 fere, the length of the curve required.

IX. *The* 185*th or* 264*th* QUESTION *ſolved by Mr.* Heath.

By the equation of the curve $axx = y^3$ (a being as yet unknown, but conſidered as fixed) we get $\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}} = \dot{y}\sqrt{\frac{9y+4a}{4a}}$ for the fluxion of the curve, whoſe fluent (corrected) is $\frac{8a}{27} \times \overline{\frac{9y+4a}{4a}}|^{\frac{3}{2}} - \frac{8a}{27} = c = 50$. Whence $aa + \frac{27cc-36yy}{16\times\overline{c-y}}a = \frac{27y^3}{16\times\overline{c-y}}$. And

$$a = \frac{\sqrt{27\times64cy^3-16y-72ccyy+27c^4}+36yy-27cc}{32\times\overline{c-y}}.$$

The fluxion of the area of the ſemi parabola $= y\dot{x} = \frac{3y\dot{y}\sqrt{y}}{2\sqrt{a}}$, whoſe fluent $\frac{3\sqrt{y^5}}{5\sqrt{a}}$ muſt be a maximum, and conſequently $\frac{y^5}{a}$; therefore

$$\frac{cy^5 - y^6}{\sqrt{3 \times \overline{64cy^3 - 16y^4 - 72ccyy + 27c^4}} + 12yy - 9cc}$$ will

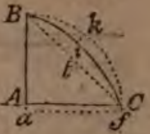
be a maximum. Put into fluxions, &c. $\overline{5c - 6y} \times$

$$\overline{\sqrt{3 \times \overline{64cy^3 - 16y^4 - 72ccyy + 27c^4}} + 12yy - 9cc} - 3$$

$$\times \frac{96cyy - 32y^3 - 72ccy}{\sqrt{3 \times \overline{64cy^3 - 16y^4 - 72ccyy + 27c^4}}} + 24yy \times \overline{c - y}$$

$= 0$; here $y = 34·89$ by a new method of ſolving equations, and conſequently $a = 35·543$ fere, and the area of the whole parabola formed thereby (which is now the greateſt) 1447·4 fere.

In anſwer to the objections by *Amicus*, a, in the equation, is as much a variable quantity as x or y, till it is determined. And in the equation $\frac{8a}{27} \times \overline{\frac{yy + 4a}{4a}}\big|^2 - \frac{8a}{27} = a$ (where $y = \frac{a^{\frac{1}{3}} \times \overline{27c + 8a}^{\frac{2}{3}} - 4a}{9}$) it has a variable relation to y, when $\frac{y^5}{a}$ or $\frac{y}{a^{\frac{1}{5}}}$ is to be determined a maximum; and ſubſtituting this way for the value of y, $\frac{a^{\frac{1}{3}} \times \overline{27c + 8a}|^{\frac{2}{3}} - 4a}{9a^{\frac{1}{5}}}$, or $a^{\frac{2}{15}} \times \overline{27c + 8a}|^{\frac{2}{3}} - 4a^{\frac{4}{5}}$, will will as properly expreſs the maximum, as if it had been denoted by relative y's; hence by making the fluxion of it $= 0$, we get by reduction $aa - \frac{9c}{16}a = \frac{27cc}{16^2}$, where the value of $a = \frac{3c\sqrt{21} + 9c}{32} = 35·5433$ &c. whence $y = 34·8909$ &c. and $\frac{6y^{\frac{5}{2}}}{5a^{\frac{1}{2}}} = 1447·4001$, the area of the greateſt parabolic ſpace, as before. Alſo when $c = 10$, then $x = 6·913$, $y = 6·978$, and $a = 7·1087$ &c. And when $c = 1000$, $x = 691·3$, $y = 697·82$, and $a = 710·87$; by which it is proved that when the figure is a maximum, the abſciſſa and ordinate will be nearly equal.



In the preſent caſe, where $c = 50$, let $x = y$, then the equation $axx = y^3$ becomes $axx = xxy$ when the ſemi-parabola is nearly the greateſt; and conſequently $a = y = x$, at that time $= \frac{27c}{13^{\frac{3}{2}} - 8} = 34{\cdot}7292$ &c. from what is done above. Whence 1447·34 will be the area of the parabola very nearly a maximum. And therefore in all queſtions of this nature, the area may be computed by taking the ordinates and abſciſſas equal; the error being inconſiderable in the maximum. Draw the parabola *ABiCA* correct, and it will approach the form of a quadrant *aBkCfa*, as near as the inequality of the curve *BiC* permits; and its double will ever be inſcribed in a ſegment of a circle ſomething leſs than a ſemicircle: but if the point *C* be made to paſs through *f*, it will be inſcribed exactly in a ſemi circle, and the area of the ſemiſpace *ABiCA* will vary from the true maximum, but by an exceeding ſmall quantity, as is evident from above. When *ABkCA* is a maximum, the ſpace *ABtCA*, or △ *ABC* is a maximum, which is when $AB = AC$: for, by the equation $x = \frac{y\sqrt{y}}{\sqrt{a}}$, and $yx = \frac{yy\sqrt{y}}{\sqrt{a}}$ or $\frac{y^5}{a}$ is a maximum as proved before: but ſince the relation of y and a can be only had from the rectification of the curve *BiC*, and its equation, with the length of curvature given, the inequality of the (*fiddleſtick*) ſpace *BiCtB*, or unequal curvature of *BiC*, involves a neceſſity of ſome little inequality betwixt *AB* and *AC*, when *ABiCA* is moſt capacious.*

The

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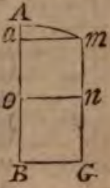
* This queſtion is the ſame as the 185th, the ſolution of which was not compleated. Both the above two methods of ſolution bring out true anſwers, but the latter is much the eaſier. By Mr. *Emerſon*'s method explained at the ſolution of the prize queſtion for 1741 the ſame concluſions are alſo eaſily obtained.

The PRIZE QUESTION *anſwered by Mr.* R. Heath.

Projecting the curve *AmnG*, which is eaſily done by making $y = 1, 2, 3, 4, 5,$ &c. and thence finding the x's by the equation $\frac{y^7}{1000000} = x$; whence it will appear that when the ſemi-bung diameter $y = 7$ inches, that x will be but ·823543 which the half length of the caſk ſhould be taken out of, and proves an anſwer to be impoſſible: but correcting the data, and making the ſemi-bung diameter $y = 8 = BG$, then $x = 16{\cdot}777216 = AB$; whence taking 15·9535 (or 15·953673) the ſemi-length of the caſk, and there remains $\cdot 823543 = Aa$; whence $am = 7$ inches, or the head diameter = 14, bung diameter = 16, and length 31·907346 inches; and being near the form of a cylinder, the mean diameter = 15 inches, and content of the ſemi-caſk by the rotation of $BoamnG$ about $aoB = 2819{\cdot}25$ &c. or 10 ale gallons, and the whole caſk 20 gallons.



The prize of 10 *Diaries was won by Mr.* Heath.

The PARADOX *anſwered by Mr.* Tho. Sparrow.

Since the velocity of the fluid is always in the ſub-duplicate ratio of the height of its ſurface above the hole, 'tis evident that when that height is infinitely ſmall, the velocity muſt be ſo too, *i. e.* in effect, *Nothing*: Conſequently the water can never be exhauſted.

Of the Eclipſes in 1746.

Calculated by Mr. Ralph Hulſe.

To the inhabitants of our terraqueous globe there will happen four eclipſes, two of each luminary. The 1ſt of the moon, February 4th, but inviſible at London, as ending 22 m. 44 s. before the moon riſes. The 2d of the ſun, on the 4th of March, early in the morning, but inviſible in the horizon of London. The 3d is a viſible eclipſe of the moon, the 19th of Auguſt, 3 quarters paſt 10 at night, and viſible at London, according to the following calculation, viz. Beg. 10h. 36m. Mid. 12h. 8m. End 1h. 24m. Total duration 2h. 38m. Digits eclipſed 8 deg. 22 m. The 4th is of the ſun, September 4th, in the afternoon, but inviſible.

New Queſtions.

I. QUESTION 265, *by Mr.* Heath,

Myſterious things, we always find,
Are moſt amuſing to mankind;
When once familiar they appear,
We look for more another year.
Juſt ſo, when men are plainly known,
We're weary of acquaintance grown;
We hug the ſtrange, and leave the true,
And ſtill are ſeeking ſomething new.
Ladies, how comes this ſtrange inconſtancy,
So viſible in you, as well as me.

The QUESTION.

If the hind wheel of a coach be ſeven feet in diameter, and a tack be driven into the middle of the ſpoke (or radius) ſtanding next the ground, and a nail touch the ground at the end of the ſaid ſpoke (or radius) when the coach ſets out to travel: Quere how many miles will the tack and nail travel reſpectively in driving the coach from London to Exeter; allowing the diſtance between thoſe two places to be 200 miles? What will be the nature of the curves they deſcribe? And their poſition, or height of tack and nail from the ground, at the end of the journey.

II. QUESTION 266, *by Mr.* Farrer.

Surveying a triangular field ABC, and ſtanding at the corner C, I took the angle included between the ſide BC and a line drawn from the angle C, to a houſe ſituated within the field, and found it 78° 10′. I then proceeded to meaſure the ſhorteſt diſtance to the oppoſite ſide AB, and having meaſured 20 chains, I obſerved the houſe and the angle A in a right line; then meaſured on 10 chains to the ſide AB: I likewiſe obſerved that the ſides AC and BC were equal, and the houſe equally diſtant from the angles A and C. Quere the area of the field?

III. QUESTION 267, *by Mr.* Powle.

Given $35x + 43y + 55z = 4000$, to find all the poſſible *values* of x, y, z, in whole numbers, and to ſhew the *method of* inveſtigation?

IV. QUES-

IV. QUESTION 268, *by* Fortunatus.

Let the ſorts of faces to be thrown on ſeven dice by four perſons, at a ſingle throw each, be as follow, viz. by *A*, a^4bcd; by *B*, a^2bcdef; by *C*, $a^2b^2c^2d$; by *D*, a^3b^2cd: Quere their reſpective chances of winning? And what throw, as to ſorts of faces, has the greateſt number of chances for coming up, at a ſingle throw, of all the ſorts which can be thrown on the ſaid number of dice?

N. B. The number of the ſame and different letters repreſent ſo many of the ſame and different ſorts of faces, viz. ſo many aces, duces, trays, cators, &c. of the ſame and different ſorts.

V. QUESTION 269, *by Mr.* J. Aſh.

A gentleman has a piece of ground, whoſe three ſides are an abſciſſa, ſemi-ordinate, and curve, the equation of which is $ax = y^3$; he has taken from thence the biggeſt oblong garden which he could poſſibly encloſe, whoſe area is 141·84243 poles; and he finds the abſciſſa longer than the ſemi-ordinate by 5 poles. It is required from thence to find the area of the whole encloſure, its perimeter, and the ſides of the garden (taken out of it) ſeparately?

VI. QUESTION 270, *by* Rhinoceros.

The perpendicular of a triangular field is 200 poles; the line equally biſecting the angle oppoſite to the baſe, drawn to the baſe, is 250 poles; and the diſtance from the ſaid obtuſe angle to the middle of the baſe is 295 poles: Quere the ſides and area of that triangular field, with the geometrical conſtruction of the ſame.

VII. QUESTION 271, *by Mr.* Chriſt. Maſon.

Lately young Chloe ſtruggling to be coy,
And ſtill prolong her Strephon's wiſh'd-for joy,
Did artfully a ſtratagem contrive,
Herſelf to ſtint, her Strephon ſtill deprive:
But he yet preſſing with the urgent *when*
Shou'd he be made the happieſt of men;
Your when, quoth ſhe (if you can make't appear)
That night the twilight's ſhorteſt in the year.
Pray lend your aid the nuptial night to fix;
The latitude is fifty, forty-ſix. *i. e.* 50° 46′

VIII. QUESTION 272, *by* F. R. S.

In what law of gravity will a projectile deſcribe a curve expreſs'd by the equation $ax^3 = y^5$, in a non-reſiſting medium?

IX. QUESTION 273, *by Mr.* Farrer.

Quere the area of a right-angled triangle whoſe hypothenuſe is x^{3x}, and the two legs x^{2x} and x^x?

X. QUESTION 274, *by Mr.* Clarke.

The thickneſs of a ring belonging to a ſhip's anchor is nine inches in circumference, and the outward circumference ſhewing the width of that ring is 50 inches: Quere the ſolid content, and weight thereof?

XI. QUESTION 275, *by* Filius Diophanti.

To find three numbers, that when each is ſubtracted from the cube of their ſum, a cube number ſhall remain?

XII. QUESTION 276, *by Mr.* J. Landen.

It is required to find the periodic time of a pendulum deſcribing a conical ſurface; the perpendicular height of the deſcribed cone being 200 inches?

XIII. QUESTION 277, *by* Crocus Metallorum.

What annuity, to continue as many years as its pounds, can I purchaſe for the ſquare of its pounds ready money, allowing me 5 l. per cent. per ann. compound intereſt for my bargain?

XIV. QUESTION 278, *by* Hurlothrundro.

Required the ratio of the diameter of the bore to the length of a piece of cannon (or other fire arms) to make it capable of throwing a ball the fartheſt poſſible; ſuppoſing the diameter of the ball nearly equal to the diameter of the bore, with a proportionable weight of powder, and the metal of the piece formed ſufficient to ſuſtain the effect?

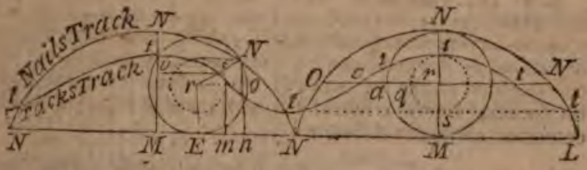
PRIZE QUESTION *by Mr.* W. Chapple.

A gentleman has a circular garden, whoſe diameter is 310 yards, in which is contained a circular pond, whoſe diameter is 100 yards, ſo ſituated in reſpect of each other, that their peripheries will inſcribe and circumſcribe an infinite number of triangles [*i. e.* whoſe ſides ſhall be tangents to the pond, and angles in the fence of the garden.] He being diſpoſed to make encloſures for different uſes, and farther ornaments on his ſcheme begun, in order thereto applies himſelf to the artiſts of Great Britain for the dimenſions of the greateſt and leaſt triangles that can be inſcribed and circumſcribed as aforeſaid? and the neareſt diſtance of the peripheries of the garden and pond? and for a demonſtration of the truth of his pond's ſituation?

1747.

Queſtions anſwered.

I. QUESTION 265 *anſwered by the Propoſer Mr.* Heath.



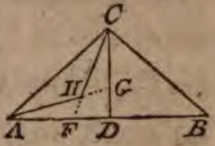
THE track of the nail will be the curve *LNNONN* &c. which is a cycloid, and that of the tack the curve *ttt* &c. both which curves are thus rectified. Put $a = NM = 7$, $x = Nr$, $y = Or$, $v = ar = \sqrt{ax - xx}$, $z =$ circular arch $Na = Oa$ (per nature curve), $\because z + v = y$, $vv = ax - xx$, $\dot{v} = \frac{a - 2x}{2v} \times \dot{x}$, $\dot{z} = \frac{a\dot{x}}{2v}$, alſo $\dot{y} = \dot{z} + \dot{v} = \frac{a - x}{v}\dot{x} =$

$\frac{a-x}{\sqrt{ax-xx}}\dot{x}$, hence $\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}=\dot{x}\sqrt{\frac{a}{x}}$, whoſe fluent is $2\sqrt{ax}=$ arch NO; therefore when $x=a$, arch $NON=2a$, conſequently $NONNtL=4a=28$ feet, the nail deſcribes in one revolution of the wheel. The nature of the curve deſcribed by the tack is expreſſed by $cq=2$ arch qt, which referred to the foregoing ſymbols, will be $y=2z+v$, parts of the leſſer circle. Conſequently, by ſubſtitution in this caſe, the fluxion of the inner curve ctt will be $\frac{\dot{x}}{2}\sqrt{\frac{9aa-8ax}{ax-xx}}$; whoſe fluent, by ſeries, is $\sqrt{ax}\times:3+\frac{x}{18a}+\frac{7xx}{216aa}+\frac{613x^3}{3\times7\times6^4a^3}$ &c. which when $x=ts=3{\cdot}5=a$, is 11·696 fere (but this fluent may be otherwiſe found). Hence the tack deſcribes 23·392 feet in one revolution, whilſt the nail deſcribes 28, and the axis 21·9911485 &c. $=7\times3{\cdot}141592653$ the wheel's circumference; by which the wheel will revolve 48019·31995 times in travelling 200 miles.——But 21·9911485 : 28 :: 200 : 254·648 miles, travelled by the nail. And 21·9911485 : 23·392 :: 200 : 212·74 miles travelled by the tack pretty nearly; the ſmall difference being only what EN (·31995 part of a revolution) differs in proportion with N, N, and t, t, part of curves deſcribed by nail and tack at the end of the journey, in the poſition of N, t, r, making an $\angle NrE$ $=\cdot31995\times360=115^\circ\ 11'$, or $Nro=25^\circ\ 11'$ with the horizon. Whence $Nn=4{\cdot}988$ feet, and $tm=4{\cdot}244$ the height of nail and tack from the ground.

N.B. The number of whole revolutions multiplied into the whole curves aforeſaid, and the rectifications of the laſt parts being reſpectively added, will exactly ſhew the diſtances deſcribed by the nail and tack, very nearly as before. Or this queſtion might be reſolved by a curve deſcribed upon the curve of a great circle of the earth, which ſolution would come out not much different.

Mr. *Bamfield* has curiouſly deſcribed theſe curves, and given an exact ſolution of their lengths; and ſo has Mr. *I. Waine*; which are the only true anſwers. Mr. *Bamfield* has found a point of inflection and retrogreſſion to be in the inner curve, when the ſtroke and tack are horizontal.

II. QUESTION 266 *anſwered by Mr.* I. Waine.



Put $x=$ ſine, and $y=$ coſ. $\angle CAD=\angle CBD$, and s and c for thoſe of the given $\angle FCB$. Then by elements of trigonometry, $sx-cy=$ ſine, and $sy+cx=$ coſ. $\angle FCD$. But $\angle ACD - FCD = \angle ACF$ whoſe tang. is $\frac{syy-sxx+2cxy}{cxx-cyy+2sxy}$, which per queſt. $=$ tang. $\angle CAG$. But $AD : GD ::$ radius : tang. $\angle GAD=\frac{x}{3y}$; whence $\frac{2xy}{3yy+xx}=$ tang. $\angle CAD - \angle GAD = \frac{syy-sxx+2cxy}{cxx-cyy+2sxy}$; reduced, $\frac{x^4}{y^4}+\frac{6xx}{yy}-\frac{8cx}{sy}=3$; ſolved, $\frac{x}{y}=\cdot 80907=$ tang. of 38° 58′ 30″; whence $AD=37\cdot0796$, and conſequently the area $=111\cdot239$ acres. *Q.E.F.*

Mr. *Kingſton*'s anſwer is the ſame.

Mr. Aſh *anſwers this* Queſtion *thus.*

Call the tang. of the given angle m; CD, a; and GD, c; alſo put x for tang. $\angle ACD$. Then tang. $\angle ACB$ will be $\frac{2x}{1-xx}$, and that of $\angle ACH$ (when $\angle ACB$ is obtuſe) $\frac{2x+m-mxx}{2mx+xx-1}$. But (by trig.) $\frac{c}{ax}=$ tang. $\angle DAG$, $\therefore \frac{ax-cx}{axx-c}=$ tang. $\angle HAC=\angle ACH$; conſequently $\frac{ax-cx}{axx-c}=\frac{2x+m-mxx}{2mx+xx-1}$; ſolved, $x=1\cdot2359=$ tang. of 51° 1′ fere. Whence the area of the field $=111\cdot23$ acres.

Mr. *John Turner* has elegantly ſolved this queſtion; ſo has Mr. *Anth. Baker*, and ſome others.

Mr. *Cuth. Cockſon* informs us that this queſtion is taken from Ronayne's Algebra, p. 273, being caſe 1 of prob. 12: We would do the propoſer all the honour due to ſo diſtinguiſhed a genius, but yet we deſire to have ſent what is new, as well as curious; being rather deſirous that the Ladies' Diary ſhould be a pattern for, than an imitation of others.

III. QUES-

III. QUESTION 267 *anſwered by Mr.* J. Waine.

For z in the given equation ($35x + 43y + 55z$) ſubſtitute its leaſt value, viz. 1, and we have $y = 91\frac{32}{43} - \frac{35}{43}x$, therefore $\frac{32 - 35x}{43}$ muſt be ſome whole number; multiplied by 2 and divided, is $1\frac{21}{43} - \frac{27x}{43} - x$, $\because$ $\frac{21 - 27x}{43} \times 2$ will be a whole number $= \frac{42}{43} - \frac{11x}{43} - x$, $\because$ $\frac{42 - 11x}{43} \times 4$ is alſo ſome whole number $= 3\frac{39}{43} - \frac{x}{43} - x$, whence $\frac{39 - x}{43}$, or rather $\frac{x - 39}{43} = m$, a whole number, $\because$ $x = 43m + 39$, which value of x ſubſtituted for x in $y = 91\frac{32}{43} - \frac{35x}{43}$, gives $y = 60 - 35m$; whence $x = 43m + 39 = 82$ or 39, $y = 60 - 35m = 60$ or 35, $z = 1$. And thus by aſſuming $z = 1$, 2, 3, 4, &c. we obtain all the poſſible values of x, y, and z, viz. z admitting of 65 different values, and x and y of 89.

Mr. *John Turner* confirms the ſame by working out all the numbers. Mr. *Farrer* has exhibited a concise method for finding thoſe numbers; and Mr. *Landen* is very explicit in finding the ſame. Mr. *Cuth. Cockſon*, Mr. *Flitcon*, and Mr. *John Williams* likewiſe anſwered this queſtion.

The following Table is a Compendium of Mr. *Turner*'s, deduced from $x = \frac{4000 - 43y - 55z}{35}$, and alſo $= \frac{20z + 8y - 10}{35}$, by finding a Submultiple of 35.

z	y	x	z	y	x	z	y	x
1	25 60 —	82 39 —	4	— 35 70	— 65 22	7	10 45 80	91 48 5
2	5 40 75	105 62 19	5	15 50 85	88 45 2	8	25 60 —	71 28 -
3	20 55 —	85 42 —	6	30 65 —	68 25 —	9	5 40 75	94 51 8
And ſo on to $z = 65$								

IV. QUES-

IV. QUESTION 268 *anſwered by Mr.* Heath.

No ſolution has appeared in any author to queſtions of this nature, which will admit of ſeveral varieties ſtill to be propoſed. Mr. *Kay*'s queſtion was the firſt propoſed, in a particular caſe, and the general method of ſolution is exhibited in the following examples, not hit upon before, that I have ſeen, by any.

Sorts of Faces.	Nº of Chances.	Combinations.	Permutations of each comb.
a^2	6	Sides revolve	
ab	30	$=\frac{6.5}{1.2}$ Prog. of equal indices. ×	$\frac{2.1 \text{ dec. dice}}{1.1 \text{ decreaſe of higheſt indices of faces.}}$
6^2 all ch. on 2 dice	$=36$		
a^3	6		
a^2b	90	$=6.5$ ×	$\frac{3.2.1}{2.1.1}$
abc	120	$=\frac{6.5.4}{1.2.3}$ Prog. eq. ind. ×	$\frac{3.2.1}{1.1.1}$
6^3 all ch. on 3 dice	$=216$		
a^4	6		
a^3b	120	$=6.5$ ×	$\frac{4.3.2.1}{3.2.1.1}$
a^2b^2	90	$=\frac{6.5}{1.2}$ ×	$\frac{4.3.2.1}{2.1.2.1}$
a^2bc	720	$=\frac{6.5.4}{1.2}$ ×	$\frac{4.3.2.1}{2.1.1.1}$
$abcd$	360	$=\frac{6.5.4.3}{1.2.3.4}$ ×	$\frac{4.3.2.1}{1.1.1.1}$
6^4 all ch. on 4 dice	$=1296$		
a^5	6		
a^4b	150	$=6.5$ ×	$\frac{5.4.3.2.1}{4.3.2.1.1}$
a^3b^2	300	$=6.5$ ×	$\frac{5.4.3.2.1}{3.2.1.2.2}$
a^3bc	1200	$=\frac{6.5.4}{1.2}$ ×	$\frac{5.4.3.2.1}{3.2.1.1.1}$
a^2b^2c	1800	$=\frac{6.5.4}{1.2}$ ×	$\frac{5.4.3.2.1}{2.1.2.1.1}$
a^2bcd	3600	$=\frac{6.5.4.3}{1.2.3}$ ×	$\frac{5.4.3.2.1}{2.1.1.1.1}$
$abcde$	720	$=\frac{6.5.4.3.2}{1.2.3.4.5}$ ×	$\frac{5.4.3.2.1}{1.1.1.1.1}$
6^5 all ch. on 5 dice	$=7776$		

Faces	Chances.	Combinations.	Permutations of each comb.
a^6	6		
a^5b	180	$= 6.5$	$\times \frac{6.5.4.3.2.1}{5.4.3.2.1.1}$
a^4b^2	450	$= 6.5.4$	$\times \frac{6.5.4.3.2.1}{4.3.2.1.2.1}$
a^4bc	1800	$= \frac{6.5.4}{1.2}$	$\times \frac{6.5.4.3.2.1}{4.3.2.1.1.1}$
a^3b^3	300	$= \frac{6.5}{1.2}$	$\times \frac{6.5.4.3.2.1}{3.2.1.3.2.1}$
a^3b^2c	7200	$= 6.5.4$	$\times \frac{6.5.4.3.2.1}{3.2.1.2.1.1}$
a^3bcd	7200	$= \frac{6.5.4.3}{1.2.3}$	$\times \frac{6.5.4.3.2.1}{3.2.1.1.1.1}$
$a^2b^2c^2$	1800	$= \frac{6.5.4}{1.2.3}$	$\times \frac{6.5.4.3.2.1}{2.1.2.1.2.1}$
a^2b^2cd	16200	$= \frac{6.5.4.3}{1.2.1.2}$	$\times \frac{6.5.4.3.2.1}{2.1.2.1.1.1}$
a^2bcde	10800	$= \frac{6.5.4.3.2}{1.2.3.4}$	$\times \frac{6.5.4.3.2.1}{2.1.1.1.1.1}$
$abcdef$	720	$= \frac{6.5.4.3.2.1}{1.2.3.4.5.6}$	$\times \frac{6.5.4.3.2.1}{1.1.1.1.1.1}$
6^6 all ch. on 6 dice	$= 46656$		
a^7	6		
a^6b	210	$= 6.5$	$\times \frac{7.6.5.4.3.2.1}{6.5.4.3.2.1.1}$
a^5b^2	630	$= 6.5$	$\times \frac{7.6.5.4.3.2.1}{5.4.3.2.1.2.1}$
a^5bc	2520	$= \frac{6.5.4}{1.2}$	$\times \frac{7.6.5.4.3.2.1}{5.4.3.2.1.1.1}$
a^4b^3	1050	$= 6.5$	$\times \frac{7.6.5.4.3.2.1}{4.3.2.1.3.2.1}$
a^4b^2c	12600	$= 6.5.4$	$\times \frac{7.6.5.4.3.2.1}{4.3.2.1.2.1.1}$
a^4bcd	12600	$= \frac{6.5.4.3}{1.2.3}$	$\times \frac{7.6.5.4.3.2.1}{4.3.2.1.1.1.1}$
a^3b^3c	8400	$= \frac{6.5.4}{1.2}$	$\times \frac{7.6.5.4.3.2.1}{3.2.1.3.2.1.1}$
$a^3b^2c^2$	12600	$= \frac{6.5.4}{1.2}$	$\times \frac{7.6.5.4.3.2.1}{3.2.1.2.1.2.1}$
a^3b^2cd	75600	$= \frac{6.5.4.3}{1.2}$	$\times \frac{7.6.5.4.3.2.1}{3.2.1.2.1.1.1}$
a^3bcde	25200	$= \frac{6.5.4.3.2}{1.2.3.4}$	$\times \frac{7.6.5.4.3.2.1}{3.2.1.1.1.1.1}$
$a^2b^2c^2d$	37800	$= \frac{6.5.4.3}{1.2.3}$	$\times \frac{7.6.5.4.3.2.1}{2.1.2.1.2.1.1}$
a^2b^2cde	75600	$= \frac{6.5.4.3.2}{1.2.1.2.3}$	$\times \frac{7.6.5.4.3.2.1}{2.1.2.1.1.1.1}$
a^2bcdef	15120	$= \frac{6.5.4.3.2.1}{1.2.3.4.5}$	$\times \frac{7.6.5.4.3.2.1}{2.1.1.1.1.1.1}$
6^7 all ch. on 7 dice	$= 279936$	Sum, &c. *ad infinitum.*	

Hence the reſpective chances of *A*, *B*, *C*, and *D* winning are obvious, as 12600, 15120, 37800, and 75600, the ſame as 1, 1·2, 3, and 6 exactly. And a^3bbcd and $aabbcde$ have equal, and the greateſt number of chances for coming up, viz. 75600. Hence alſo is inferred that the beſt throw for winning will be when the ſorts are within a place or two of being all different. *Q. E. F.*

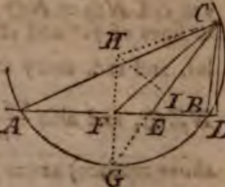
V. QUESTION 269 *anſwered by Mr.* John Turner.

Put $x = HG$, $y = GF$, $b = AD$, $b - x = GD$, [*ſee fig. page* 109] $a =$ parameter, the equation of the curve being $ax = y^3$, $\because y = \sqrt[3]{ax}$, and $\overline{b-x} \times \sqrt[3]{ax}$ is to be a maximum; which in fluxions, &c. gives $-\sqrt[3]{ax} + \frac{b-x}{\sqrt[3]{aaxx}} \times \frac{a}{3} = 0$; hence $3x = b - x$, and $x = \frac{1}{4}b$ exactly. To proceed, put $z = HD$, $\frac{z}{4} = HG$, $z - 5 = DA$, $\frac{3z}{4} = GD$, $m = 141{\cdot}84243$; then $GF = \sqrt[3]{\frac{az}{4}}$, $DA = \sqrt[3]{az} = z - 5$, $d = \sqrt[3]{4} = 1{\cdot}5874$, and per queſt. $\sqrt[3]{\frac{az}{4}} \times \frac{3z}{4} = m$. Hence $\sqrt[3]{az} = \frac{4dm}{3z} = z - 5$; $3zz - 15z = 4dm$; here $z = 20$ exactly. And $a = 168{\cdot}75$; alſo the length of the curve $HFA = 27{\cdot}014$. So that the fences of the garden are $GF = 9{\cdot}4494$, and $GD = 15$: Moreover $HD = 20$, $DA = 15$. And the area $HFADH = \frac{3}{4} HD \times DA = 225$ ſquare poles, or 1 acre, 1 rood, 25 perches.

This queſtion was conciſely ſolved by Mr. *Farrer*. And alſo ſolved by Mr. *J. Waine*, and by Mr. *Kingſton* the ſame; the Rev. Mr. *Baker*; the propoſer, Mr. *Aſh*; and others.

VI. QUESTION 270 *anſwered by Mr.* Farrer.

At the point *D*, on the line *AD* raiſe the perp. $CD = 200$ poles; make $CE = 250$, and $CF = 295$; produce *CE* to cut the perp. *FG* in *G*, draw *CH* making the $\angle GCH = \angle CGH$; then upon *H*, as a center, with the rad. *HG* deſcribe the arch *CBGA*. Join *CB* and *CA*, and the $\triangle ABC$ is that required.

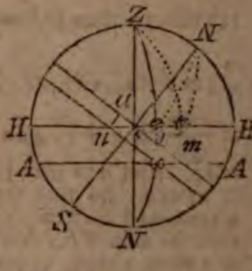


CALCULATION. Having CD, CE, and CF given, find $ED=150$, $FD=216·85$, and $EF=66·15$. But $ED:DC :: EF:GH=89·14$; ∵ $GE=111·42$; and $GF:GE :: GI:GH=225·89$; hence $AB=359·58$, $BC=203·41$, and $AC=441·21$. The area 35958 ſquare poles, or 224 a. 2 r. 38 perches.

Mr. *Aſh* has elegantly conſtructed and ſolved this queſtion, and ſo has Mr. *John Turner*, Mr. *Joſeph Orchard*, Mr. *John Hunter*, (and Mr. *James Tercy*, who objects to the ſolution of queſtion 260 by Mr. *Heath*, as ſuppoſing him not to know that three perpendiculars let fall from the angles of a triangle would interſect, in one point, within or without the triangle; his expreſſion of data not inferring ſuch thing.) It is alſo analytically ſolved by Mr. *Richard Gibbons*, Mr. *Baker*, Mr. *J. Waine*, Mr. *Kingſton*, and others.

VII. QUESTION 271 *anſwered by Mr.* J. Turner.

If N repreſent the north pole, Z the zenith, H☼ an arch of the horizon, A☼A a parallel of the ſun's depreſſion 18° below it; and ☼☉ a parallel of declination deſcended; then the angle ☉N☼ will be a minimum: but 'tis alſo evident that when the crepuſculum is the ſhorteſt, the diſtance ☼☉ deſcended in the parallel of declination will be a minimum: being the arch of a leſſer circle cutting the ſame azimuth in the points of ſetting and end of twilight. And this will be when the motion of the ſun is moſt perpendicular, and therefore deſcends the faſteſt to 18°, or the parallel of A☉A; which he does by touching at the points ☼ and ☉ in the ſame azimuth Z☼☉ with his ſetting, inſtead of making an oblique angle with the ſetting azimuth, as in the poſition of ☼m☉ with Za☼. This being admitted, put $y=$ coſ. N☉ $=N$☼; m and n ſine and coſ. Z☉ $=108°$, or ſines of 72° and 18°; p and q ſine and coſ. $NZ=$ comp. lat. and rad. unity $=Z$☼. Then per ſpherics $\frac{y}{p}=$ coſ. NZ☼, and alſo $\frac{nq-y}{mp}=$ coſ. NZ☉, which (for the above reaſons) are $=$ to each other, i. e. $\frac{y}{p}=\frac{nq-y}{mp}$, therefore



fore $y=\frac{nq}{m+1}$, which is a general theorem for all queſtions of this nature. And the ſun's declination in the preſent caſe is found by it $=7°\ 2'\ 42''$ ſoutherly, anſwering to Feb. 19, or Sept. 30; $\because m+1 : n :: q : y$.

N. B. If the lat. is north, the declination muſt be ſouth, and vice verſa.

Mr. Farrer *anſwers thus.*

Let ⊙ be the point in the circle bounding twilight 18° below the horizon, where the ſhorteſt twilight happens (without ſaying why the bounding point falls in the ſame azimuth circle with the ſun's ſetting.) Then in the triangle bc☼, ∠☼ $=90°$, $\angle c=$ co-lat. ☼$b=9°$, c☼ $=cn$, whence rad. : tang. ☼b :: ſine lat. : ſine arc $=7°\ 3'$ fere, the ſun's declination when Strephon is allowed to wed his Cloe.

Mr. *J. Turner* (from Dr. Gregory's Elem. of Aſtron.) confirms the ſame proportions.*

VIII. QUESTION 272 *anſwered by Mr.* Landen.

Let ArC repreſent the curve; AB the axis thereof; Bm and Hr ordinates indefinitely near each other. [*See fig. to Q.* 253.] Call AH, x; Hr, y; and the gravity G. Then ſince the velocity in the direction Hr is always the ſame, that in the direction rn will be $\frac{\dot{x}}{\dot{y}}$, whoſe fluxion $\frac{\ddot{x}}{\dot{y}}$ ($\dot{y}$ being conſtant) will be as $G\times\dot{y}$; that is, as the force by which the body is accelerated at r drawn into the time of deſcribing rm. Hence putting $G\times\dot{y}=\frac{\ddot{x}}{\dot{y}}$ we have $G=\frac{\ddot{x}}{\dot{y}\dot{y}}$. In which expreſſion if for $\ddot{x}$ we put its value $\frac{15\dot{y}\dot{y}\sqrt{y}}{4\sqrt{a}}$, found by the equation of the curve ($axx=y^5$) we get $G=\frac{15\sqrt{y}}{4\sqrt{a}}$; i. e. the gravity in this caſe muſt be in a ſubduplicate ratio of the ordinates, or in a ſubquintuplicate ratio of x, the diſtance of the ordinate from the vertex.

Mr. *Farrer* attempted this ſolution by another method, as alſo did Mr. *Aſh.*

IX. QUES-

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* A conſtruction to this queſtion is given at queſtion 564.

* IX. QUESTION 273 *anſwered by Mr.* I. Aſh.

Let $y=x^x$, then (per 47 E. 1, and per qu.) $y^2+y^4=y^6$, $y^2=\frac{\sqrt{5}+1}{2}=1\cdot618034$, $y=1\cdot272019$, and the area required $=1\cdot02908$.

Mr. *Richard Gibbons* ſolved this queſtion in the ſame elegant manner. The Rev. Mr. *Baker* likewiſe gave a curious ſolution, and ſo did Mr. *Farrer* the propoſer, and others.

X. QUESTION 274 *ſolved by Mr.* Landen.

Put $d=$ diff. of the given circumferences, $r=$ rad. of the leſſer circumference, and $x=$ any abſciſſa of the circle whoſe radius is r. Then $d\times2\sqrt{2ax-xx}\times\dot{x}$ is the fluxion of half the required ſolidity. But the fluent of $2\sqrt{2ax-xx}\times\dot{x}$, when $x=r$, is the area of the ſemi-circle, whoſe radius is r; therefore, the area of the circle ſhewing the thickneſs of the ring, multiplied by d, the diff. of the given circumferences, will give the ſolid content of the ring $=264$ inches fere. Whence the weight thereof, according to Dr. Wiberd, is nearly $=77$ pounds avoirdupois.

Mr. *John Turner*, and ſeveral others, have proved the ſolidity of the ring to be equal to a cylinder whoſe length is equal to the middle circumference, and the area at the baſe equal to the area of its circular ſection, or of the circle whoſe diameter expreſſes the thickneſs; moſt agreeing in the ſolidity to be $=264$ &c. inches, and the weight 73 pounds, &c. according to Ward's proportions.

N. B. There are ſeveral methods of inveſtigating the fluxion of this ring, whoſe fluents reſpectively give the ſolid content as above.

XI. QUES-

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* IX. QUESTION 273.

Of this triangle, the perpendicular from the right angle on the hypothenuſe is $=1$. For it is $=$ the double area divided by the hypothenuſe $=$ the product of the two legs divided by the hypothenuſe $=\frac{x^x\times x^{2x}}{x^{3x}}=\frac{x^{3x}}{x^{3x}}=1$.

XI. QUESTION 275 *anſwered by Mr.* J. Hampſon.

The numbers are $\frac{12851}{85184}$, $\frac{19467}{85184}$, and $\frac{18954}{85184}$; theſe fractions are in lower terms than thoſe given by Dr. Wallis from Dr. Pell, where the method of ſolution may be ſeen.

Mr. Turner *gives this Anſwer from Dr. Wallis's Algebra.*

1ſt, $a = \frac{494424}{2352637}$; 2d, $b = \frac{472696}{2352637}$; 3d, $c = \frac{448000}{2352637}$, whoſe ſum $= \frac{1415120}{2352637} = \frac{80}{133}$. The cube of their ſum $= \frac{512000}{2352637}$, from which numbers taking ſeverally the values of a, b, and c, there will remain theſe three cubes, viz. $\frac{17576}{2352637}$, $\frac{39304}{2352637}$, and $\frac{64000}{2352637}$, whoſe roots are $\frac{26}{133}$, $\frac{34}{133}$, and $\frac{40}{133}$. *Q. E. F.*[*]

† XII. QUESTION 276 *anſwered by Mr.* I. Aſh.

Suppoſe the ſine of the vertical angle of the cone ·0507; then by trigon. the ſide or length is found = 3944·77 inches or 328·73 feet = length of the pendulum's ſtring. Hence

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[*] This is the ſame with queſtion 51, which ſee for a ſolution.

† XII. QUESTION 276.

The principle uſed in the above ſolution is not general for any angle at the vertex, but only for that one particular angle there uſed, as may be ſeen in the Prop. of *Keil* there referred to. But the times of gyration do not depend on the length of the ſtring but only on the altitude of the cone, they being univerſally as the ſquare roots of the altitudes; and when the altitudes are equal, the times will be equal alſo, whatever the lengths of the pendulums may be.

By Prop. IX. *Emerſon's* Centrip. Forces, the proportion is univerſally thus, as $\sqrt{16\frac{1}{12}}$ feet : $\sqrt{400}$ inches (twice the given ⊥) :: 3·1416 : $3{\cdot}1416\sqrt{\frac{400}{193}} = \frac{62{\cdot}832}{\sqrt{193}} = 4{\cdot}5228$ ſeconds, the time required.

(per Keil's Introd. Theor. 11, p. 302) the time of one revolution is equal to the time of the perpendicular fall of a heavy body from a height equal to the pendulum's length, $\because 16\frac{1}{12} : 1'' :: 328{\cdot}73 : 20{\cdot}439442''$ the ſquare time, whoſe root $= 4{\cdot}521$ ſeconds required.

This queſtion was elegantly ſolved by Mr. *Farrer*, Mr. *Turner*, Rev. Mr. *Baker*, and others, which agree with the propoſer's ſolution.

XIII. QUESTION 277 *anſwered by Mr.* Heath.

Square is printed inſtead of *Square Root* in this queſtion; correcting which, and putting $a =$ the pounds of an annuity, $r = 1{\cdot}05$ the amount of a pound and its intereſt for one year at 5 per cent. $r =$ the year's continuance; then $\frac{a}{r-1} - \frac{a}{r^a \times \overline{r-1}} = z$, the preſent worth. And if the conditions of the queſtions be ſubſtituted therein, the equation becomes $\frac{a}{a-1} - \frac{a}{r^a \times \overline{r-1}} = \sqrt{a}$. Whence $1{\cdot}05^a = \frac{\sqrt{a}}{\sqrt{a} - {\cdot}05}$; here $a = 1{\cdot}034$ fere $=$ 1 l. 0s. 7 d. the annuity required.

N. B. As the queſtion was printed, the final equation is $1{\cdot}05^a = \frac{1}{1 - {\cdot}05a}$, where a is evidently $= 0$, or the leaſt money poſſible.

* XIV. *Obſervation on* QUESTION 278, *by the* Editor.

This queſtion was propoſed with an intent to improve gunnery, of which there are ſeveral things wanting. In particular, a treatiſe on the ſubject by an experienced hand: For to be treated on by any other perſon, will only be compiling of matters already known. And in order for this improvement, and the ſolution of this queſtion, experiments ſhould be made in a general way, which we have not yet received.

The

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* XIV. QUESTION 278.

The Editor obſerves above that this queſtion was propoſed with *an intent* to improve practical gunnery, though it does not appear from

The PRIZE QUSTION *anſwered by Mr.* R. Heath.

The biggeſt two circles, within one another, admitting of an infinite number of triangles to be drawn with their angles and ſides terminating in their outer, and touching their inner peripheries, are thoſe which are concentric, and their diameters exactly as 2 to 1; in which caſe the triangles will be all equal and equilateral. This is ſo evident as to appear upon the ſlighteſt examination. And if the diameters be in proportion leſs than 2 to 1, then no triangle whatſoever can be drawn as aforeſaid. But if the proportion be greater than 2 to 1 (as 310 to 100, or 3·1 to 1 as in the preſent caſe) their peripheries being eccentric and at a proper diſtance, will admit of an infinite variety of triangles to be drawn in and about them, from the iſoſceles $\triangle ACC$ to that of BDD, which are the greateſt and leaſt triangles: Becauſe the area of every triangle ſo drawn, being equal to the ſum of all the ſides

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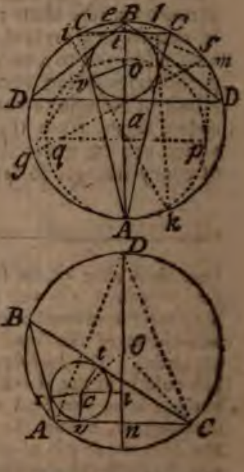
from the nature of the ſcience that it would at all have anſwered that purpoſe. It is well known that *ſhort pieces* are requiſite at *ſea*, both for the convenience of working them in an engagement, and on account of the ſmall ſpace they muſt ſtand in when the ports are cloſed. The *land-ſervice*, on the contrary, requires *long pieces*, particularly in the attack of a place, in order to preſerve the embraſures from the blaſt of the powder, which ſhort pieces would ſoon deſtroy, beſides the danger of ſetting fire to a faſcine battery. Not only the *lengths* of pieces are *limited* by the nature of the ſervice, but alſo the *diameters of the bores*; for pieces which carry balls from 24 to 42 pounds comprehend the limits of *battering* cannon, and thoſe from 3 to 12 pounds limit *field* pieces. Experience proves that balls of a leſs weight than 24 pounds are inſufficient to make a moderate breach; and that pieces carrying 42 pound balls, or upwards, become unmanageable from their great weight; ſo that in general the 32 pounder is the moſt common battering piece. When the field piece exceeds 12 pounds, it, in like manner, becomes too unwieldy for *that* ſervice.

Mr. *Robins* is the only author, that I know of, who has ſolved a propoſition exhibiting the relation between the *velocity* of the ball and the *dimenſions* of the piece. Another author, who proceeds in a very different manner, makes the *velocity* always increaſe with the *length* of the piece.

If any perſon however think it worth his while to go through the calculation of this problem, he may eaſily do it, making the expreſſion for the velocity found by prop. 7 of *Robins*'s Principles of Gunnery, a maximum, then its fluxion being taken, &c. there will be determined the relation between the diameter and length.

ſides into half the given radius of the inſcribed circle, the more the ſides of any triangle are ſituated about the center, or diameter, or the farther removed from them, the greater or leſs will the periphery, and conſequently area, of that triangle be: the ſides of the triangles *ACC* and *BDD* being the moſt near and remote to ſuch a ſituation.

For the property of drawing triangles as aforeſaid, in and about eccentric circles, there is this demonſtration.



Firſt to inſcribe the iſoſceles, or one $\triangle$, ACC in the greater circle, which at the ſame time ſhall circumſcribe the leſſer circle. Put $m=$ rad. greater circle $=Aa$, $n=$ rad. leſſer $=Ov$, and $x=AO$; then $\sqrt{xx-nn}=Av$; per ſim. $\triangle$s, $Av:AO::At:AC=\frac{x+n}{\sqrt{xx-nn}}$ xx; again, $AO:Av::AB:AC$ $=\frac{2m\sqrt{xx-nn}}{x}$; hence $\frac{x+n}{\sqrt{xx-nn}}$ $xx=\frac{2m\sqrt{xx-nn}}{x}$, and $x-m$ $=\sqrt{m^2-2mn}=ao$, ſhewing the diſtance of the centers to be leaſt, or o, when $2n=m$; both circles being then concentric, and radius of the greater $=$ twice the radius of the leſs. Whence 12·269074 $=Bt$ the neareſt diſtance of the peripheries. And area $\triangle$ ACC $=$ 18242·89 yards, the greateſt; and that of $\triangle$ $BDD=$ 16799·82 &c. the leaſt.

The diſtance of theſe centers of the circles being known, as $d=cO$ capable of having one triangle drawn as aforeſaid: Suppoſe any chord $AC=x$ touch the inner circle at v, then it is plain another chord can touch it ſomewhere at t, and if another can touch it at r, the property is proved. But, in general, the $\angle ABC=\angle ADC=\angle nOC$ its ſine $=\frac{x}{2m}$; and $\angle rBc=\angle cBt=\angle nDC$ (per 33 and 20 Euc. 3) $=\frac{x}{2\sqrt{2mn+m\sqrt{4mm-xx}}}$; the comp. or $\angle rcB=\angle tcB=\angle ncD$, its ſine $=$ $\frac{x}{\sqrt{2mm\times m\sqrt{4mm-xx}}}$; whence, by trig. $rB=Bt=$

2mm

$\frac{2nm+2n\sqrt{4mm-xx}}{x}$. Again $iO=\frac{1}{2}\sqrt{4mm-xx}-n$, and (per 47 Euc. 1) $ci=vn=\sqrt{dd-\overline{\frac{1}{2}\sqrt{4mm-xx}-n}|^2}$; hence $vC=\frac{1}{2}x+vn$, $Av=Ar=\frac{x}{2}-vn$, and AB and BC, alſo $\angle vCc=\angle cCt$, and $\angle vCt$ are expreſſed. Say $\frac{x}{2m}$ (s. $\angle ABC$) : x (AC) or $1:2m$ :: s. $\angle vCt:AB$, which gives an equation ſhewing the general value of x, whilſt two other chords are tangents likewiſe; and triangles *ilk*, *gef*, will revolve as in the 1ſt fig. Hence, if any chord or ſide of a triangle is given, the others follow by trigonometry; having firſt found the diſtance of the centers aforeſaid. This property of drawing triangles about circles I diſcovered ſome years ago, as may be ſeen in the Monthly Oracle; though the propoſer has greatly deſerved in a long account of it from Scilly, printed in a book called the Quarterly Miſcellanea Curioſa.

Mr. *Landen* puts $b=$ rad. leſſer circle, $a=$ diam. greater, and $x=$ altitude iſoſceles triangle, and gets $x=\frac{a+2b}{2}\pm\sqrt{\frac{aa-4ab}{4}}=297{\cdot}33$ or $112{\cdot}67$ &c. Whence he infers this conſtruction [*ſee fig.* 1]. From the center of the garden, along the diameter, ſet off the diameter of the pond, find a center (at n) betwixt that point and the other end, deſcribing a circle Apq, draw pm parallel, and mo perpendicular to AB, and the point o will be the center of the pond.

Mr. *Aſh* finds the diſtance of the centers by a theorem like the former; and ſo does Mr. *Bamfield*, who has given a conciſe and elegant ſolution.

The Prize of 10 *Diaries was won by Mr.* R. Gibbons.

The Eclipſes calculated for 1747, *ſhewing in what Parts of the World they will be viſible. By Mr.* Ralph Hulſe.

1. On January 29, at 3 in the afternoon, the ſun will be eclipſed in ♒, 21 deg. vertical to Brazil, lat. 14 deg. ſouth, long. 50 deg. weſt. This eclipſe will be very ſmall, and viſible near the antartic circle.

2. Feb. 14, at 5h. 2 m. in the morning the moon will be eclipſed viſible and total at London: Beginning 3h. 15 m. Middle 5h. End 6 h. 45 m. Total duration 3 h. 30 m. Digits eclipſed 20⅓.

Calculated by		Begin. h. m. s.	Mid. h. m. s	End h. m. s.	Dur. h. m. s.	Dig. h. m.
Mr. C. Cockſon, by Leadbetter's Tables,	London	3 13 20	5 1 30	5 50 29	3 36 9	19 43
	Durham	3 8 18	4 56 28	5 45 27	† End	of
	Lambton	3 8 10	4 56 20	5 45 19	Total	Dark.
	Cornwen	2 56 9	4 44 17	5 33 16		
-----New Tables from Sir Iſaac Newton'sTheo.	Durham	3 2 10	4 44 9	5 40 8		
	Lambton	3 2 2	4 44 1	5 40 0	3 40 18	19 54
	Cornwen	2 50 1	4 32 0	5 27 59		
By Mr. Bulman,	London	3 2 0	5 5	7 5 *	* End	of the
	Edinburgh	2 50	4 53	6 53	Eclipſe.	
	Dublin	2 34	4 37	6 37	4 3	19 58
	Carliſle	2 51	4 54	6 54		
	Rocheſter	3 3	5 6	7 6		
Mr. Cowper, London		3 24 34	5 13 19	7 2 4	3 37 30	19 43
Mr. Farrer, Sunderland		2 41	4 30	6 19	3 38	20 12

3. Feb. 28, at 5 in the morning, the ſun will be eclipſed 1 digit in ♓ 20 deg. He is then vertical to the Indian ſea, between Bornio and Java, lat. 4 deg. ſouth, long. 110 deg. and will be ſeen in Greenland, and the places adjacent.

4. July 26, at 8 in the forenoon, the ſun will be 1 digit eclipſed in ♌, 13 deg. vertical to Arabia Fœlix, lat. 17 deg. long. 41 deg. eaſt, viſible in the north frozen ſea, lat. 80 deg.

5. Auguſt 9th, at 10 in the forenoon, the moon will be totally eclipſed, viz. 27 deg. in ♒. She is then vertical to Mardelzur, lat. 12 deg. S. long. 146 deg. W. viſible to the Japan and Philippine iſlands, to all the weſt ocean between Aſia and America, from S. to N. including Jamaica, Cuba, Carolina, and Virginia.—And the horizon of the viſible diſk paſſes through Penſylvania.

6. Auguſt 24, at 9 at night, when the ſun will be eclipſed in ♍, 12 deg. This will be a very ſmall eclipſe, and viſible only in the unknown ſouthern parts of the world.

New

The 2d Eclipſe was obſerved at *St. Angelo, Paragua*; whoſe lat. is 18° 17′ ſouth, and long. weſt of the Ferro iſle 36° 30′.

The end of the eclipſe 15 h. 16 m. 4 s.

The 5th Eclipſe was obſerved at *St. Maria Major*, lat. 27° 51′ ſouth, and long. 37° 20′ weſt of the Ferro iſle.

The beginning of the eclipſe	14 h.	15 m.	14 s.
Total obſcuration — —	15	53	16
Beginning of the emerſion	17	34	48

New Queſtions.

I. QUESTION 279, *by Mr.* Landen.

A charming briſk maid has aſſur'd me, and ſaid,
Since I'm ſuch a fine mathematician,
(Laying puzzling aſide, for the joys of a bride)
She will wed me——but on this condition:
That I firſt ſhall unfold what * pieces of gold
Her father has for her in ſtore:
And theſe I muſt find from the data ſubjoin'd,
And then I'm to puzzle no more.

* The pieces are half-guineas, guineas, moidores, and three-pound-twelves. The whole number is 4000. And if v, x, y, and z be put for the number of each ſort reſpectively, $v^4x^3y^2z$ is a maximum. Quere what is the lady's fortune?

II. QUESTION 280, *by Mr.* John Williams.

Going along a river's ſide, on an even and direct road *ABC*, I obſerved a tower on the other ſide of the river, whoſe angle of altitude at *A* was 5° 24′; going farther on to *B*, 100 yards, the angle of altitude was 6° $27\frac{1}{2}'$; and intending, again, to take an obſervation when directly oppoſite to the tower, but was prevented by an iſland in the river (over-grown with furz), I then came to *C*, 400 yards from *B*, where I found the angle of altitude was 8° 36′. Quere the tower's height?

III. QUESTION 281, *by Mr.* J. May, *jun. of Amſterdam.*

It is required to find (by a general theorem) the number of fractions of different values, each leſs than unity, ſo that the greateſt denominator be leſs than 100?

IV. QUESTION 282, *by the Rev. Mr.* Anth. Baker.

A gentleman would have a ſilver punch-bowl made in the form of a parabolic conoid, containing exactly two gallons, but being frugally inclined, deſires firſt to know what ought to be its inſide dimenſions ſo that, cæteris paribus, it may require the leaſt quantity of ſilver poſſible?

V. QUESTION 283, *by Mr.* N. Farrer.

Quere the axis and parameter of a parabola and ſemi-ellipſis, when the latter is circumſcribed by the former, whoſe ordinate is equal to the conjugate axis, and abſciſſa equal to the ſemi-tranſverſe: both curves having the ſame focus, the difference of their parameters 2, and the length of the parabolic curve being 28·68?

VI. QUESTION 284, *by Mr.* Cuth. Cockſon.

Quere with what part of a cylindrical ſtick ſhould a perſon ſtrike, to give the greateſt blow; the length of the arm being 20 inches, and that of the ſtick 50?

VII. QUESTION 285, *by Mr.* Landen.

I am about building a houſe, the breadth whereof I deſign ſhall be 24 feet, and the perpendicular height from the ground to the ridge 42 feet. The ends, which are to point directly eaſt and weſt, will be ſheltered by neighbouring tenements; but the ſides will be expoſed to the fury of the north and ſouth winds. I therefore would be ſatisfied what the angle of the ridge muſt be, and how high the ſide walls, that the wind blowing from either of thoſe quarters ſhall have the leaſt effect on the building?

VIII. QUESTION 286, *by Mr.* J. Aſh.

If a parabolic conoid, whoſe altitude is 9, and baſe 6 inches, be cut by a right line at ſome diſtance from, but parallel to its axis, what is the ſolidity and convex ſurface of that ſegment, or part cut off, when the height of the plane of that ſection is 5 inches?

IX. QUESTION 287, *by Mr.* Cuth. Cockſon.

Given $a^3\dddot{y} + \dot{y}\dot{y}x^2 - aa\dot{y}\ddot{y} = 0$, the equation of a curve, whoſe radius of evolution at the vertex is $140 = a$; to find the value of y, the abſciſſa, when its correſponding ſemi-ordinate $x = 50$?

X. QUESTION 288, *by Mr.* John Hampſon.

Required to find three numbers, that when each is ſeverally added to the cube of their ſum, their reſpective ſums *ſhall* be a cube number?

XI. QUES-

XI. QUESTION 289, *by Mr.* Heath.

On what days of the year do our ſhadows move ſloweſt and faſteſt in London? And at what times do they move ſloweſt and faſteſt on any day?

XII. QUESTION 290, *by Mr.* Bulman.

Being at a town in Kent, I obſerved three objects on the other ſide of the river Medway, (a caſtle, wind-mill, and ſpire) whoſe diſtance from one another are known: From the caſtle (the neareſt object ſeen) to the ſpire, is 10 furlongs; from the caſtle to the wind-mill 23 furlongs; and from the wind-mill to the ſpire is 25 furlongs. I alſo obſerved the town angle between the caſtle and ſpire = 28° 34′, and the town angle between the caſtle and wind-mill = 57° 45′. What diſtance did I ſtand from each of thoſe objects? And give a geometrical conſtruction of the ſame.

XIII. QUESTION 291, *by Mr.* J. Aſh.

A lady of important ſpeculation,
Would gladly know her age from this * equation.

$$* \; 1\cdot05^{A} = \frac{\sqrt{A}}{1 - \cdot05\sqrt{A}}.$$

XIV. QUESTION 292, *by Mr.* Heath.

The pounds power of their Napier's logarithm be
Equal that logarithm power of ſhillings left to me.
From myſtic words, artiſts, the truth extract,
And tell what is the legacy exact?

XV. QUESTION 293, *by Mr.* Bulman.

A ſpheroidal ullage lies upon the ground with the bung uppermoſt, from whence to the ſurface of the liquor (which is exactly the height of the upper ends of the caſk) is 9 inches, and its diagonal either way from the bung to the lower ends = 55 inches, its ullage is a maximum: Quere the content of the caſk and ullage, brother gaugers?

XVI. QUESTION 294, *by Mr.* Chr. Maſon.

In an evening lately, hearing the noiſe of guns at a ſmall diſtance off at ſea, I ſtraight repaired to the Strand, where I loomed a man of war's tender giving chace to a French privateer; I perceived a flaſh of a gun from the tender S.W.

by S. at 33 half ſeconds of time heard the report: Eight minutes after I obſerved another flaſh at S. S. W. from the ſame, and at 25 half ſeconds heard the report. The chace kept her courſe, making equal way; in 35 minutes more was drove on ſhore, being but two minutes of time a-head of the tender. What was the bearing and diſtance between my ſtation, the tender, and pilf'ring poltroon when ſtranded?

PRIZE QUESTION *by Mr.* Bamfield.

If the diameter of Syſiphus's cylindrical ſtone be two feet, which he continually rolls upon the ſurface of a ſemi-globular mountain, half a mile high: Quere what ſpace will a ſpot on the convex ſurface of that ſtone travel through in rolling directly up and down the ſaid mountain? And what will be the time of its deſcent from the top, by the force of gravity?

1748.

Queſtions anſwered.

I. QUESTION 279 *anſwered by Mr.* J. Turner.

LET $v =$ the half guineas, $x =$ guineas, $y =$ moidores, and $z =$ 3l. 12s. By the queſtion $v + x + y + z = 4000 = b$: Now $v^4x^3y^2z$ being a maximum, or (expunging z) $b - v - x - y = \frac{1}{v^4x^3y^2}$. In fluxions $-\dot{v} - \dot{x} - \dot{y} = \frac{-2\dot{y}}{y^3x^3v^4} - \frac{3\dot{x}}{y^2x^4v^4} - \frac{4\dot{v}}{y^2x^3v^5}$; whence $\dot{y} = \frac{2\dot{y}}{y^3x^3v^4}$; $\dot{x} = \frac{3\dot{x}}{y^2x^4v^4}$; $\dot{v} = \frac{4\dot{v}}{y^2x^3v^5}$: And $\frac{1}{v^4x^3y^2} = \frac{y}{2} = \frac{x}{3} = \frac{v}{4} = b - v - x - y$. Conſequently $x = \frac{3}{2}y$; $v = \frac{4}{3}x = 2y$: Alſo $\frac{1}{2}y = b - 2y - \frac{3}{2}y - y$; or $y = \frac{1}{5}b$; $z = \frac{1}{10}b$; $x = \frac{3}{10}b$; $v = \frac{4}{10}b$; and the lady's fortune is 4620l. Hence this

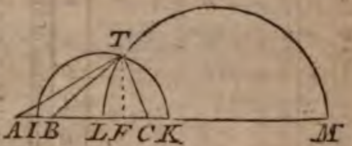
GENERAL RULE.

Make the ſum of the exponents a denominator, and each *particular* exponent (of the quantities) the numerator of a fraction, theſe reſpectively multiplied into the whole number of pieces, will ſhew the particular number of pieces of each ſort.

Mr. *James Waine* has ſolved this queſtion curiouſly and conciſely, as alſo did Mr. *Landen* (the propoſer), Mr. *Kingſton*, Mr. *Jepſon*, Mr. *Bamfield*, Mr. *Farrer*, Mr. *Collingridge*, Mr. *Garrard*, Mr. *Dun*, Mr. *Cowper*, Mr. *Cockſon*, &c. Though it has been obſerved that this queſtion is taken from Emerſon's Fluxions (p. 128), yet the new mode of language and application of it, is a merit which muſt be acknowledged.

II. QUESTION 280 *anſwered by Mr.* James Terey.

Put $x=$ tower's height, $AC=500=b$, $AB=100=d$, $BC=400=a$. Co-ſec. $\angle A=t$, of $B=v$, of $C=y$. Then (rad. $=1$) $tx=AT$, $vx=BT$, and $yx=CT$. Now (geometrically) making $t:v::AI:IB::AK:KB$, and deſcribing the ſemi-circle ITK; alſo $v:y::BL:LC::BM:MC$, deſcribing the ſemicircle LTM; the interſection T, being the place of the tower, for which ſee Univerſal Arith. prob. 26) let down $TF\perp AC$. Per fig. $b:tx+yx::tx-yx:\frac{ttxx-yyxx}{b}=AF-FC$; alſo $BT^2-BF^2=AT^2-AF^2$; whence $attxx+dyyxx-bvvxx=abd$, and $x=\sqrt{\frac{abd}{att+dyy-bvv}}=44'4609$ &c. yards, the tower's height. Q.E.F.



The ſame anſwered by Mr. J. Aſh.

Call the co-ſecants of the three angles of altitude a, b, and c; AB, s; BC, $4s$; the tower's height, x. By trigon. (T being the top) $AT=ax$; $BT=bx$; $CT=cx$; coſ. $\angle CBT=\frac{ccxx-bbxx-16ss}{8sbx}$; coſ. $\angle ABT=\frac{ss+bbxx-aaxx}{2sbx}$; which two laſt expreſſions made equal, $x=\sqrt{\frac{20ss}{cc-5bb+4aa}}=44{\cdot}4557$ yards, the tower's height required.

This queſtion was alſo anſwered by Mr. *J. Turner*, Mr. *Waine*, Mr. *R. Hall*, Mr. *Kingſton*, Mr. *Jepſon*, Mr. *Bamfield*, Mr. *Pitches*, Mr. *Hampſon*, Mr. *Walker*, and Mr. *Collingridge*.

III. QUESTION 281 *anſwered by Mr.* Heath.

The number of fractions of different and like values, each being leſs than unity, and the greateſt denominator being any number, (from 2 upwards) will appear by the following ſeries, continued to $\frac{98}{99}$, &c.

Sum = 27 = fractions of diff. values.		Sum = 9 = fractions of like value.
6	$\frac{8}{9}$ $\frac{7}{9}$ $\frac{6}{9}$*$\frac{5}{9}$ $\frac{4}{9}$ $\frac{3}{9}$*$\frac{2}{9}$ $\frac{1}{9}$	2
4	$\frac{7}{8}$ $\frac{6}{8}$*$\frac{5}{8}$ $\frac{4}{8}$*$\frac{3}{8}$ $\frac{2}{8}$*$\frac{1}{8}$	3
6	$\frac{6}{7}$ $\frac{5}{7}$ $\frac{4}{7}$ $\frac{3}{7}$ $\frac{2}{7}$ $\frac{1}{2}$	0
2	$\frac{5}{6}$ $\frac{4}{6}$*$\frac{3}{6}$*$\frac{2}{6}$*$\frac{1}{6}$	3
4	$\frac{4}{5}$ $\frac{3}{5}$ $\frac{2}{5}$ $\frac{1}{5}$	0
2	$\frac{3}{4}$ $\frac{2}{4}$*$\frac{1}{4}$	1
2	$\frac{2}{3}$ $\frac{1}{3}$	0
1	$\frac{1}{2}$	0
27	36	9

Here it is evident, that the greateſt numerator, or greateſt denominator leſs unity, will be the number of terms, which will always be equal to the laſt term; therefore $\frac{n+1}{2} \times n =$ the ſum of the ſeries, or the number of different and like fractions in all caſes. The fractions of like value with ſome of the reſt, are all thoſe not in their loweſt terms: As thoſe of different value are all the incommenſurable ones: For the more ſpeedy determining of which, all the incommenſurable denominators, from the leaſt in the ſeries to the greateſt, muſt be taken, viz. 3, 5, 7, 11, 13, &c. to 97, which are 25; and the ſum of their different fractions will be the ſeries 2, 4, 6, 10, 12, &c. to 96; each row being one leſs than the denom. To theſe if the different fractions (in loweſt terms) of the commenſurable denominators 2, 4, 6, 8, 10, &c. to 99, be added, the ſum will be all the different fractions. But as there is more trouble than art to diſcover them, I ſhall leave it to perſons of leiſure to purſue the computation, the method being here planned out.

Mr. *Aſh*, upon the ſame principles, computes the number of different fractions to be 3055; but doubts the truth.

N. B. By the above theorem, the whole number of fractions = 4851 (as Mr. *Bamfield* made it) from which ſubducting thoſe of like values, all the different fractions will remain.

A General

A General Method of ſolving this Queſtion *from the Diary for* 1751, *by Mr.* Flitcon.

Each diff. let. in 1ſt ſcheme denotes a diff. prime.

	Steps	Denominators	The ſeveral quotients ariſing by dividing any denominator, by				Numb. of diff. frac. in each ſeries.
			Each ſingle prime	The prod. of each 2	Product of each three	Product of each four	
	1	x	-1				n
1 × x	2	x^2	$-x$				xn
2 × x	3	x^3	$-x^2$				$x^2 n$
	4	vx	$-v$ $-x$	$+1$			n
4 × v	5	v^2x	$-v^2$ $-vx$	$+v$			vn
5 × x	6	v^2	$-v^2x$ $-vx^2$	$+vx$			vxn
	7	vxy	$-vx$ $-xy$ $-yv$	$+y$ $+v$ $+x$	-1		n
	8	$vxyx$	$-vxy$ $-xyz$ $-yzv$ $-zvx$	$+vz$ $+xy$ $+yz$ $+zv$ $+yv$ $+xz$	$-z$ $-v$ $-x$ $-y$	$+1$	n

II.

Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns	Denominator	Component primes	Diff. fractns
2	P	1	16	—	8	30	2·35	8	44	—	20	58	2·23	28	72	—	24	86	2·43	42
3	P	2	17	P	16	31	P	30	45	—	24	59	P	58	73	P	72	87	3·29	56
4	—	2	18	—	6	32	—	16	46	2·23	22	60	—	16	74	2·37	36	88	—	40
5	P	4	19	P	18	33	3·11	20	47	P	46	61	P	60	75	—	40	89	P	88
6	2·3	2	20	—	8	34	2·17	16	48	—	16	62	2·31	30	76	—	36	90	—	24
7	P	6	21	3·7	12	35	5·7	24	49	—	42	63	—	36	77	7·11	60	91	7·13	72
8	—	4	22	2·11	10	36	—	12	50	—	20	64	—	32	78	23·13	24	92	—	44
9	—	6	23	P	22	37	P	36	51	3·17	32	65	5·13	48	79	P	78	93	3·31	60
10	2·5	4	24	—	8	38	2·19	18	52	—	24	66	2·3·11	20	80	—	32	94	2·47	46
11	P	10	25	—	20	39	3·13	24	53	P	52	67	P	66	81	—	54	95	5·19	72
12	—	4	26	2·13	12	40	—	16	54	—	18	68	—	32	82	2·41	40	96	—	32
13	P	12	27	—	18	41	P	40	55	5·11	40	69	2·23	44	83	P	82	97	P	9
14	2·7	6	28	—	12	42	2·37	12	56	—	24	70	2·5·7	24	84	—	24	98	—	2
15	3·5	8	29	P	28	43	P	42	57	3·19	36	71	P	70	85	5·17	64	99	—	6
		71			198			314			416			564			666			774

Third columns collected.

71
198
314
416
564
666
774
3003

The ſum of all the different fractions

N. B. P ſignifies each prime number in the ſecond column in ſcheme 2, with a new number to each in the 3d column, deduced from the ſeries in the firſt ſcheme, according to the general method: the different denominators being placed in the firſt column all along.

For the multipliers, or powers of each. Againſt 3 in the 1ſt column, ſtands 2 in the 3d column, each being drawn into 3 gives 6 to be ſet in the 3d column againſt 9 in the firſt collection. Againſt 6 in the 1ſt col. is ſet 2·3 in the 2d col. whence (by 4th ſtep) is found 2 for the 3d collection. Alſo for its multiples, into either, or both its parts, as 2 × 3 = 6, drawn into 6 and 2 gives 12 (by 6th ſtep) to be ſet in the 3d col. againſt 36 in the firſt: dotting through all the ſecond columns all ſuch multiples, by which the fractions are found. Proceeding thus, the whole number of different fractions are truly determined in a ſhort time.

IV. QUESTION 282 *anſwered by Mr.* J. Turner.

When the convex ſuperficies is the leaſt, the conoid muſt come neareſt to the form of a ſemi-ſphere (being under the leaſt ſurface of all ſolids of given ſolidity formed by the rotation of a curved ſpace about its axis) which is when the abſciſſa and ſemi-ordinate of the generated parabola are nearly equal: the conoid then being nearly moſt capacious. Putting $b = 462$ inches in two wine gallons, $p = 3{\cdot}1416$, $y =$ ordinate $x =$ abſciſſa of the parabola. Then the ſolidity of conoid $= b = \frac{pyyx}{2} =$ (when $y = x$) $\frac{py^3}{2}$; where $y = 6{\cdot}65$ inches. But accurately, $\frac{p}{6a} \times \overline{aa + 4yy}|^{\frac{3}{2}} - \frac{paa}{6}$ is the ſuperficial content of the ſame (*ſee p.* 202 *of Mr. Emerſon's Fluxions*). Now if the value of yy be ſubſtituted therein, and the fluxion of it made $= 0$, then $a = 6{\cdot}1$; $y = 6{\cdot}54$; $x = 6{\cdot}94$; and the convex ſurface $= 235{\cdot}5$ ſquare inches.

The ſame anſwered by Mr. Aſh.

The conoid's convex ſurface is $\frac{p}{3a} \times \overline{aa + 4yy}|^{\frac{3}{2}} - \frac{paa}{3}$, a minimum $(p = 1{\cdot}5708)$; $\because \frac{\overline{aa + 4yy}|^{\frac{3}{2}}}{a} - aa$ is alſo a minimum. Put $cc =$ double ſolidity $= 4$ wine gall. $qq =$ area of a circle whoſe rad. $= 1$. Then, by the curve, $ax = yy$, and $qqyyx = cc$, whence $4yy = \frac{4c\sqrt{a}}{q}$; and ſubſtituting for $4yy$ in the min. it becomes $\overline{\frac{qaa+4c-a}{a}}\big|^{\frac{3}{2}} - aa$. Whence by fluxions, $a = \overline{\frac{15}{24} + \sqrt{\frac{11cc}{192qq}}}\big|^{\frac{3}{2}} = 6{\cdot}03$; whence ſemi-axis $= 6{\cdot}98$, and ſemi-ordinate $= 6{\cdot}49$ maxime.

It was elegantly ſolved by the propoſer, alſo by Mr. *Landen*, Mr. *Farrer*, Mr. *Dixon*, Mr. *Waine*, Mr. *Kingſton*, Mr. *Bamfield*, &c.

V. QUESTION 283 *anſwered by Mr.* Aſh.

'Tis inconſiſtent with the nature of the parabola and ſemi-ellipſis that the latter ſhould be circumſcribed by the former, when the ordinate = conjugate, and axis = ſemi-tranſverſe (which Mr. *Turner* alſo obſerves); ∵ ſemi-ellipſis muſt be read for parabola, & contra. Then from the properties of the curve and conditions of the queſtion, the parameter of the parabola is found = 2. Suppoſe $AH = c = 12{\cdot}5$; the correſponding curve $Ar = 13{\cdot}90375$ &c. ∵ the remaining part rm $(= z)$ of the curve $= 0{\cdot}43625$. [*See fig. to Q.* 253.]

Put $y = nm$, $x = nr$, and we ſhall have $\dot{y} = \frac{a\dot{x}}{\sqrt{4ac+4ax}}$, and $\overline{\dot{x}^2 + \dot{y}^2}\big|^{\frac{1}{2}} = \dot{z} = \sqrt{1 + \frac{a}{4c+4x}} \times \dot{x}$; whoſe fluent is $x + \frac{ax}{8c} - \frac{aax}{128cc} - \frac{axx}{16cc} + \frac{aaxx}{128c^3}$ &c. $= z$; or $ax - bx^2 + cx^3$ &c. $= z$ [by ſubſtituting a for $1 + \frac{a}{8c} - \frac{aa}{128cc}$; b for $\frac{-a}{16cc} + \frac{aa}{128c^3}$; &c.] And by reverſion $x = \frac{z}{a} + \frac{bz^2}{a^3} + \frac{2bb-ac}{a^5}z^3$ &c. $= 0{\cdot}4289$; which added to AH makes $12{\cdot}9289$ for the required axis of the parabola. *Q. E. F.**

VI. QUES-

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* V. QUESTION 283.

As the method of finding the parameter is not inſerted, I ſhall here ſupply it, and at the ſame time give another very eaſy method of ſolving the latter part of the queſtion.

Since then $ax = yy$ by the nature of the parabola, putting $y =$ the ordinate, $x =$ the abſciſſa, and $a =$ the parameter of the parabola; and, by the nature of the ellipſe, $x : y :: 2y : \frac{2yy}{x} =$

parameter

VI. QUESTION 284 *anſwered by Mr.* J. Turner.

Put $a = 70$, the length of the arm and ſtick; $b = 20$, the length of the arm; then the diſtance of the center of percuſſion of the ſtick from the upper end of the arm will be $= \frac{2aa + 2ab + 2bb}{3a + 3b} = 49·63$ inches, and from the fartheſt end of the ſtick $= 20·37$ inches (vide Stone's Flux. p. 177).

The ſame anſwered by Mr. J. Aſh.

Let $20 = c$ repreſent the length of the arm, and put $x =$ the diſtance of the center of percuſſion from the hand. Then $\overline{c + x}x \times \dot{x} =$ fluxion of the momentums; and $\overline{c + x}x \times \dot{x} =$ that of the forces; the fluent of which divided by that of the momentums is $\frac{6cc + 6cx + 2xx}{6c + 3x} =$ the diſtance of the center of percuſſion of the part xp, from the point of ſuſpenſion; which (when $x = 50$) will be $= 49·6296$ &c. Conſequently the part of the ſtick required is $20·37037$ &c. inches from the top.

Mr. *Walker*, Mr. *Kingſton*, and Mr. *Cuth. Cockſon* gave a ſolution to the ſame.

VII. QUES-

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parameter of the ellipſe $= a + 2$ by the queſtion; or $ax + 2x = 2yy$; ſubtract the former equation from this, then $2x = yy = ax$, and therefore $a = 2$ the parameter of the parabola.

The parameter being thus found, by p. 309 my *Menſuration*, the length of the double curve will be $y\sqrt{1 + yy} +$ hyp. log. of $y + \sqrt{1 + yy} = 28·68$, by the queſtion, $= c$; call the hyp. log. of $y + \sqrt{1 + yy}$, v; then $y\sqrt{1 + yy} = c - v$. Now it is eaſy to perceive that $y = 5$ nearly; then ſince a ſmall difference in the value of y will make a ſtill ſmaller and inconſiderable difference in the value of v, and when y is ſuppoſed 5 then $v = 2·312456$, and then the equation $y\sqrt{1 + yy} = c - v$ becomes $y\sqrt{1 + yy} = 26·367564 = d$, and $y = \sqrt{\sqrt{dd + \frac{1}{4}} - \frac{1}{2}} = 5·08$. With this value of y find a new value of $v = 2·3286281$, and thence of $d = 26·3513719$; and this ſame theorem will give $y = 5·08501$, which is very exact.

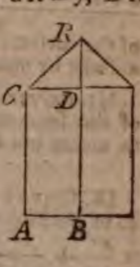
Then $x = \frac{yy}{a} = \frac{yy}{2} = 12·92806$.

VII. QUESTION 285 *anſwered by Mr.* Landen *the Propoſer.*

Put $a = 42$ the height of the building; $b = 12$, half the breadth; $x =$ perpendicular height of the roof. The force of all the particles of the air impinging againſt the wall, to blow upon the building, will be as $aa - 2ax + xx$. The effect of any particles ſtriking againſt the roof (found by mechanics) is to the effect the ſame would have had, ſtriking directly againſt an upright plane, as the ſquare of the ſine of the angle of incidence to the ſquare of radius; therefore the force of all the particles againſt the roof, to blow down the building, will be as $\frac{2ax^3 - x^4}{bb + xx}$. And by the queſtion $\frac{2ax^3 - x^4}{bb + xx} + aa - 2ax + xx$ muſt be a minimum. Let the fluxion of it be put $= 0$, then $x = \frac{b\sqrt{bb + 4ax} - bb}{2a}$ $= 10{\cdot}4076$ feet: Whence the height of the ſide-walls muſt be $31{\cdot}5924$ feet, and the ridge angle 98° 7′.

Mr. *Tho. Cowper* anſwered this queſtion exactly as above. Mr. *Jepſon* makes the ridge angle = 90°, and ſide walls = 30 feet, as does Mr. *Aſh*, but by a method like the propoſer's firſt conſideration, who then made the ridge angle and ſide walls agree the ſame. Mr. *Farrer*, by a different method, makes the ſide walls $= 32{\cdot}566$ feet, and the ridge angle = 103° 39′.

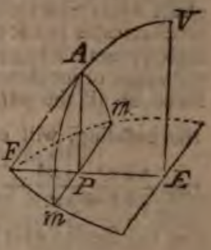
Mr. Jepſon's *Solution is as follows.*



Let $BR = 42 = a$, $AB = CD = 12 = b$, $CR = y$, $DR = x$, then $AC = BD = a - x$. By p. 255 of Emerſon's Fluxions, the reſiſtance of the plane DR, to the reſiſtance of the plane CR, is as yy to xx (ſuppoſing the wind to blow in a direction parallel to the plane of the horizon); but the reſiſtance of the plane DR is $= cx$ (putting $c =$ the length of the building) and (per 47 Euc. 1) $yy = bb + xx$, therefore the reſiſtance of the plane CR is $= \frac{cx^3}{bb + xx}$, and that of the ſide wall AC $= ca - cx$, $\because \frac{x^3}{bb + xx} + a - x$ is a minimum; which in fluxions, &c. gives $bbxx - b^4 = 0$, and $x = b = 12$. Hence the height of the ſide walls = 30 feet, and ridge angle 90°, which will always be the ſame, let the height and length of of the building be what they will.

VIII. QUESTION 286 *anſwered by the Propoſer Mr.* Aſh.

Let $x=FP$, $n=2FE=6$. By the nature of the circle we get $\sqrt{nx-xx}=Pm=\sqrt{PA}$ by the nature of the parabola (becauſe 9 the altitude of the whole parabola $=3^2$ the ſquare of its ſemi-ordinate, and all the perpendicular ſections are ſim. parabolas); and $\frac{4a}{3}\times\overline{n\dot{x}-xx}|^{\frac{3}{2}}=$ $=$ area mAm; conſequently $\frac{4}{3}\times\overline{nx-xx}|^{\frac{3}{2}}\dot{x}=$ flux. of the ſolidity; whoſe fluent is $\frac{4\sqrt{nx}}{3}\times:\frac{2nx^2}{5}-\frac{6x^3}{34}+\frac{6x^4}{72n}$ &c. $=6{\cdot}1122$ the ſolid content.



Again ſince $n\dot{x}+xx=Pm^2=AP$, whence the fluxion of the curve is $\dot{x}\sqrt{mm+4mx+4xx+1}$, or putting cc for $nn+1$) $\dot{x}\sqrt{cc+4nx+4xx}$; and becauſe $2\sqrt{mx+xx}$ $=$ chord of the arch mFm; and $\sqrt{mx+2xx}=$ chord of half that arch; therefore (by Huygens's Theor.) the arch $mFm=\frac{8\sqrt{4x+4xx}-2\sqrt{nx-xx}}{3}$ nearly. Conſequently $\frac{8\sqrt{4x+2xx}-2\sqrt{nx-xx}}{3}\cdot\dot{x}\sqrt{cc+4nx+4xx}=$ fluxion of the ſurface. The fluent of which (when $x=1$) is (if no miſtake be made) $=16{\cdot}34$ fere. *Q.E.F.*

Mr. *Farrer*, by a different method, only finds the ſolidity of the ſegment (without the content of the ſurface) which he makes but 6·845 cubic inches.

IX. QUESTION 287 anſwered by Mr. *Emerſon*, at p. 39 and 40 of his Fluxions, from whence it was taken; y being there $=\frac{x^2}{2a}+\frac{x^4}{24a^3}+\frac{x^6}{720a^5}-\frac{x^8}{4480a^7}$ &c. $=9{\cdot}0238$ &c.

Mr. *Farrer*, Mr. *Kingſton*, Mr. *Waine*, and Mr. *Collingridge* confirm the ſame.

We wiſh the invention of our worthy contributors may be more prolific, than to ſend us queſtions already ſolved, in *authors*, to hand.

X. QUES-

* X. QUESTION 288 *anſwered by Mr.* Hampſon *only.*

1. Anſwer, $a=\frac{23625}{157464}$, $b=\frac{1538}{157464}$, $c=\frac{18577}{157464}$.

2. Anſwer, $a=\frac{18954}{132651}$, $b=\frac{4184}{132651}$, $c=\frac{271}{132651}$.

† XI. QUESTION 289 *anſwered by Mr.* Heath *only.*

It is evident, that, when the ſun's motion is moſt vertical, his increaſe of azimuth is leaſt, and, conſequently of altitude greateſt; and when his motion is moſt horizontal, his increaſe of azimuth greateſt, and of altitude leaſt.

At

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* This queſtion may be ſolved after the manner of the 51ſt or 275th.

† XI. QUESTION 289 *otherwiſe ſolved by the Rev. Mr.* Cha. Wildbore.

'Tis manifeſt from my ſolution to the 654th diary queſtion, that at all places within the polar circles, the velocity of the ſhadow of the ſummit of an object, increaſes from paſſing due north (or ſouth in the Antartic) till it attains the maximum there determined, after which the velocity continually decreaſes till noon, when it is the leaſt poſſible. And with regard to the different greateſt velocity on different days, 'tis evident that the expreſſion for the maximum $PE^2 \times PE \times rS + Pb - rP \times PE \times rS - Pb + rP$ will continually increaſe, by increaſing rS or diminiſhing the declination; conſequently the greateſt velocity on the longeſt day will be leſs than the maximum on any other day: the ſaid maximum increaſing till its time and that of riſing coincide, when it will be infinite.

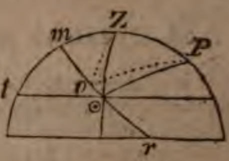
There is likewiſe a time in problems of this kind, when the velocity of the ſhadow, after being the greateſt, decreaſes the faſteſt, which may be found by taking the ſecond fluxion of the above maximum $= 0$. But in our latitude both theſe times coincide at ſun-riſing, when the velocity being always infinite, decreaſes, and that more and more ſlowly till noon, when it is always the leaſt poſſible. And the ſame may much more be ſaid of the velocity with which the altitude alone increaſes. As to the velocity with which the azimuth alone increaſes, what Mr Simpſon has done upon the ſubject is right; for it muſt increaſe the ſloweſt

when

At 12 each day, the ſun's motion is parallel to the horizon, or leaſt vertical, being then perpendicular to the meridian, or vertical circle: therefore the circular motion of ſhadows, in general, is at noon the faſteſt.

To find when the increaſe of azimuth is leaſt on any day, and in any place.

Put $p =$ s. $\odot P$, the ſun's diſtance from the pole; $d =$ its coſ. or ſine of declination; $c =$ s. ZP; $b =$ coſ. or s. of latitude; $y =$ s. azim. $\odot ZP$. Now if $r \odot om$ be the ſemi-diurnal arc of a leſſer circle, it is evident that the ſun's motion is moſt vertical when angle $o \odot Z$ is leaſt, or its coſ. $Z \odot P$ greateſt (the leſſer circle coinciding, at the point $\odot$, with a greater,



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when the fluxion of the hour angle P bears the greateſt ratio poſſible to that of the azimuth Z, the reaſon for which is no more, than if any ſmall diſtance $\dot{x}$ be divided by the time $\frac{\dot{x}}{v}$ in which it is gone over, it will give v the velocity of uniform motion at that time. Therefore the fluxion of the azimuth, divided by that of the time, muſt give the velocity with which the azimuth increaſes at that time, or $\frac{\text{coſ.} \odot}{\text{s.} Z \odot} =$ (vide my ſolution above referred to) $\frac{Pb . PE - rP}{rS . EO . aO}$ or (becauſe $aO = \frac{EO}{EP}$) $\frac{Pb . PE^2 - rP . PE}{PE^2 - 1}$ or $\frac{Pb - rP . PE}{PE^2 - 1}$ is a minimum. Which will evidently be leſs, the greater rP the ſine of declination is taken; and therefore the longer the days are, the leſs will the ſloweſt increaſe of azimuth be, and conſequently it will be leſs on the longeſt day than on any other day. Moreover, when the ſun is in the equinoctial, the minimum becoming $\frac{Pb}{PE^2 - 1}$, will be leſs as PE is greater, and therefore will be the leaſt poſſible when PE is greateſt, and the altitude $= 0$, or at ſun-riſing; therefore on the equinoctial days the azimuth will increaſe the ſloweſt at riſing due eaſt. Conſequently when the declination is ſouth, the minimum will be before ſun-riſing, and therefore in that caſe the azimuth will increaſe faſter and faſter from ſun-riſing till noon, when it will in all caſes increaſe the faſteſt, but on the ſhorteſt day ſlower than on any other day, and on the longeſt faſter.

greater, at right angles with $\odot P$); ſay, $p : y :: c : \frac{cy}{p} =$ a maximum, which is evidently when $y = 1$, the ſine of 90°. Whence, the azimuth increaſes ſloweſt and altitude faſteſt when the ſun is due eaſt, on all days, and in all places whatſoever; and therefore ſhadows in general move ſloweſt when the ſun is due eaſt; or at ſun-riſe (the moſt eaſterly) in the ſouthern declinations.

In the maximum $\frac{cy}{p}$, when p is leaſt (the ſine of 66° 30′ or 113° 30′) the azimuth on thoſe days increaſes the ſloweſt of all other days, and altitude faſteſt: Whence, our ſhadows move ſloweſt of all on the longeſt and ſhorteſt days of the year: the ſun due eaſt, or at riſing. But when $p = 1$, the increaſe of azimuth, on that day, is leaſt ſlow (at ſun due E.) and the increaſe of altitude the leaſt faſt; and as the azimuth increaſes its ſwiftneſs till noon, therefore our ſhadows move faſteſt of all at noon, on the day of the year when the ſun is in the equinoctial.

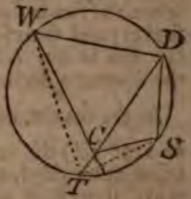
By erecting a wire perpendicular to an horizontal plane, and marking the circular increaſe of the ſhadow at equal diſtances of time, the truth of the foregoing will appear by inſpection. And by this circular motion, and the ſhortening one of the ſhadow, or by the increaſe of azimuth and altitude together, the ſummit of it is made to deſcribe a curve, whoſe nature is determinable, according to the latitude of the place and ſun's declination.

N. B. The increaſe of azimuth being always as the angle $\sigma \odot Z$, and the increaſe of altitude as its comp. the $\angle \sigma \odot t$, the ſum of both increaſes (viz. of azim. and alt.) will be at all times and all places alike; *i. e.* the ſum of the circular and ſhortening motions of ſhadows on a horizontal plane will be every where equal.

The increaſe of an azimuth is but the fluxion of it, and the ſloweſt increaſe of an azimuth is but the leaſt fluxion of that fluxion of azimuth; therefore if the angle of time be ſubſtituted for (which flows equally) and an expreſſion be raiſed of the azimuth angle (by trigon. and ſeries) the fluxion of that fluxion made $= 0$ will exhibit an equation, ſhewing the azimuth properties as aforeſaid. *Q. E. F.**

XII. QUESTION 290 *anſwered by Mr.* I. Aſh.

The $\triangle CSW$ being formed, make $\angle WSD = 57^\circ\ 45'$, and $\angle DWS = 28^\circ\ 34'$; then circumſcribe $\triangle SWD$ with a circle: draw DT through C, and the point T is the town's ſituation. From whence, by trigonometry, $TC = 0{\cdot}65707$, $TW = 23{\cdot}3439$, and $TS = 10{\cdot}5721$ fere.



Mr. *James Terey* has given a conciſe and elegant conſtruction in the ſame manner; and ſo has Mr. *Farrer*. And it is ſolved by Mr. *Bamfield*, Mr. *T. Cowper*, Mr. *Collingridge*, Mr. *Walker*, Mr. *Hampſon*, Mr. *Fitches*, Mr. *Elgar*, Mr. *Waine*, Mr. *Kingſton*, &c.

XIII. QUESTION 291 *anſwered by Mr.* Aſh.

By trials A is found between 48 and 49; put $\overline{7-x}|^2 = A$, $1{\cdot}05^{\overline{7-x}|^2} = \frac{7-x}{0{\cdot}65 + 0{\cdot}5x}$ per queſt. (which ſubſtitution is neceſſary to make the following ſeries converge). Let $c = 0{\cdot}5$, $n = 7$, $q =$ hyp. log. 7, $m = {\cdot}65$, and $p =$ its log. alſo $a =$ log. $1{\cdot}05$; then (by log. ſeries) $\overline{n-x}|^2 \times a = q - \frac{x}{n} - \frac{x^2}{2n^2} - \frac{x^5}{3n^3}$ &c. $+ p - \frac{cx}{m} - \frac{ccxx}{2mm} - \frac{c^3x^3}{3m^3}$ &c. And by reverſion, $x = 0{\cdot}030409$; $\because \overline{n-x}|^2 = A = 48{\cdot}5752$ &c. the lady's age.

Mr. *Turner* exactly ſolved the ſame by a table of logarithms (which is the eaſieſt and quickeſt way). Mr. *Farrer*, Mr. *Dixon*, Mr. *Waine*, and Mr. *Hall* ſolved it by another method.

XIV. QUESTION 292 *anſwered by Mr.* J. Turner.

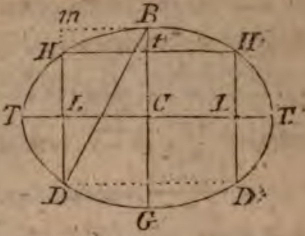
Put $x =$ the pounds of the legacy; then $\overline{\text{l.}\,x}|^x = \overline{20x}|^{\text{l.}\,x}$ per queſtion; or putting $b = 2{\cdot}30258$ &c. $\overline{b\,\text{l.}\,x}|^x = \overline{20x}|^{b\,\text{l.}\,x}$ by common logarithms: And by a few trials $x = 15{\cdot}615 =$ 15 l. 12 s. 3 d. required.

Mr. *Farrer* ſolved the ſame by ſeries; as did ſeveral others: But as the ſolving of all ſorts of exponential equations by the tables of logarithms is preferable to all other methods, the general method of ſolution ſhall be exhibited in ſome future Diary.

XV. QUES-

XV. QUESTION 293 *anſwered by Mr.* Heath.

As the contained liquor always bears a proportion conſiderably greater than the vacuity, in the ſeveral variations of the form of the caſk (which ſuppoſing but 2 to 1) the greater the vacuity, the greater will be the quantity of liquor, and conſequently the ſum of both; therefore when the ullage is a maximum, the whole caſk will be alſo a maximum.



Firſt, *to find the content of ſpheroidal fruſtums.* Let $t = TT$, the tranſ. $b = BG$, the conjugate of the gener. ellipſis; and $x = CL$. By the property of the curve, $tt : bb :: \frac{tt}{4} - xx : \frac{bb}{4} - \frac{bbxx}{tt} = HL^2$; conſequently $bb - \frac{4bbxx}{tt} = HD^2 = hh$, (whence $tt = \frac{4bbxx}{bb - hh}$), and $n\dot{x} \times \frac{bbtt - 4bbxx}{tt}$ = flux. fruſt. $BGDHB$ (n being ·7854) whoſe fluent is $nx \times \frac{bbtt - 4bbxx}{3tt}$; in which ſubſtituting for tt, we have $nx \times \frac{2bb + hh}{3}$ = the content $BGDHB$, which holds true when it is the ſegment or fruſtum of a ſphere.

THEOREM. Multiply the length of any ſpheroidal caſk into the ſum of twice the ſquare of the bung diameter added to the ſquare of the head diameter, and that product multiplied by ·2618 will give the content in inches; or divided inſtead by 1077·158, or 882, 3529 will give the content in ale or wine gallons: or multiply inſtead by ·00092837, or ·0011333, &c.

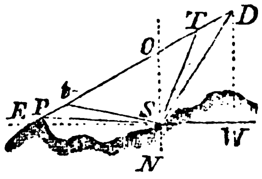
N. B. If x be put for TL; then the flux. ſegment $DTH = n\dot{x} \times \frac{bbtt - 4bbxx}{tt}$, whoſe fluent gives the ſame theorem as above for the content.

Now if $c = Bt$, $d = BD$, and x be put $= Dm$; then $x - c$ = head diam. $x + c$ = bung diam. $2\sqrt{dd - xx}$ = length of the caſk, and $\frac{3cc + 2cx + 3xx}{\text{Diviſor}} \sqrt{dd - xx}$ =

the content, a maximum. In fluxions and reduced, $9x^3 + 4cxx + 3ccx = 6ddx + 2cdd$, where $x = 44{\cdot}1615$. Whence head diam. $= 35{\cdot}1615$, bung diam. $= 53{\cdot}1615$, length $= 65{\cdot}0567$ inches, and content $= 419{\cdot}313$ ale gallons. Again, put $x = Bt$; the ſection or ſurface of ullage, HtH, is an ellipſis, whoſe conjugate $= 2\sqrt{bx - xx}$; and tC being $\frac{b}{2} - x = HL$, if $y = LC$, then $tt : bb :: \frac{tt}{4} - yy : \frac{bb}{4} - bx + xx$, and $2y = \frac{2t}{b}\sqrt{bx - xx} =$ tranſverſe axis. The area ſection is $4nt \times \frac{bx - xx}{b}$, and flux. ſegment HBH $4nt\dot{x} \times \frac{bx - xx}{b}$, whoſe fluent $4ntx \times \frac{3b - 2x}{6b}$ is its content (which when $b = t$ becomes $4nxx \times \frac{3t - 2x}{6}$ for the ſpherical-ſegment, as the other ſegment * becomes $nx \times \frac{3tt - 4xx}{3}$, proved alſo in Diary p. 45, 1747). But, by above, $t = \frac{2bx}{\sqrt{bb - bb}}$ $= 87{\cdot}4207$; whence the content of the fruſtum, or ſegment $HBH = 34{\cdot}99$ gallons; conſequently $384{\cdot}322$ ale gallons of liquor remain in the caſk. *Q. E. F.*

Mr. *Waine* was curious in his method of ſubſtitution for finding the content of the whole caſk, and ſolution of the ſame; but ſent no theorem for the ullage. The Rev. Mr. *Baker* ſent the ſame theorem for finding the content of the ullage, with the above; but came too late to be inſerted.

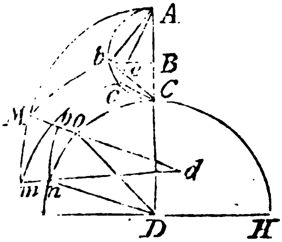
XVI. QUESTION 294 *anſwered by Mr.* T. Cowper.



Let D and T be the places of the tender at the firſt and ſecond obſervations; t and P the places of the tender and privateer when the latter is ſtranded; and S the ſtation of the obſerver. By the queſt. $\angle TSD = 11^\circ\ 15'$, and found moving 1142 feet per ſecond, $SD = 18843$ feet, and $ST = 14275$; whence by trigon. $TD = 5586$, and $\angle STo = 41^\circ\ 9\frac{1}{4}'$. Now the interval between the firſt and ſecond flaſhes at D, and T, being $8'\ 16\frac{1}{2}''$, and and from the flaſh at T, till the privateer was drove aſhore at P, by the tender at t, 35′ 12½″; by proportion of motion, Tt will be found 23767·22, and $tP = 1350·09$ feet. Whence $\angle oSt = 80° 31'$, the bearing of the tender about E. by S. diſt. $St = 3·04061$ miles; alſo the $\angle oSP = 84° 10'$, the privateer's bearing about E. ½ S. diſt. $SP = 3·2514$ miles required.

The PRIZE QUSTION *anſwered by Mr.* J. Landen.

Let Md be perpendicular to the epicycloid AMm, and md another perpendicular ſuppoſed infinitely near the former; then the concentric arches Mb, mc, being deſcribed from the center D of the immoveable circle; the chords Cb, Cc of the moveable one will be reſpectively $= Mo$, mn.



From the center d (the point where the perpendiculars Md, md touch the evolute) deſcribe the ſmall arch nv; and the ſmall right lines be, ce, being conſidered as ſmall arches, deſcribed from their reſpective centers A and C; the ſmall right-angled triangles bce, vno, will be ſimilar and equal; bc being $= no$, and $ce = nv$; ∵ $Cc : dn :: \angle ndv : cCe$. But $ndv = nDo + cCe$. And calling AC, $a = 2$; CD, $b = 2640$ feet; AB, x; we get $nd = \frac{b\sqrt{aa-ax}}{a+b}$; now, as $\frac{b\sqrt{aa-ax}}{a+b}$ (nd) : $\frac{ax}{2\sqrt{ax}}$ $(vn = ce)$ :: $\frac{a+2b}{a+b} \times \sqrt{aa-ax}$ $(mn + nd)$: $\frac{a+2b}{2b\sqrt{ax}} \times ax$ (Mm) the fluxion of the epicycloid; the fluent whereof is $\frac{a+2b}{b}\sqrt{ax}$ = half the ſpace a ſpot on the convex ſurface of the ſtone will go through in one revolution (when $x = a = 2$) = 4.001515, &c. Conſequently 8·00303, &c. feet is the whole ſpace deſcribed at each revolution; and the ſtone revolving exactly 1320 times, going up and down the mountain, the whole ſpace gone through by the ſpot will be 10564 feet, or 2·00075, &c. miles.

Removing the ſtone to the next point from the vertex, the gravity will make it deſcend along the ſide of the mountain, till its velocity, in an horizontal direction, is the greateſt it can acquire; when it will fly off, and deſcend to the bottom, in the curve of a parabola. Putting $a = 2640$ feet, the mountain's height, $x =$ diſt. perp. deſcended, $s = 16\frac{1}{12}$ feet, a ſpace perp. deſcended in the 1ſt ſecond of time, then $\sqrt{s} : 2s :: \sqrt{x} : 2\sqrt{sx}$ the velocity in the curve, which is to the velocity in an horizontal direction, as the tangent is to the ordinate, i. e. $\frac{a}{a-x} : 1 :: 2\sqrt{sx} : \frac{2\sqrt{s}}{a}\sqrt{aax - 2axx + x^3}$. the velocity in a direction parallel to the horizon, whoſe fluxion reduced is $xx - \frac{4ax}{3} + \frac{aa}{3} = 0$, where $x = \frac{1}{2}a$ the diſtance deſcended when the ſtone ceaſes to touch the mountain.

The velocity $2\sqrt{sx}$ applied to $\frac{a\dot{x}}{\sqrt{2ax - xx}}$, the fluxion of the arc, gives $\frac{a}{2\sqrt{s}} \times \frac{\dot{x}}{x\sqrt{2a - x}}$, the fluxion of time, whoſe fluent is $\frac{\sqrt{a}}{2\sqrt{2s}} \times$ hyp. log. $x + \frac{\sqrt{a}}{2\sqrt{2s}} \times : \frac{x}{2.2.a} + \frac{3x^2}{2.2.4.2^2a^2} + \frac{3.5x^3}{3.2.4.6.2^3a^3}$ &c. $= 31''$ (when $x = \frac{1}{3}a$). If to which $6''\ 18'''$, the time of deſcending through the parabolical arch, be added, the ſum is $37''\ 18'''$, the whole time of deſcent required; and the ſtone will fall 326 feet from the foot of the mountain. *Q.E.F.*

Mr. Stone obſerves in his Mathematical Dictionary, that 'as ſemi-diam. reſting circle : ſum diameters of both circles :: double ſine of half the arc, touching the circle at reſt : length of the part of the epicycloid deſcribed by a point in the revolving circle; if upon the convexity of the reſting circle. But if upon the concavity; as ſemi-diam. reſting circle, to the difference of diameters of the touching circle.'

In a ſemi-revolution, 180° of the moving circle touched the circle at reſt; the double ſine of its half arc $= 2$ (its rad. being unity); ∵ 2640 : 5282 :: 2 : 4·001515, &c. = half the epicycloidal curve, as before.

Mr. *Nicholas Dixon* ſent us (from cor. 2 prop. 49 lib. 1 Principia Newtonian.) the ſame proportions, who gave the ſpace run through by the ſpot 10564 feet; computing it from 1320 revolutions of the ſtone. Our ingenious correſpondent Mr. *Aſh* gave the fluxion and fluent of the curve, by two different methods; confirming the truth of the above ſolutions. He refers to Mr. Stone's Fluxions (p. 127 cor. 1) in his firſt and beſt method; finding the fluent of the ſemi-curve $\frac{2bx+2ax}{b}$; $b=$ rad. reſting circle; $a=$ rad. moving circle; and $x=$ a variable chord, till it becomes $=$ diam. moving circle, or $2a$.

Mr. *William Hounſell*, and Mr. *Cottam*, are true in the diſtance travelled by the ſpot.

The Prize of 10 *Diaries was won by Mr.* Landen.

The Eclipſes calculated for 1748, *by Mr.* Bulman.

1. The ſun eclipſed Jan. 19, at 3 h. 27 m. in the morning. Conſequently inviſible to our part of the globe.

2. The moon eclipſed Feb. 3, at 11 h. 53 m. morn. inviſible.

3. A great viſible ſolar eclipſe July 14, at 10 h. 43 m. morn.

Calculated by		Beg.	Mid.	End	Dur.	Dig.	
		h. m.	h. m.	h. m	h. m.		
Mr. Turner, by a geom. conſtruct. and calculation for	Hull	9 0	10 32	12 6	3 6	$10\frac{3}{4}$	☉'s upper limb.
	London	—	10 34	—	—	$10\frac{1}{8}$	
	Paris	—	10 49	—	—	$9\frac{5}{8}$	
	Boſton N.E.	—	4 53	—	—	$9\frac{7}{8}$	
	Dundee	—	10 19	—	—	Central.	
	Stockholm	—	12 14	—	—	$9\frac{5}{8}$	☉'s slow. limb.
	Peterſburgh	—	1 15	—	—	$8\frac{1}{2}$	
Mr. Bulman, By Aſtro. Carol.	London	9 13	10 43	12 17	3 4	10 20	
Leadbet. Tables,	Idem	9 6	10 40	12 9	3 13	10 26	
Aſtro. Carolina,	Rocheſter	9 14	10 44	12 18	3 4	10 21	
Leadbet. Tables,	Idem	9 7	10 41	12 20	3 13	10 28	
Mr. Hawkins,	London	9 3	10 40	12 17	3 14	10 31	
Mr. Cockſon,	London	8 56	10 31	12 6	3 10	10 18	
	Lambton	8 50	10 31	12 12	3 22	11 14	
	Durham	8 50	10 31	12 12	3 21	11 14	
	Feckenh.	8 46	10 28	12 10	3 24	11 6	
Mr. Cowper,	London	8 57	10 28	12 1	—	10 1	
	Northamp	9 1	10 32	12 6	—	10 4	
Mr. Farrer,	Sunderland	9 15	10 48	12 24	3 9	11 38	
Late Mr. Beighton,	Coven.	9 8	10 38	12 12	3 4	10 16	

Mr. *Turner* curiouſly obſerves that the penumbra, in the great eclipſe of the ſun, firſt touches the earth's diſk, in lat. 44° 15′ north, and long. 75° 47′ weſt from London. That the central eclipſe firſt enters upon the river St. Lawrence, a little to the ſouthward of Quebec; from whence directing its courſe about E. N. E. it paſſes over St. Lawrence Bay, the northermoſt part of Newfoundland, and the Atlantic ocean; and arriving in lat. 57° 32′ and long. 18° 30′ weſt (where it approaches the neareſt to the north pole) it then paſſes about E. S. E. traverſing the weſtern iſles of Scotland, over the midſt of Barra, and the north part of Mull; ſo over Fort William and Dundee; then croſſes the German ocean, and paſſes over Lubec, Glogaw, and Breſlaw, being centrally eclipſed in the meridian in lat. 52° 5′ N. and long. 15° E. in which ſame longitude, and in lat. 18° 3′ N. the lower limb of the moon juſt touches the upper limb of the ſun in the meridian. Then the central ſhadow enters Poland, goes near Cracow, and croſſing the ſouthern parts thereof, enters Turkey in Europe, and croſſes over the Euxine ſea, the Grand Signior's Aſiatic dominions, Perſia, and the eſtates of the Great Mogul; and finally quits the earth near Tranquebar, upon the coaſt of Coromandel, in lat. 10° 54′ N. long. 77° 49′ E. Laſtly, in long. 55° 30′ E. under the equator, the penumbra quits the earth's diſk, and the eclipſe totally ends at the top of the ſun's vertical diameter, whilſt his laſt rays are hiding under the horizon.

The breadth of the annular ſhade is about 166 geographical, or 192 Engliſh miles; and the velocity of which, over the earth's diſk, 40 ſtatute miles in a minute.

But the velocity whereby the ſhade recedes from any given place on the earth's ſurface, is leſs than that with which it paſſes over the diſk: Becauſe while the ſhadow moves from weſt to eaſt, all the places of the earth are carried by its rotation the ſame way, which following the motion of the ſhadow with a ſlower pace, they diminiſh the velocity of it; moving from them; making it move not above 30½ ſtatute miles per minute.

Hull, April 11*th*, 1737. *J. Turner.*

Mr. *Bulman* ſays according to the Chaldean ſaros, that this eclipſe will return again July 25, 1766; and that it is the greateſt viſible ſolar eclipſe which will happen before March 21, 1764. Mr. *Sam. Owen*, of Kent, ſent an exact calculation for London. Some other perſons, beſides thoſe inſerted in the tables, ſent calculations of this year's eclipſes, which for want of being correct are omitted. And we muſt acknowledge the favour of Mr. *Cowper* in offering us a correct and annual calculation of the lunations; as we are ſorry his lunations for laſt year were omitted for thoſe which were groſly falſe and inconſiſtent.

4. The moon viſibly eclipſed July 28, at 11h. 34m. at night.

Calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h.m.	Dig.
Mr. Bulman,	London	10 24	11 34	12 44	2 20	4 37
	Edinburgh	10 12	11 22	12 32		
	Dublin	9 56	11 6	12 16		
	Carliſle	10 13	11 23	12 33		
	Rocheſter	10 25	11 35	12 45		
Mr. Hawkins,	London	10 32	11 37	12 41		4 45
	Hanover	11 12	12 17	1 21		
	Oxford	10 27	11 32	12 36		
Mr. C. Cockſon,	London	10 32	11 36	12 40		
	Lambton	10 45	11 49	12 53	2 8	4 40
	Durham	10 45	11 49	12 53		
	Feckenham	10 38	11 42	12 46		
Mr. T. Cowper,	Wellingbor.	10 6	11 13	12 19		5 near
Mr Farrer,	Sunderland	10 43	11 48	12 52	2 9	5 2

Mr. Hawkins has confirmed the above eclipſes, and given (very judiciouſly) the appearance of the ſolar eclipſe on Jan. 19, in lat. 43° 38′ S. long. 64° 8′ E. And alſo that of the moon's eclipſe for Feb. 3, at Boſton, Carthagena, and Hiſpaniola.

New

The beſt obſervations of the Eclipſe of July 14, are as here below.

Places,	Obſerver	Beginning	End
		h. m. s.	Between
Marlborough Houſe	Dr. Bevis	9 3 48	12 h. 9 m. 15 s. and 12 9 35 Ap.t.
Lufwick, Northampt. lat. 52° 27½′	Mr. Mark Day	9 1 0	12 5 25 Ap.t.
Madrid	A. N. Grexhow	8 49 6	True time
Aberdour Caſtle, N. Britain	Lord Morton, Mr. Short, and Mr. le Monnier	8 51 18	11 48 18

Aberdour Caſtle is in lat. 56° 4′, and long. 6° 15′ weſt of Edinburgh College.——The Eclipſe was obſerved to be annular in N. Britain, Berlin, Francfort, and many other places.

A COMET

Was this year obſerved, and an account of its motion taken at Pekin in China, in the months of April, May, and June.

New Queſtions.

I. QUESTION 295, *by Mr.* Heath.

With guineas and moidores the feweſt, which way
Three hundred and forty-one pounds can you pay?
If paid ev'ry way 'twill admit of, what ſum
Do the pieces amount to?——My fortune to come.

II QUESTION 296, *by* Amanda.

To find the leaſt number of guineas, which being divided by 6, 5, 4, 3, and 2, reſpectively, ſhall leave 5, 4, 3, 2, and 1, reſpectively remaining?

III. QUESTION 297, *by Mr.* John Turner.

What is the day of the month, and hour of the day, at Rocheſter, in Kent, (at which time a couple are to be married this year) when the degrees of time from noon, to the degrees of time from ſun-riſe, are in the proportion of 3 to 1, and the reſpective ſines of thoſe degrees in the proportion of 9 to 4?

IV. QUESTION 298, *by Miſs* Manlove.

All the different ways poſſible, in which a gentleman can place his ſervants, combining them by 1, 2, 3, &c. at a time are,960799; what number of ſervants does he keep?

V. QUESTION 299, *by Mr.* Bulman.

Three to two, top and bottom, a tub's width I've found,
Seven yards the diag'nal, as it ſtands on the ground,
With its gallons the moſt; but in regions below,
Where 'twill take in three more, brother conjurers, ſhow.

VI. QUESTION 300, *by* Rodomontado.

What is the leaſt degree of velocity with which an iron ball of 12 pounds weight muſt be projected from the ſurface of our earth, at an angle of 40° elevation, whereby it ſhall not return.

VII. QUES-

VII. QUESTION 301, *by Mr.* Landen.

If an infinite number of perpendiculars be let fall from one end of the diameter of a ſemicircle, upon an infinite number of tangents drawn about it, and a curve paſſes through all thoſe angular points, what will be the length of that curve? The area of the ſpace included betwixt it and ſemi-circle? With the dimenſions of the greateſt ordinate, when the ſaid diameter is = 20 inches?

VIII. QUESTION 302, *by* Upnorenſis.

Obſerving a horſe tied to feed in a gentleman's Park, with one end of a rope to his fore foot, and the other end to one of the circular iron rails, incloſing a pond, the circumference of which rails being 160 yards, equal to the length of the rope, what quantity of ground, at moſt, could the horſe feed?

IX. QUESTION 303, *by Mr.* John Turner.

If the axis of the penumbral cone, falling upon the diſk the earth, makes an angle with the earth's diameter at the ſurface of 24°, (the angle at the cone's vertex being 32′ 46″) and from a point in that axis, at the diſtance of 58·5 ſemi-diameters of the earth from the vertex, it is 64 ſemi-diameters to the earth's center, how much of the earth's ſurface is included in the penumbral ſhadow?

X. QUESTION 304, *by Mr.* John Hampſon.

Required to find three ſuch fractional numbers, that when each is leſſen'd by the cube of their ſum, three cube numbers ſhall remain?

XI. QUESTION 305, *by* Roſamond.

To find two (or more ſuch pair of amiable, but unequal) numbers, that each ſhall be mutually equal to the ſum of the aliquot parts of the other? And alſo to find the leaſt number, whoſe aliquot parts ſummed up, ſhall exceed it by 7?

XII. QUESTION 306, *by Mr.* Heath.

If the long-diſputed prize money, between the officers of the ſhip Centurion and Glouceſter, had been divided in the proportion of two numbers [each of which being raiſed to a power expreſſed by the natural logarithm of the other, shall be equal to the ſum and difference of thoſe numbers]. What would be the odds of advantage allotted to Lord Anſon's officers belonging to the ſaid ſhip the Centurion?

XIII. QUESTION 307, *by Mr.* Landen.

Let a ball of heavy metal be laid upon one end of an horizontal plane, of an indefinite length, round which end let the plane be made to revolve downwards, with ſuch an uniform motion, that the angle of inclination may increaſe at any given rate: It is required to find what length the ball will deſcend along the plane, before it acquires ſuch a velocity as will cauſe it to fly off, and ceaſe touching the plane?

XIV. QUESTION 308, *by* Upnorenſis.

A lady paid twice as much a-piece for geeſe, as ſhe paid for ducks; and twice as much a-piece for ducks, as ſhe paid for chickens, which coſt together 1 l. 13 s. 4 d. the ſum of the ſquares of the number ſhe bought of each ſort was 326. What number of geeſe, ducks, and chickens did ſhe buy? And what was the price of each?

XV. QUESTION 309, *by* Hurlothundro.

What are the odds of battle, or the different probabilities of ſucceſs of two armies going to engage, the chances of each army for victory being reſpectively equal to the ſum raiſed to a power expreſſed by the difference, and the difference raiſed to a power expreſſed by the ſum, of thoſe chances?

XVI. QUESTION 310, *by Mr.* Aſh.

A ſpider, at one corner of a ſemi-circular pane of glaſs, gave uniform and direct chace to a fly, moving uniformly along the curve before him: The fly was 30° from the ſpider at their firſt ſetting out, and was taken by him at the oppoſite corner. What is the ratio of both their uniform motions?

PRIZE QUESTION *by Mr.* R. Heath.

On what days of the year does the city of London travel the greateſt and leaſt number of miles, by the diurnal and annual motion of the earth? And how many miles per day, and alſo per hour about noon does it travel, when the days are longeſt and ſhorteſt in that place?

Queſtions

1749.

Queſtions anſwered.

I. QUESTION 295 *anſwered by Mr.* John Turner *of* Heath, *near* Wakefield.

IT is evident, that with the number of pounds printed (by miſtake) the queſtion is impoſſible; but ſuppoſing 351 l. intended (as the author informs me of) inſtead of 341, it is thus anſwered.

Put $x =$ number of guineas, $y =$ number of moidores $= \frac{7020 - 21x}{27}$ by conſequence, which muſt be a whole number. Whence $\frac{21x}{27}$, or $\frac{x}{9}$ is a whole number; conſequently the leaſt value of $x = 9$; whence $y = 253$, whoſe ſum is 262 the leaſt number of pieces; becauſe there are taken the moſt moidores, except paying the whole with them. Let $\frac{x}{9} = m$, then $x = 9m$, and $y = 260 - 7m$ by ſubſtitution; by which the greateſt value of $m = 37$, when $x = 333$ guin. and $y =$ 1 moid. the greateſt number of pieces. The firſt term of an arithmetical progreſſion being 9 (a), the laſt 333 (y), and the difference 9 (e) as it is evident, then the number of terms $= \frac{y + e - a}{e} = 37$, the ways to pay the ſum of 351 l. which being 37 times taken is $= 12987$ l. due to Mr. Heath.

N. B. The intermediate numbers are found by continually adding 9 to the guineas, and ſubtracting 7 from the moidores. There will be but 36 ways, if the payment with one moidore is not admitted. *Q. E. F.*

We deſire all thoſe who anſwered this queſtion, or any other wrong, not to be diſobliged at our omiſſion of their ſolutions; being not againſt their ſhining in a proper place.

II. QUESTION 296 *anſwered by Mr.* Heath.

It is a maxim, *That whole numbers added to, ſubtracted from, or multiplied into whole numbers, ſhall produce whole numbers*: Upon which fundamentals theſe kind of queſtions are naturally reſolved; though I have not ſeen the method clearly explained. Let $x =$ the number ſought, then $\frac{x-5}{6}$, $\frac{x-4}{5}$, $\frac{x-3}{4}$, $\frac{x-2}{3}$, $\frac{x-1}{2}$ are all whole numbers by the queſt. Put $\frac{x-5}{6} = m$, then $x = 6m+5$ a whole number, and by ſubſtitution in the ſecond whole number for the value of x, $\frac{6m+1}{5} = m + \frac{m+1}{5}$ a whole number, ∵ $\frac{m \quad 1}{5} = n$ a whole number, and $m = 5n-1$; which ſubſtituted for m, in the firſt value of x, is $x = 30n-1$ for the ſecond value; ∵ $\frac{30n-4}{4}$ by ſubſtitution is the value of the 3d number, and $7n - 1 + \frac{2n}{4}$ is a whole number, and $n =$ a whole number; ∵ ſubſtituting again the 2d value of x in the 4th whole number, $\frac{30n-3}{3} = 10n - 1 =$ a whole number, ∵ n is a whole number (firſt ſuppoſed); and ſubſtituting again the 2d value of x in the 5th whole number, $\frac{30n-2}{2} = 15n-1$ a whole number; ſo that $x = 30n-1$ is the general value, after aſſuming $n = 2$, by which the leaſt value of $x = 59$.

Mr. *Richard Gibbons* of Plymouth has ſhewn an eaſy method of finding the leaſt number only, to anſwer theſe kind of queſtions. For he obſerves that $2 \times 3 \times 4 \times 5 \times 6 = 720$ is a number, which being divided reſpectively by 6, 5, 4, &c. will leave no remainder: and therefore $720 - 1 = 719$ will leave the required remainders, of 1 leſs than each factor. But 4 being a ſquare of 2, and 6 a multiple of 2 and 3, ∵ $2 \times 3 \times 2 \times 5 : - 1 = 59$ the leaſt number required.

Mr. *Flitcon* has made exactly the ſame obſervation; and alſo Mr. *James Terey* of Portſmouth.

Mr. *Farrer* gives $x = 60q - 1$, where q is a whole number, for the general value of x; which it ſeems is according to Mr. *Robinſon*'s method: but the next value of x to 59 is *89, which* Mr. *Farrer*'s equation does not find, and therefore not true, as is proved above. This gentleman obſerves, that this queſtion is of like kind with one in another kind of place; to which we did not deſign any reſemblance, nor conſulted ſuch pattern, if we have done any honour. Mr. *Collingridge*, Mr. *Hare*, and ſome others, ſolved this queſtion.

III. QUESTION 297 *anſwered by Mr.* Nich. Farrer.

Let $x =$ s. time from ſun-riſe, then $3x - 4x^3 =$ s. time from noon. By queſt. $9 : 4 :: 3x - 4x^3 : x$; hence $x = \frac{\sqrt{3}}{4}$ $= \cdot 4330127 =$ s. 25° 39′ 32″, and $3x - 4x^3 = \cdot 9742786 =$ s. of 76° 58′ 36″ = 5h. 7′ 54″ time. Hence the ceremony begun at 6h. 52′ 6″ in the morning. ☉ riſes 5 h. 9′ 27″, aſcenſional diff. = 12° 38′ 8″ lat. Rocheſter 51° 28′, from which the ſun's declination = 9° 53′, anſwering to April 4th, or Auguſt 16th. *Q.E.F.*

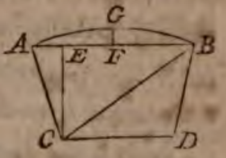
Mr. *Gibbons* has ſolved this queſtion by the method of trial-and-error; thereby proportioning the truth very exactly. He obſerves the advantages of trial above the uſe of ſeries, in many caſes. The ſame was ſolved by the Rev. Mr. *Baker*, and ſome others.

IV. QUESTION 298 *ſolved by Mr.* Collingridge.

Let $x =$ the number of ſervants; then, by Mr. Stone's Mathematical Dictionary, $\frac{x^{x+1} - x}{x - 1} = 960799$, all the poſſible variations of the ſervants; which equation ſolved, $x =$ 7 ſervants exactly. *Q.E.F.*

Mr. *Turner* ſolved the ſame; referring to Wolfius' Elementa Matheſeos, p. 305, for the equation; which however is eaſy to be deduced from the principles of combination. Mr. *Farrer* ſolved it likewiſe.

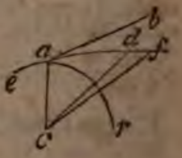
V. QUESTION 299 *anſwered by Mr.* James Terey *of* Portſmouth.



Let $CB = 7 = d$, $AB = x$, then $CD = \frac{2}{3}x$, per queſt. alſo $BE = \frac{5}{6}x$. Then $\sqrt{dd - \frac{25}{36}xx} = CE$. Now $\frac{19}{9}xx \times \sqrt{dd - \frac{25}{16}xx} \times 43{\cdot}313864$ = content in ale gallons, a maximum; or $x^4 \times \overline{36dd - 25xx}$, a maximum; in fluxions and reduced, $x = \frac{d\sqrt{24}}{5} = 6{\cdot}858571279$ yards $= AB$; and $CD = 4{\cdot}572380852$, $CE = 4{\cdot}04145188$ yards. The content in ale gallons $= 17383{\cdot}721448$ &c. But to hold 3 gallons more AGB muſt be a ſpherical ſegment, containing 3 gallons. Let $FG = z$, $AF = FB = b$. By a theorem, ${\cdot}523598z$ &c. $\times \overline{3bb + z^2} =$ cont. ſpher. ſeg. in numbers $z^3 + 35{\cdot}28\,z = {\cdot}0346309367$, where $z = {\cdot}00098160248$ yards. The diameter of which ſphere $= 11980{\cdot}410811$ &c. yards. Conſequently, if the brim of the tub is placed $5990{\cdot}205405$ yards from the earth's center, it will hold 3 gallons more. *Q.E.F.*

Mr. *Bamfield* has found the diſtance from the earth's center exactly the ſame to a yard; but his figure was drawn ſo large, that we were obliged to omit inſerting his anſwer: as we omit other anſwers to queſtions on account of the unfit ſize of the ſchemes drawn, which require too much time and trouble to alter. Mr. *Baker*'s ſcheme was very proper, and his ſolution very elegant to this queſtion; but we cannot inſert all good performances. Mr. *Farrer*'s figure was too large. Mr. *Hampſon* and Mr. *Himbol* ſent us their ſolutions. Mr. *Bulman* the propoſer died near Rocheſter, in Kent, about Michaelmas 1747, who may have given a practical ſolution himſelf before this time; though he was (like moſt of the brother conjurers) a very honeſt, odd kind of perſon.

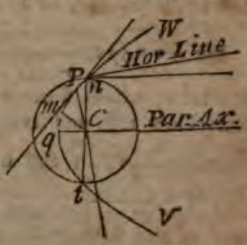
VI. QUESTION 300 *ſolved by the Excellent* J. Landen, *near* Peterborough.



Let c be the earth's center, ear the ſurface, ab the projectile's direction, and adf its trajectory. Suppoſe cd, cf indefinitely near each other, and call ca, (the earth's radius $= 21000000$ feet) a; cd, x; $32{\cdot}2$ feet, the velocity generated in a ſecond at the earth's ſurface, b; v the velocity in d; V the required velocity. Then the centripetal force in d will be $\frac{a^2b}{x^2}$ (being reciprocally as the ſquare of the diſtance from the earth's center) and the force to retard the motion in the direction df, $\frac{a^2b\dot{x}}{x^2 \times \dot{df}}$; this retarding force drawn into the fluxion of the time, being equal to the fluxion of the velocity, $\frac{a^2b\dot{x}}{vx^2}$ will be $= -\dot{v}$;

$-\dot{v}$; therefore $v\dot{v} = -\frac{a^2 b\dot{x}}{xx}$, and the fluent $\frac{vv}{2} = \frac{a^2 b}{x}$: But in a, (v being $= V$, and $x = a$) the correct fluent gives $v = \sqrt{VV - 2ab + 2a^2 bx^{-1}}$. After an infinite time, x will be infinitely great, and $\sqrt{VV - 2ab + 2a^2 bx^{-1}}$ infinitely ſmall, and therefore may be put $= 0$, in which equation a is nothing in reſpect of the value of x: and therefore $V = \sqrt{2ab}$. Hence, without regard to the angle of direction, if a body be projected from the earth's ſurface, in any direction whatever above the horizon, with ſuch a velocity as will carry it above 7 miles per ſecond, it will never return. *Q. E. F.*

Mr. John Turner *of* Heath, *ſends us his Solution as follows.*

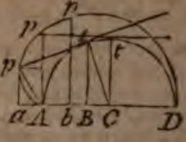


Let C repreſent the earth's center, and let a body be ſuppoſed to revolve in a circle at the ſuperficies, the angle of the projectile's elevation being 40°, and a tangent to the parabolical curve qPW at P, the $\angle CPm$ will be 50°, (letting fall Cm perpendicular to Pm) whoſe complement to $180° = 130° =$ angle mPb. By prop. 17 and corol. of prop. 16, Newton's Principia, the axis of the parabolic trajectory will be parallel to Pb, paſſing through C; and likewiſe the latus rectum of the orbit $= 2CP + 2Pn = 2{\cdot}3472$ ſemi-diameters of the earth (letting fall Cn perpendicular to Pb) and the focal diſtance from the vertex $= {\cdot}5868$ ſemi-diameters. Laſtly, the velocity of a body moving in a circular orbit at the earth's ſuperficies (by the prop. aforeſaid) is ſuch as would carry it through 4·92 miles uniformly, in each ſecond; therefore, if $x =$ velocity in the parabolic curve at P, we have (by prop. 15, 16, and 17 of the ſaid Principia) As $24{\cdot}2064 : {\cdot}5868\,xx :: 2 : 2{\cdot}3472$; whence $x = 6{\cdot}958$ miles, or about 7 miles per ſecond, the uniform velocity with which the body muſt be projected from P in the given angle of elevation, not to return. Or, ſince it is demonſtrated (by the writers on phyſics) that the velocity of a body moving in a parabola is to the velocity of a body moving in a circle, at the ſame diſtance from the center of force, as $\sqrt{2}$ to 1, $\because 4{\cdot}92 \times 1{\cdot}4142 = 6{\cdot}958$ miles, the projectile's velocity per ſecond, as before. Q. E. F.

VII. QUES-

VII. QUESTION 301 *anſwered by Mr.* Landen.

It is eaſily proved that the perpendicular Ap is always $=$ its reſpective abſciſſa AB of the ſemicircle, and the $\angle pAe = \angle tCt$. Then, if Aep be ſuppoſed infinitely near Ap, and Ap or AB be called x, and AD, $2a$; it follows, as $a : \frac{a\dot{x}}{\sqrt{2ax-xx}} ::$ $x : \frac{x\dot{x}}{\sqrt{2ax-xx}} =$ the infinitely ſmall arch pe, and the fluxion of the curve $= \sqrt{2a} \times \frac{\dot{x}}{\sqrt{2a-x}}$, the fluent of which corrected is $4a - 2\sqrt{4aa-2ax} =$ length of the curve; which, when $x = 2a$, in the preſent caſe, is equal to twice the diameter AD, or 40 inches.

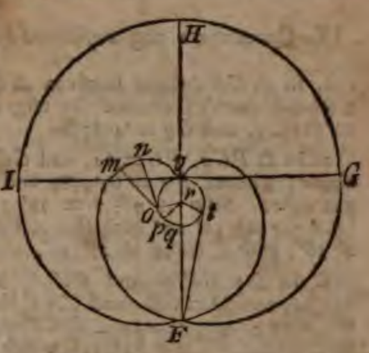


But $\frac{1}{2}x \times (pe)\ \frac{x\dot{x}}{\sqrt{2ax-xx}} = \frac{\frac{1}{2}x^{\frac{3}{2}}\dot{x}}{\sqrt{2a-x}}$, the fluxion of the area; whoſe fluent, by Mr. Cotes' 6th form, or Mr. Emerſon's 10th and 11th forms, is $\frac{3}{2}aa \times$ arch of a circle whoſe radius $= 1$ and nat. ſine $\sqrt{\frac{x}{2a}}$, $-\frac{3}{4}ax^{\frac{1}{2}}\sqrt{2a-x}$ $-\frac{1}{4}x^{\frac{3}{2}}\sqrt{2a-x} =$ area ſought; in the preſent caſe $= \frac{3}{2} \times$ area of the ſemicircle, when $x = 2a$. Conſequently the ſpace $ApDtA =$ quadrant $AttCA = 78{\cdot}54$ inches.

For the greateſt ordinate, the proportion is $a : \sqrt{2ax-xx}$ $:: x : \frac{\sqrt{2ax^3-x^4}}{a} =$ any ordinate. Its fluxion being made $= 0$, $x = \frac{3}{2}a = 15$ (though $= bD$), which being ſubſtituted for x in the expreſſion of the ordinate above, then $\frac{3\sqrt{3}}{4} \times a$ will be the greateſt ordinate in all caſes; but in this caſe $=$ 12·9903 inches $= bp$. *Q.E.F.*

We cannot help admiring this gentleman's art and ſagacity, in performing what Sir Iſaac Newton muſt have commended. And as we know him to be acquainted with moſt of Sir Iſaac Newton's excellencies, great things are promiſed from his genius in the ſciences. Mr. *Farrer* ſent us a fluxionary ſolution to the ſame, with the true numbers; but as the general fluent to the fluxion was not found, we could not ſee the uſe of it.

VIII. QUESTION 302 *anſwered by Mr.* Heath.



Let all the rope be wrapt round the rails of the pond *vopqtv*, and the horſe begin to move, or unwrap his rope from *v* (where it is fixed) in the track of *vnm FGHIFv*; then the ſpace *FGHIF*, deducting the pond's area, is the greateſt he can feed. When his rope is completely unwrapt at *G*, he deſcribes the ſemicircle *GHI*, with rad. *vG* = 160 yards, the area of which = 40212·48 yards ſquare. Then he begins to wind his rope round the rails the contrary way to the former winding; deſcribing the track *IFv*, ſimilar and equal to the track *vnmFG*, at the unwinding of his rope. *To find the area he deſcribes, at unwinding, or winding?* Put r = pond's rad. = 35·464731 yards; z = any arch of the rails unwound, ſuppoſe $vo = on$, and $\dot z$ its fluxion = op. Now, little ſector rop is the fluxion or next increment of circular ſpace rvo, as little ſector pnm (with radii of curvature pn, pm) is the next increment of the ſpace von; both ſectors, when infinitely little, being ſimilar, ſay, r (ro) : $\dot z$ (op) :: z (on) : $\frac{z\dot z}{r} = nm$, the fluxion of the involute arch vn, whoſe fluent is $\frac{z^2}{2r}$: but ſector $pnm = \frac{z^2\dot z}{2r}$, the flux. ſpace von, whoſe fluent is $\frac{z^3}{6r}$ = (when $z = 160$) to 26808·317 yards ſquare, being area ſpace *vnmFGvtqpov*, when the rope is quite unwound. Alſo *Ft* = *tqpov* = 114·42 yards, beſt found by trial and a table of logarithms (ſeries and the reverſion of ſeries being leſs certain and expeditious); whence area *vnmFtqpov* = 9804·2 ſquare yards; whence area *GvtFG* = 17004·117 by ſubſtitution, to which adding area *tqF* = 1018·58 gives 18022·69 = area *GvtqFG*, which doubled and added to ſemicircle *GHI* make 76257·86 ſquare yards = 15a. 2r. 12p. And length of the whole track = 1507·96 yards. N. B. *vnmFG* = GHI = 502·656. Q.E.F.

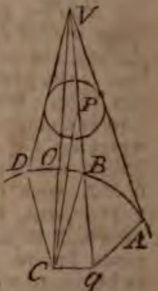
Mr. *Landen* ſent us an exact ſolution by another method; which are all the ſolutions given.

IX. QUESTION 303 *anſwered by the Rev. Mr.* Baker.

1. In $\triangle CBq$ there is given $\angle CBq = 24°$, and $CB =$ 1 ſemi-diameter; whence (by trigon.) $Bq = {}^{\cdot}913545$, and $Cq = {}^{\cdot}406736$.

2. In $\triangle PCq$, $PC = 64$, and $C\mathcal{Q} =$ as above; whence $Pq = 63{}^{\cdot}9987$ and $Vq = 122{}^{\cdot}4987$. Now $\angle qVA = 16' \; 23'' \mp$ $(\angle CVq =)$ $11' \; 27''$ give the $\angle CVD = 4' \; 56''$, and $\angle CVA = 27' \; 50''$.

3. In $\triangle$s CVD and CVA, $CV = 122{}^{\cdot}4993$, $CD = CB$, and $\angle$s CVD and CVA as above; thence $\angle DCV = 9° \; 59'$ fere, and $\angle VCA = 81° \; 12'$, which added $= 91° \; 11' =$ arch $Do +$ arch oA, the quantity of the earth's ſurface included in the penumbral ſhadow. *Q. E. F.*



Mr. *J. Turner* has likewiſe ſent a ſolution; being the only ones we received.

X. QUESTION 304 *anſwered by Mr.* Landen.

This gentleman puts zy for the 1ſt number, zx for the 2d, and zv for the 3d, p^2z for the ſum of the three; zs for the ſide of the 1ſt cube, zr for the ſide of the 2d, and zq the ſide of the 3d, and ſo proceeds to equations; from whence, in a curious manner, he determines the 3 cube numbers $\frac{341}{4913}$, $\frac{854}{4913}$, and $\frac{350}{4913}$. But we muſt reſerve his method for more room than we have at preſent.

Mr. *Farrer* informs us, that this queſtion was taken from Branker's Algeb. where the ſolution may be ſeen at large; and ſays that one ſet of numbers is $\frac{494424}{2352637}$, $\frac{242696}{2352637}$, $\frac{448000}{2352637}$. Mr. *Hampſon*'s numbers are $\frac{854}{4913}$, $\frac{341}{4913}$, $\frac{250}{4913}$, beſides other of his numbers ſent.

XI. QUESTION 305 *anſwered by Mr.* Landen.

Put $4x$ for one of the amiable numbers, and $4yz$ for the other, x, y, and z being primes; we ſhall have $7+3x=4yz$, or $x=\frac{4yz-7}{3}$, and $4x=7+7y+7z+3yz=\frac{16yz-28}{3}$; hence $z=3+\frac{16}{y-3}$. Now taking $y=5$, z will be $=11$, and $x=71$, $4x=284$, and $4yz=220$, the firſt and leaſt amiable pair.

This gentleman proceeds in a new method of ſubſtitution (ſhewing the defect of the preſent method) and finds 18416 and 17296 the next amiable pair, and 9437056 and 9363584 for a third amiable pair. And then gives this general rule: If 2^n (n being an affirmative integer) be taken ſuch, that $3\times2^n-1$, $6\times2^n-1$, and $18\times2^{2n}-1$ be primes, then $2^{n+1}\times18\times2^{2n}-1$ ſhall be an amiable number, and $2^{n+1}\times3\times2^n-1\times6\times2^n-1$ its amiable correſpondent or partner.

N. B. Mr. Stone's theorem, in his Mathematical Dictionary, for finding amiable numbers, is erroneous. The above true rule is ſaid by Mr. Landen to be firſt ſhewn by the famous Deſcartes.

The ſame ingenious gentleman proceeds to the ſolution of the 2d part of the queſtion, putting $4x^n=$ the numb. ſought, x being a prime, and n an affirmative integer.

$7+7x+7x^2+\ldots.7x^{n-1}+3x^n$ is the ſum of its aliquot parts, which muſt be $=4x^n+7$. Therefore $x^n-7x-7x^2-\ldots7x^{n-1}=\frac{x^{n+1}-8x^n+7x}{x-1}=0$. Whence $x^n-8x^{n-1}+7=0$. Herein, if $n=2$, x will be $=7$. Whence $4\times7^2=196$ the leaſt whole number, whoſe aliquot parts ſummed up ſhall exceed it by 7.

XII. QUESTION 306 *anſwered by Mr.* Heath *only*.

Inſtead of each proportional number raiſed to the power expreſſed by the natural logarithm of the other, it ſhould have been expreſſed *root extracted*, or each proportional number raiſed to a power expreſſed alſo by the logarithm root of the other. Then putting x and z for the numbers, $x^{\frac{1}{\mathrm{l}.z}} = x + z$, and $z^{\frac{1}{\mathrm{l}.x}} = x - z$; which reduce to $x = \overline{x+z}|^{\mathrm{l}.z}$ and $z = \overline{x-z}|^{\mathrm{l}.x}$; and if $b = 2{\cdot}3025$ &c. then $x = \overline{x+z}|^{\mathrm{l}.z \times b}$, and $z = \overline{x-z}|^{\mathrm{l}.x \times b}$; whence, by the table of common logarithms (performing beyond the art of ſeries) $x = 3{\cdot}8761$, and $z = 2{\cdot}1292$ fere. And hence the odds of proportion of payment are as 1·8204 to 1. As the queſtion was expreſſed $x^{\mathrm{l}.z} = x + z$, and $z^{\mathrm{l}.x} = x - z$, where, if $z = 0$, the Glouceſter's ſhare of prize money, in the firſt equation, then x, in the ſame equation has an impoſſible value for the Centurion's lot. *Q. E. F.*

We thank Mr. *Farrer* for his obſervation on this queſtion; truth being always welcome to us.

XIII. QUEST. 307 *anſwered by Mr.* Landen *the Propoſer.*

When the velocity of the ball, in a direction parallel to the horizon, is the greateſt it can acquire by ſuch a deſcent, the ball will fly off, and deſcribe a portion of a parabola, to which the plane, at that inſtant of time, will be a tangent. *To find the ſaid Velocity.* Put a = angular velocity of the plane per ſecond, meaſured by the arch of a circle, whoſe rad. $= 1$; $A = 32\frac{1}{6}$ feet, the abſolute force of gravity, computed by the velocity generated in a ſecond; x = ſine of angle of inclination; rad. being $= 1$. The force of gravity to accelerate the motion of the ball along the plane $= Ax$; the abſolute gravity being to the relative gravity of the deſcending ball, as rad. to ſine of the angle of inclination. The fluxion of the velocity along the plane $= \frac{A}{a} + \frac{x\dot{x}}{\sqrt{1-xx}}$, i. e. the accelerating force drawn into the fluxion of the time. The fluent of which corrected is $\frac{A}{a} \times \overline{1 - \sqrt{1-xx}} = \frac{A}{a} \times$ verſed ſine of the plane's inclination, for the velocity itſelf: But this velocity is to the velocity in a horizontal direction as rad. to coſ. angle of inclination. Therefore $1 : \sqrt{1-xx} :: \frac{A}{a} \times \overline{1 - \sqrt{1-xx}} : \frac{A}{a} \times \overline{\sqrt{1-xx} - \overline{1-xx}}$, the velocity of the ball in the ſaid direction; the fluxion of which being made $=0$, and reduced $x=\sqrt{\frac{3}{4}}$, the ſine of the angle of inclination, when the ball quits the plane. *To find the length then deſcended by the ball along the plane.* Multiply the velocity $\frac{A}{a}\times\overline{1-\sqrt{1-xx}}$ by $\frac{\dot{x}}{a\sqrt{1-xx}}$ the fluxion of time, and we have $\frac{A}{aa}\times\overline{\overline{1-xx}|^{-\frac{1}{2}}-1}\times\dot{x}$ the fluxion of the ſaid length, whoſe fluent is $\frac{A}{aa}\times$ exceſs of the arch of a circle (rad. being unity) above its ſine x; which when $x=\sqrt{\frac{3}{4}}$, will be the length required.

COROLLARY 1. The velocity of the ball along the plane, at its becoming inclined in any given angle, is directly as the time it has been in motion, or reciprocally as the angular velocity of the plane.

COROLLARY 2. The length deſcended by the ball along the plane, at its becoming inclined in any given angle, will be directly as the ſquare of the time it has been in motion, or reciprocally as the ſquare of the angular velocity of the plane.

COROLLARY 3. The ball will quit the plane when its angle of inclination becomes equal to 60°, let the uniform angular velocity and the force of gravity be what they will.*

XIV. QUESTION 308 *anſwered by Mr.* James Terey.

All the numbers whoſe ſquares equal 326, are eaſily determined. $\left\{\begin{array}{ccc}18&1&1\\17&6&1\\15&10&1\\14&11&3\\14&9&7\\13&11&6\end{array}\right\}$ Putting x, y, and z for the number of geeſe, ducks, and chicken, and v the pence paid for the chicken; then, among ſome of thoſe 3 numbers (varied for geeſe, ducks, and chicken) $4vx+2vy+vz=400$; whence $v=\frac{400}{4x+2y+z}$ a whole number; which only admits of $\left\{\begin{array}{l}\text{13 geeſe}\\ \text{11 ducks}\\ \text{6 chicken}\end{array}\right\}$ price of each $\left\{\begin{array}{r}\text{1s. 8d.}\\ \text{10}\\ \text{5}\end{array}\right\}$. And $\left.\begin{array}{l}\text{9 geeſe}\\ \text{7 ducks}\\ \text{14 chicken}\end{array}\right\}$ price of each $\left\{\begin{array}{l}\text{2s. 1d.}\\ \text{1 } 0\tfrac{1}{2}\\ \text{ } 6\tfrac{1}{4}\end{array}\right\}$. Amounting each way to 1l. 13s. 4d.

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* Mr. *Landen* remarks that this queſtion is not right, but that a true ſolution may be made out from *Simpſon's* Fluxions.——He farther obſerves that his ſolution to the prize queſtion Diary 1747 is not entirely right. The fluent for the time of deſcending along the mountain's ſide would be infinite if properly corrected. The ſtone ſhould be laid at ſome diſtance from the top of the mountain, otherwiſe it will not move.

Mr. *J. Turner*, Mr. *Collingridge*, Mr. *Sam. Atkin*, Mr. *Flitcon*, and others, ſolved the ſame; but only Mr. *Terey* and Mr. *Hampſon* in the variety.

XV. QUESTION 309 *anſwered by the Propoſer.*

Raiſed to a power, &c. ſhould have been expreſſed *root extracted, &c.* Then, if v and y repreſent the reſpective chances of each army for victory, $\overline{v+y}|^{\frac{1}{v-y}} = v$, and $\overline{v-y}|^{\frac{1}{v+y}} = y$; whence $v+y = v^{v-y}$, and $v-y = y^{v+y}$; where $v = {}^{\cdot}5806$ and $y = {}^{\cdot}2590$ very correctly, as may be proved by a table of logarithms; which numbers were not a little curious to determine. Hence the odds of battle are as 2·2417 to 1.

As moſt perſons of ſcience are but little converſant with theſe ſort of equations, it may not be improper here to unfold the myſtery of raiſing powers of all ſorts. And 1. A decimal raiſed to a decimal power produces a greater value than the root. 2. A decimal raiſed to an integral power produces a leſs value than the root. 3. An integer raiſed to a decimal power produces a leſs value than the root.

$x^0 = 0^0 = 1$; all very ſmall powers of quantities approaching the value of unity. x^x is leaſt $= {}^{\cdot}6922$ correctly, when $x = {}^{\cdot}3678798\ \&c. = \frac{1}{2{\cdot}30258509\ \&c.}$. The logarithm of a decimal is negative, or ſo much leſs than nothing.

To determine the value of the unknown quantities in all exponential equations, it is convenient to ſuppoſe the leaſt quantity unknown $= 0$, and thence to find the value or values of the next greater, noting the error in the next equations. Again, ſuppoſe the value of the leaſt unknown quantity $= 1$, and thence find the value or values of the next greater; noting the quality of the error, in the next equations, as before; and ſo on to 10, 20, &c. for the leaſt value, if need requires. Or ſuppoſing the value of the greateſt unknown quantity to be 1, 10, 100, &c. determining the value of the next leſs unknown quantity to each ſuppoſition; at the ſame time always denoting the quality of errors, by which the true values of the unknown quantities are determined by a table of logarithms very exactly.

N. B. It is often very eaſy and convenient to ſuppoſe the value of one unknown quantity in two equations, in ſuch a manner as that by it the other may be determined in a whole number. *Hurlothundro.*

XVI. QUESTION 310 *anſwered by* Nobody.

Mr. *Farrer* ſent us a ſolution which was not true. Mr. *Landen* ſent us a true method; but the calculus being ſo operoſe, it was not wrought out. And no method appearing to us yet elegant enough for a place, it will be next year before we ſhall have time to catch the ſolution to this famous ſpider and fly queſtion. The ratio of the motion of the inſects are little different from an equality; though a certain gentleman makes the motion of the chaſing inſect the ſloweſt, to overtake the fly.*

PRIZE QUESTION *anſwered by Mr.* Heath *the Propoſer.*

Anno 1748, June 18d. 10h. 17m. and Dec. 18d. 1h. 22m. by authentic tables the earth was in the aphelion and perihelion points of her orbit, or at the greateſt and leaſt diſt. à ⊙; her motion (by deſcribing equal areas in equal times round him) being at thoſe times of aphelion and perihelion, (later by 9 hours than according to Street's Tables) the sloweſt and quickeſt reſpectively.

By the equation table of the earth's orbit, her true diurnal motion round ⊙ at coming to aphelion is $57' \; 12''$ (earth retarding variouſly from aph. to perih.) and true diurnal motion coming to perihelion $61' \; 10''$ (earth accelerating variouſly from perih. to aph.) Now allowing $10''$ one-half ⊙'s mean parallax, or angle at ⊙, ſubtended by earth's ſemidiameter, then by trigon. 19644 earth's ſemi-diameters is her mean diſt. à ⊙; being then at the conjugate end of her orbit; which diſt. à ⊙ = length of the ſemi-tranſverſe.

Mr. Flamſteed's eccentricity of the earth's orbit, or ⊙'s focal diſt. à center of her orbit is 1692 ſuch parts as ſemi-tranſverſe or mean diſt. is 100000, conſequently 100000 : 1692 :: 19644 : 332·37 earth's ſemi-diameters, the ⊙'s true diſt. à center earth's orbit: whence 19976·37 and 19311·63 ſemi diameters are the earth's greateſt and leaſt diſt. à ⊙. And by her deſcribing equal elliptical ſectors round him each day, with the reſpective angles 57′ 12″ and 61′ 10″, as before obſerved, the correſpondent elliptical arches, which may be conſidered as circular for a day, will be 332·28 and 343·6 &c. ſemi-diameters which the earth's center goes over in 24 hours, when her motion is ſloweſt and faſteſt. Whence, by allowing the earth's ſemi-diam. = 3967 miles, her center goes over at faſteſt 56794 miles per hour, 946 per minute, and almoſt 16 miles per ſecond! an amazing ſwiftneſs! Alſo at her ſloweſt rate, in aphelion, 54923 miles per hour, 915 per minute, and 15 per ſecond.

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* A ſolution to this queſtion may be ſeen in the *Mathematician.*

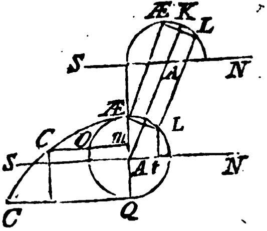
Anno 1748, June 9d. 16h. 34m. 46s. and Dec. 9d. 21h. 8m. equal time, ⊙ enters Cancer and Capricorn, the reſpective mean anomalies being then 11 S. 21° 23′ 15″ and 5 S. 21° 56′ 28″ (by Halley's and Flamſteed's numbers); whence the proportional diſtances of earth à ⊙ are 101675 and 98322 reſpectively, and thence the true diſtances 19973 and 19314 earth's ſemi-diameters (by multiplying 19644, a conſtant multiplier, and cutting off 5 fig.); and the earth's diurnal angular motion round ⊙ being 57′ 11″ and 61′ 9″ reſpectively (by tables of the earth's orbit) to the ſaid radii; conſequently the arches reſpectively moved over by the earth's center, when the days are longeſt and ſhorteſt, are 332·230 and 343·554 &c. ſemi diameters of the earth; nearly equal to her ſloweſt and faſteſt motions as above; ſhe being at thoſe times but a few days from the aphelion and perihelion points.

N. B. Anno 1748, March 19d. 2h. 41m. and Sept. 17d. 17h. 51m. the mean anomalies of the earth are reſpectively 9 and 3 degrees when her mean and true places differ the moſt, viz. 1° 56′ 20″; about which times ſhe neither accelerates nor retards for ſome days.

To find the diſtance travelled over by a ſpot on the earth's ſurface for a day.

The earth's radius bearing ſo ſmall a proportion to her diſt. à ⊙, ſhe may be conſidered as revolving forward in the direction of a ſtreight line, for a ſmall interval of time, with a progreſſive motion as p, to 1 rotatory, or over 332·23 and 343·554 ſemi-diameters reſpectively, on the days about the ſolſtices of Cancer and Capricorn; to each of which diſtances *gone forward* ſhe makes but one revolution.

Put



Put $a = ÆQ$, the diameter of a circle generating a cycloidal curve, with progreſſive motion p, and rotatory 1; $x = Æm$; then $om = \sqrt{ax - xx}$ per circle. And fluxion arch $Æo = \frac{a\dot{x}}{2\sqrt{ax-xx}}$; whence $1 : p :: \frac{a\dot{x}}{2\sqrt{ax-xx}} : \frac{pa\dot{x}}{2\sqrt{ax-xx}} =$ flux. Co; and flux. $om = \frac{a-2x}{2\sqrt{ax-xx}} \times \dot{x}$, whoſe ſum $\frac{ap+a-2x}{2\sqrt{ax-xx}} \times \dot{x} =$ fluxion Cm. But $\sqrt{\overline{\text{flux. } Cm}|^2 + \overline{\text{flux. } Æm}|^2} =$ fluxion arch $CÆ = \frac{\dot{x}}{4x} \times \frac{\sqrt{\overline{p+1}|^2 \times a^2 - 4pax}}{a-x}$; whoſe fluent is $\sqrt{ax} \times : \dot{p}+1 + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 3 \cdot p+1} \times \frac{x}{a} + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 5 \cdot p+1} - \frac{\overline{\overline{p+1}|^2 - 4p}|^2}{2 \cdot 4 \cdot 5 \cdot \overline{p+1}|^3} \times \frac{x^2}{a^2} + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 7 \cdot p+1} - \frac{\overline{\overline{p+1}|^2 - 4p}|^2}{2 \cdot 2 \cdot 7 \cdot \overline{p+1}|^3} + \frac{\overline{p+1}|^2 - 4p}{2 \cdot 2 \cdot 4 \cdot 7 \cdot \overline{p+1}|^5} \times \frac{x^3}{a^3}$ &c..

The poles of the earth S, N, moving nearly parallel, for a ſhort time, the earth's center A is carried with an oblique direction AK, in an angle of 23° 30′ with the poles of the equator $ÆQ$; and the city of London, on the ſurface, at L, in the parallel rotatory direction to $ÆQ$, on the ſurface; the diſtance betwixt the finiſhing of each revolution being $LL = AA = ÆÆ = kK$; each town on the earth's ſurface deſcribing cycloidal curves whoſe baſes are equal. Lt the diſt. of the city of London from the earth's axis $=$ ·622059 ſemi-diam. being nat. s. co lat. London 88° 28′ (vid. curve of the tack, D. 1747). Now, when $x = 2$, $Lt =$ 1·244118 earth's ſemi-diam. $= a$, $p = 85$, and $p = 87{\cdot}9$ fere (dividing 332·230 and 343·554 by 3·90852 $=$ leſſer circle's circumference whoſe diam. $= a = 2Lt$) the ſemi-cycloids $=$ 166·12 and 171·78 fere; whence 332·24 and 343·56 earth's ſemi-diam. $=$ diſtances travelled by the city of London in 24 hours, when the days are longeſt and ſhorteſt: conſequently it travels about 54916 and 56787 miles per hour, reſpectively, at thoſe times. *Q.E.I.*

N. B. This being but in conſequence of theory, thoſe who are more curious may rectify the cycloidal curve deſcribed by the progreſſive motion of a point on a revolving globe, proceeding uniformly, in a circular or elliptical direction, forward as *p*, with a rotatory motion as 1, at the ſame time: though a real acceleration or retardation in either, would perplex the motion ſo, as to render the ſolution of the track next to impoſſible.

The Prize of 10 *Diaries was won by Mr.* John Turner.

The Eclipſes calculated for 1749, *by Mr.* John Smith.

There will happen five eclipſes, three of the ſun, and two of the moon; but only one of each luminary will be viſible to the inhabitants of Great Britain.

1. The ſun eclipſed on ſaturday January 7th, paſt 7 at night, but inviſible, becauſe of the ſun being ſet.

2. The moon eclipſed on monday June 19th, at 9 h. 7 m. in the morning.

3. The ſun eclipſed on monday the 3d of July, at 3 h. afternoon.

4. The moon eclipſed on tueſday the 12th of December. Beginning 6 h. 48 m. Middle 7 h. 59 m. End 10 h. 22 m. Duration 2 h. 23 m. Digits eclipſed 5° 3′.

Calculated by		Beg.	Mid.	End	Dur.	Dig.
		h.m.	h.m.	h. m.	h.m.	° ′
Mr. T. Cowper,	Wellingborough	6 55	8 4	9 13		4 42
Mr. J. Brown	London	6 58	8 8	9 18	2 20	
	Witton-le-wear	6 52	8 2	9 12		4 45
Mr. W. Caile	Great Muſgrave	6 51	8 1	9 11	2 20	4 35
	Etherly	6 52	8 2	9 12		
Mr. A. Man	Rome	7 48	9 0	10 12		
	Liſbon	6 19	7 31	8 43	2 24	5 6
	Paris	7 6	8 18	9 30		5 6
Mr. Hawkins	London	7 0	8 7	9 14	2 14	4 36
	Hanover	7 40	8 47	9 52		
	Oxford	6 55	8 2	9 9		
	Rome	7 52	8 59	10 46		
Mr. J. Hampſon,	Leigh, Lancaſh.	6 39	7 50	9 1	2 22	4 50
Mr. Joſ. Walker,	Kettering	6 52	8 1	9 10	2 18	4 42
Mr. Farrer,	Sunderland	6 41	7 53	9 4	2 23	5 10

5. The ſun eclipſed on thurſday Dec. 28th, 9h. 11 m. morning. Beginning 8 h. 4 m. Middle 9h. 11 m. End 10h. 22 m. Duration 2 h. 18 m. Digits eclipſed 7° 8′.

Calculated by		Beg.		Mid.		End		Dur.		Dig.	
		h.	m.	h.	m.	h.	m.	h.	m.	°	′
Mr. T. Cowper,	Wellingborough	8	12	9	8	10	18	2	16	6	49
Mr. J. Brown,	Witton-le-wear	8	4	9	10	10	16	2	12	7	9
Mr. W. Caile,	Great Muſgrave	8	3	9	9	10	15				
	Etherly	8	4	9	10	10	16	2	12	7	9
Mr. A. Man,	Rome			10	9	11	29	2	20	7	39
	Liſbon	8	5								
Mr. A Hawkins,	London	8	5	9	12	10	22	2	17	7	9
Mr. J. Hampſon,	Leigh, Lancaſh.	8	1	9	6	10	16	2	5	7	1
Mr. Joſ. Walker,	Kettering	8	2	9	8	10	18	2	16	6	47
Mr. Farrer,	Sunderland	8	1	9	5	10	7	2	6	6	25

Mr. *Abraham Clark* ſent us the firſt three eclipſes right.

We cannot but be pleaſed with the ingenùity and pains of Mr. *Aaron Hawkins*, who has calculated the appearances of the ſolar eclipſes on Jan. 7th, in New Spain, Terra Firma, Jamaica, Cuba, and places adjacent; all the penumbra falling within the earth's diſk. And this gentleman has given the times of the appearances with reſpect to the meridian of London. He has alſo ſhewn that the ſolar eclipſe on July 3d, 27′ P. M. inviſible in England, will be ſeen in the more ſouthern parts of the globe, where all the penumbra will fall within the earth's diſk, and be central and annular. And at

long.

The Eclipſe of the moon on the 12th of December was obſerved (as below) at Mr. *Graham*'s in *Fleetſtreet*, by *John Bevis*, M. D. and Mr. *James Short*, F. R. S.

App. time			
6 h.	46 m.	36 s.	A ſenſible penumbra
6	50	56	Eclipſe began
9	12	38	Eclipſe ended
9	17	3	Penumbra gone.

The ſolar Eclipſe of the 28th of December was thus obſerved:

	Beginning			End			
At *Rome*, by Mr. *Chr. Maire*	8 h.	34 m.	35 s.	11 h.	11 m.	31 s.	
At the Obſervatory at *Berlin*, by M. *Griſchow*, jun. and M. *Kies*	8	59	19½	11	20	5¼	tr. t.
At *Berlin*, near the Obſervatory, by M. *Euler*	8	58	30	11	19	50	tr. t.

the middle of the eclipſe the ſun will be vertical at St. Anthony's river, on the weſtern coaſt of Barbary, lat. 21° 42′ north; will be ſeen very great at guinea; and ſet at St. Helena, and places adjacent. Alſo the ſolar eclipſe on Dec. 28, viſible, will appear eclipſed at his riſing in 14° 3′ S. lat. and long. 23° 9′ weſt, at Ethiopia. Centrally eclipſed as he riſes lat. 32° 10′ S. long. 50° 49′ weſt, at St Domingo. Centrally eclipſed in nonageſimal lat. 26° N. long. 14° 53′ E. at Barbary. Centrally eclipſed in the meridian lat. 28° 4′ N. long. 32° E. at the Red Sea. Sets centrally eclipſed lat. 54° 37′ S. long. 144° 25′ E. at the South Sea. Eclipſe ends at ſun-ſetting lat. 38° 10′ S. long. 109° 23′ E. at Holland Nova.

Our worthy correſpondent Mr. *J. Turner*, of Heath, near Wakefield, writes as follows: ' On monday, April 18th at midnight, as I was looking towards the north part of the heavens, I accidentally caſt my eye upon a comet, near the chain of Andromeda. Its ſplendour is not very great at preſent, yet the tail is perfectly diſtinct, ſtretching towards Lyra. The motion of it is very ſwift, amounting to near 4 degrees of a great circle in a day, and tending almoſt towards the north pole. It comes to the north part of the meridian about 9 at night, being then about 10° high. On thurſday the 21ſt, half an hour paſt 10 at night, the comet was in a right line with ε and β in Caſſiopeia, and with the pole ſtar and γ, in Cepheus's foot, or rather the line paſſed between γ and τ. Alſo with the bright ſtar of the ſwan's tail, and Caſſiopeia's head; by which its place may be exactly determined. Its aſcending node is in about 25° of Piſces: and the inclination of its orbit to the ecliptic about 52° 0′." This comet we alſo had the pleaſure to obſerve; and ſhould be glad to diſcover the certain paths and periods of comets; the ſubſtances, quantities, and qualities of which they are compoſed; as well as their proper deſigned uſes, and laws of continuation and ſupport: And the like of all the other celeſtials wandering in infinite ſpace.

Mr. *T. Cowper*, of Wellingborough, has communicated the following occultation of the ſcorpion's heart, by the moon, 1749.

		d.	h.	m.	s.	
Apparent time at Wellingborough	Immerſion, March	27	12	50	9	P.M.
	Middle of Occulation	28	1	24	32	Mane
	Viſible conjunction —	—	1	25	10	
	Emerſion — —	—	1	59	40	
	Duration — —	—	1	9	31	

New

New Queſtions.

I. QUESTION 311, *by Mr.* Landen, *near* Peterborough.

To find three ſuch numbers, that the ſum or difference of any two of them ſhall be a ſquare number?

II. QUESTION 312, *by the Rev. Mr.* Baker, *at* Stickney, Lincolnſhire.

A bowl, by its byas, deſcribes a ſpiral, expreſſed by z, whoſe equation is $\frac{2}{3}y^{\frac{5}{3}} - \frac{3}{2}z^{\frac{3}{2}} + y^{\frac{2}{3}}z^{\frac{3}{3}} = 0$. In what direction muſt the bowl ſet out to fall upon the jack, when the length of the caſt is 47 yards?

III. QUESTION 313, *by Mr.* Landen.

There are four remarkable high trees growing in a ſtrait hedge-row, the diſtance of the 1ſt and 2d is 60 yards, of the 2d and 3d 40 yards, and of the 3d and 4th 20 yards. Where muſt I ſtand to obſerve them, ſo that the three intervals may appear equal?

IV. QUESTION 314, *by the Rev. Mr.* Baker.

A hare ſets out 50 yards before a grey hound, at the rate of 31 yards per ſecond, and continues a ſtrait courſe in the ſubquadruplicate inverſe ratio of the time taken up in running: The dog ſets forward only at the rate of 26 yards per ſecond, and maintained his pace in the ſubquintuplicate inverſe ratio of his time ſpent in running: How far had the dog run when his ſpeed was equal to that of the hare's? Alſo when he was again as near to the hare as at firſt? And laſtly, when he killed her?

V. QUESTION 315, *by Mr.* Philip Stevens, *of* Briſtol.

If the diurnal rotation of the earth was ſtopt from the 10th of December, 1748, at midnight, to the 10th of December, 1749, what time of the year would it be day-break, ſun-riſe, and mid-day, at London?

VI. QUESTION 316, *by Mr.* John Hampſon, *of* Leigh, Lancaſhire.

At Bedford mill, near old Leigh town, is found,
In form triangular, a piece of ground,
Whoſe ſides and area none can yet explain,
Tho' theſe ſubſequent hints may them obtain.

One angle makes degrees juſt ſeventy-nine,
Which being, as three to ten, cut by a line,
Of chains eleven, drawn to its ſide oppos'd,
The area is the leaſt can be inclos'd.
The miller thus—'Who beſt explains the truth,
'Wins for reward our buxom daughter Ruth.'

VII. QUESTION 317, *by Mr.* James Collingridge.

A marble table of ſix equal hexagon ſides, each $1\frac{1}{2}$ feet length, and the table $1\frac{1}{3}$ inch thickneſs, is levelly ſuſpended by a point on the under ſurface, and there hangs 7, 11, 15, 19, 23, and 27 pound weights, in a ſucceſſive order, from each corner of the table. Quere the point of ſuſpenſion?

VIII. QUESTION 318, *by Mr.* James Terey.

Two roads, perpendicular to each other, iſſue from the extremity of a ſemi-elliptical encloſure, next to a gentleman's ſeat; one road runs along the tranſverſe cloſe by the ſide of the ſtrait paling, ſome diſtance beyond the other end of the encloſure, and the other road proceeds forward next the crooked paling: A ſtrait diagonal viſto of 20 yards breadth is to be made, ſo as to communicate between the two roads, with one of its fences touching the curved fence of the elliptic encloſure; how muſt the viſto fences be drawn, and of what dimenſions, ſo that the viſto may take up the leaſt quantity of ground poſſible; the tranſverſe axis of the elliptic encloſure being 100, and its ſemi-conjugate 40 poles?

IX. QUESTION 319, *by* Harmonicus.

There are 13 muſical cords of equal thickneſs and tenſion, each an inch longer than the other, from the ſhorteſt of 12, to the longeſt of 24 inches; the tone of the longeſt cord to the ſhorteſt is as 2 to 1. Quere the proportion of the tones of all the intermediate cords?

X. QUESTION 320, *by Mr.* Heath.

At what time (next enſuing) will Mars and Venus, Sol and Terra, be conjunctly in a right line?

XI. QUESTION 321, *by Mr.* Farrer.

A muſket ball being ſhot $3\frac{1}{2}$ furlongs perpendicularly upwards, at what diſtance in its return will the force of the ball be equal to the weight of 9 pounds? And what will be its force at coming to the earth's ſurface, ſuppoſing the ball *to weigh* an ounce when at reſt?

XII. QUESTION 322, *by Mr.* Landen.

A cylindric pillar of ſtone, of 2 yards in circumference, being drawn up 20 yards above ground at a building, a rope being at the ſame time wrapt ten times round its convexity, with its lower end fixed to a hole upon the middle part where it begun to wrap; but before the pillar could be lodged upon the ſcaffolding, as it was drawn up with the rope's upper part paſſing through pulleys above, the rope unfixed its ſecurity at the tenth round, and the ſuſpended pillar by its weight then unwrapped itſelf of the rope, and deſcended to the ground. How long was the time of its deſcent?

XIII. QUESTION 323, *by Mr.* John Corbet, *Surveyor.*

How many acres of the moon's ſurface are ſeen enlightened 10 days after her conjunction with the ſun? And how many acres are contained on the convex ſuperficies of a lunar mountain (part of a gentleman's eſtate) its height being 3 furlongs, and its ſuperficies equal to that by the rotation of the ſemi-cycloid of that height about its axis?

PRIZE QUESTION, *by the Excellent Mr.* J. Landen.

An eagle (200 yards above) ſtooped to a kite, then taking flight from the ground with a chicken, at an angle of 60°, the eagle ſoaring directly towards the kite, then flying from her, at an uniform rate of ſwiftneſs of 3 to 1, the ratio of both their uniform motions; when, after ſome time flying, the kite finding herſelf cloſely purſued, quits her little captive, which fell to the ground at the inſtant the bird of Jove ſeized her priſoner, who was then juſt as high from the ground as at her firſt ſtooping. The eagle's diſtance from the kite, at firſt ſetting out? Her neareſt approach to the ground during the purſuit? Her height above it, and diſtance from the kite, when the chicken begun to fall? And alſo the time of flight, are from hence required?

A PARADOX, *by Mr.* Landen.

Two ivory balls of five inches diameter, each being placed at the diſtance of two inches from one another, and both ſtruck by another ivory ball of the ſame ſize, in a perpendicular direction to their line of diſtance, with any given velocity, they will move ſwifter after the ſtroke, than if they had been placed cloſe together, or at any other diſtance.

Queſtions

1750.

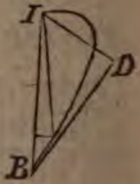
Queſtions anſwered.

I. QUESTION 311 *anſwered by Mr.* C. Bumpkin.

BY a method of ſubſtitution, too tedious to inſert, he finds 1873432, 2399057, 2288168, the three numbers, anſwering the conditions of the queſtion.

II. QUEST. 312 *anſwered by Mr.* W. Jepſon *of* Lincoln.

In the equation $\frac{2}{3}y^{\frac{5}{3}} - \frac{3}{2}z^{\frac{3}{2}} + y^{\frac{2}{3}}z^{\frac{3}{5}} = 0$, if we write v^5 and v^2 for $z^{\frac{3}{2}}$ and $z^{\frac{3}{5}}$ it will become $v^5 - 8{\cdot}68245v^2 = 272{\cdot}05$ when $y = 47$; whence, by converging ſeries, $v = 3{\cdot}252325$, and thence $z = 50{\cdot}9695$.



To the tangent *BD*, which is the line of direction, draw the perpendicular *ID*. Then, $\dot{z} : \dot{y} :: y : \frac{y\dot{y}}{\dot{z}} = BD$. And the above equation in fluxions is $\frac{10}{9}y^{\frac{2}{3}}\dot{y} - \frac{9}{4}z^{\frac{1}{2}}\dot{z} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{1}{3}}\dot{y} + \frac{3}{5}y^{\frac{2}{3}}z^{-\frac{2}{5}}\dot{z} = 0$, which multiplied by $y^{\frac{1}{3}}z^{\frac{2}{5}}$ gives $\dot{y} = \frac{\frac{9}{4}y^{\frac{1}{3}}z^{\frac{9}{10}} - \frac{3}{5}y}{\frac{10}{9}yz^{\frac{2}{5}} + \frac{2}{3}z} \times \dot{z}$, whence $BD = \frac{\frac{9}{4}y^{\frac{1}{3}}z^{\frac{9}{10}} - \frac{3}{5}y}{\frac{10}{9}yz^{\frac{2}{5}} + \frac{2}{3}z} \times y$. Now $BI (= y) : 1 :: BD : \frac{\frac{9}{4}y^{\frac{1}{3}}z^{\frac{9}{10}} - \frac{3}{5}y}{\frac{10}{9}yz^{\frac{2}{5}} + \frac{2}{3}z} = {\cdot}879268$ &c. the coſine of 28° 27′ (nearly) the angle of direction required.

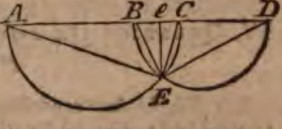
The ſame anſwered by Mr. J. Powle.

It is evident that the line of direction will be a tangent to *the curve* where the bowl is delivered, the poſition whereof, with a line drawn from thence to the jack is what is required? The equation of the curve in fluxions is, $\frac{10}{9}\dot{y}y^{\frac{2}{3}} - \frac{9}{4}\dot{z}z^{\frac{1}{2}} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{1}{3}}\dot{y} + \frac{3}{5}y^{\frac{2}{3}}z^{-\frac{2}{5}}\dot{z} = 0$, therefore $\frac{\dot{y}}{\dot{z}} = \frac{\frac{9}{4}z^{\frac{1}{2}} - \frac{3}{5}z^{-\frac{2}{5}}y^{\frac{2}{3}}}{\frac{10}{9}y^{\frac{2}{3}} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{1}{3}}}$, conſequently $\frac{\dot{y}y}{\dot{z}}$ the ſubtangent $BD = \frac{\frac{9}{4}z^{\frac{1}{2}} - \frac{3}{5}z^{-\frac{2}{5}}y^{\frac{2}{3}}}{\frac{10}{9}y^{-\frac{1}{3}} + \frac{2}{3}z^{\frac{3}{5}}y^{-\frac{4}{3}}}$. But y being given, z is known from the equation of the curve. Therefore $BD = 41\cdot47$ nearly. Then, by trigonometry, as $BI : BD :: \text{rad.} :$ coſine angle of direction, $IBD = 28° 8'$ nearly.*

Theſe are the only ſolutions that we received to this curious queſtion, except one from the propoſer.

III. QUEST. 313 *anſwered by Mr.* Terey *of* Portſmouth.

Let A, B, C, D, repreſent the four trees, and E the ſtation ſought of the obſerver.



$AB = 60$ yards $= a$, $BC = 20 = b$, $CD = 40 = c$. *Geometrically*. In this particular caſe, draw two ſemicircles AEC and BED, on the diameters AC and BD, and the point of interſection E will be the obſerver's ſtation: For then $AB : BC :: AD : CD$; agreeing with the data; and $AB : BC :: AE : EC$; alſo $CD : BC :: ED : EB$, 3 Euc. 6. (vid. Univerſal Arith.) Letting fall the perpendicular $Ee = y$, and putting $x = eC$, per property of the circle $\frac{2cb}{c-b}x - xx = \frac{2ab}{a-b} \times \overline{b-x} - \overline{b-x}|^2$, whence $x =$

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* *Mr.* LANDEN (in the Diary for the year following) ſays, that Mr. *Jepſon* and Mr. *Powle* have both abſurdly conſidered the bowl running backwards to the point where the ſpiral begins, and calls their ſolutions erroneous; though the propoſer, Mr. *Baker*, meant that it ſhould do ſo, and ſolved the queſtion himſelf the ſame way. Mr. *Landen* thinks it too eaſy a queſtion, in the caſe of drawing a tangent to a curve, whoſe equation is given; and therefore, correcting the fault, propoſes it ſhould be ſolved by drawing a tangent to the ſpiral, at the point where it begins, which will make it the more hard: and ſays it is a caſe not taken notice of by authors. He has gone through part of the ſolution this way himſelf.

$x = \frac{1}{2}b \pm \frac{c-a}{2ac-bb} \times bb = 12$ or 8 yards; whence $y =$ 24 yards, and each of the angles of interval 45°.

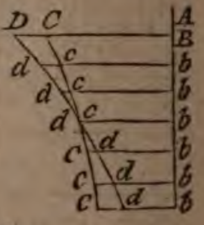
G. Bumpkin's ſolution to this queſtion was of the like nature.

General Solution by Mr. Ch. Smith.

Let E be the place of the obſerver; A, B, C, D, the places of the trees; put $y =$ the perpendicular Ee, $a = 20 = BC$, $x = eC$, then $4a - x = Ae$, $a - x = Be$, and $2a + x = eD$; whence the tangents of the angles AEB, BEC, and CED are $\frac{3ay}{yy+4aa-5ax+xx}$, $\frac{ay}{yy-ax+xx}$, and $\frac{2ay}{yy+2ax+xx}$, which muſt be all equal by the queſtion. Whence $yy = 4ax - xx = 2aa - ax - xx$, $x = \frac{2}{5}a = Ce = 8$ yards, $Ee = 24$, and each angle $= 45°$. *Q.E.F.*

Mr. *John Turner* ſolved this queſtion in an elegant and general manner; ſo did Mr. *Garrard*, the Rev. Mr. *Baker*, Mr. *William Spicer*, Mr. *Eneſer*, and ſome others.

IV. QUESTION 314 *anſwered by the Rev. Mr.* Baker, *propoſer.*

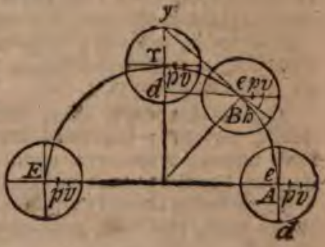


Let Bb, bb &c. repreſent equal portions of time, indefinitely ſmall; BC, bc &c. the celerities of the dog; BD, bd &c. the celerities of the hare, at each of thoſe times. Then, by mechanics, the areas $BCcb$ and $BDdb$, will denote the reſpective diſtances run by the dog and hare, in any given time. Putting $d = 50$, $AB = 1$, $BC = 26$ yards $= a$, $BD = 31 = b$, $Bb = x$, $bc = y$, and $bd = v$, we have, by the queſtion, $\overline{1+x}|^{\frac{1}{5}} : 1^{\frac{1}{5}} :: a : bc = y = \frac{a}{\overline{1+x}|^{\frac{1}{5}}}$, $\because \dot{x}y = a\dot{x} \times \overline{1+x}|^{-\frac{1}{5}} =$ fluxion of the ſpace $BCcb$, whoſe corrected fluent is $\frac{5a}{4} \times \overline{1+x}|^{\frac{4}{5}} - \frac{5a}{4} =$ diſt. run by the greyhound in the time x. After the ſame manner we get $v = \frac{b}{\overline{1+x}|^{\frac{1}{4}}}$, and $\frac{4b}{3} \times \overline{1+x}|^{\frac{3}{4}} - \frac{4b}{3} =$ *the ſpace* $BDdb$, run by the hare in the ſame time. Hence, by making $\frac{a}{\overline{1+x}|^{\frac{1}{5}}} = \frac{b}{\overline{1+x}|^{\frac{1}{4}}}$, we have $x = \overline{\frac{b}{a}}|^{20} - 1 =$ 32·7107 ſeconds, and 509·623 yards, the diſtance run by the dog, when his pace equalled the hare's. Again, making $\frac{5a}{4} \times \overline{1+x}|^{\frac{4}{5}} - \frac{5a}{4} = \frac{4b}{3} \times \overline{1+x}|^{\frac{3}{4}} - \frac{4b}{3}$, we have $x =$ 106·8167 ſeconds, and 1341·577 yards run by the greyhound, when he had regained his loſt ground. Laſtly, making $\frac{5a}{4} \times \overline{1+x}|^{\frac{4}{5}} - \frac{5a}{4} = \frac{4b}{3} \times \overline{1+x}|^{\frac{3}{4}} - \frac{4b}{3} + d$, we have $x =$ 181·3635 ſeconds, and conſequently 2059·846 yards = diſtance run by the dog when the hare died.

We have received no other anſwer to this queſtion, nor was the application of the inverſe ratio, as expreſſed, clearly underſtood by any. One of the greateſt mathematicians of the age, and a fluxioniſt, has aſſerted it unintelligible, as he does of the exhalation queſtion, this year inſerted.*

V. QUESTION 315 *anſwered by Mr.* W. Sutton.

Let e, e, ♈, E repreſent the pole of the earth's ecliptic, in the reſpective poſitions of the earth, when the required appearances happen, as the earth moves along in the plane of the ecliptic, from A to B, ♈, and E about ⊙, the ſun's center. Let A be the place of the earth on Dec. 10, 1748, at midnight, p the pole of the equator, and v the vertex of London, in the obſcure hemiſphere of the diſk, at which time the vertex of London is diſtant 61° 57′ from the horizon of the diſk (noted b, e, d) = ſum of the co-latitude and ſun's greateſt declination.



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* *Mr.* LANDEN corrects this queſtion (in the year following) by adding *ſpeed*, for rate of going forward; which ſpeed, at the end of the firſt ſecond, according to his Commentary, was 31 yards, &c. ſolving this queſtion throughout, except giving the numbers; and ſays, the propoſer's ſolution would have agreed exactly the ſame with his, had he brought the ſpaces paſſed over in the firſt ſecond

2. Conceive the earth with its axis, ſtill keeping its parallelism, carried to B, where the diſtance of the vertex and horizon is 18° $= vh$, which ſides vh, ev, and eh, conſtitute a right-angled ſpherical triangle, right-angled at h, in which is given $vh = 18° \, 0'$, and $ev = 61° \, 57'$, to find the $\angle veh$, $= yed = d \odot e$, the coſ. of the diff. of long. from Dec. 10, $= 69° \, 31'$, which added to the ſun's longitude on Dec. 10, gives the longitude for the day 11 s. 10° 8′ when 'tis day-break firſt at London, according to the queſtion, anſwering to Feb. 17.

3. 'Tis evident, that when the earth comes to the right angle in the ecliptic, $A \odot$ ♈ or 90°, from its place Dec. 10, at ♈, the vertex of London at v will firſt arrive in the horizon of the diſk, where the ſun will firſt appear to riſe: Therefore the longitude anſwering that appearance 0s. 0° 37′ anſwering with March 10.

4. When the ſun comes to the oppoſite point of the ecliptic, at E, or 180° from its long. Dec. 10, the vertex of London will tranſit the meridian at v, whence the ſun's longitude then is 3s. 0° 37′, to which agrees June 11, 1749.

Mr. *J. Powle*, drawing a ſcheme, ſays, that ſince on Dec. 10. the ſun enters Capricorn, and that ſign being on the meridian at midnight, it is evident, on the earth's rotation being ſtopt, that when the ſun is depreſſed below the horizon 18° in his progreſs through the ecliptic, day will break.

In Aries he will riſe; in Cancer it will be mid-day, *i. e.* ſun-riſing and Mid-day are on the 10th of March and 10th of June reſpectively.

To find day-break. Say, ſine ſum complement lat. and declin. 61° 58′ : ſine ſun's depreſſion 18° :: radius : ſine ſun's diſtance from vern. equinox 20° 29′, anſwering to the 20th of April, the time of day-break required.

Theſe being the only anſwers received, we thought fit to inſert both, that each gentleman may be convinced of the truth.*

VI. QUES-

ſecond into conſideration. He ſays, it will be as $1 : 31 :: \frac{1}{x^{\frac{1}{4}}} : \frac{31}{x^{\frac{1}{4}}}$ the hare's ſpeed; and $1 : 26 :: \frac{1}{x^{\frac{1}{5}}} : \frac{26}{x^{\frac{1}{5}}}$ the dog's ſpeed. And the ſpeed into $\dot{x}$, the fluxion of the time, will be equal to the fluxion of the diſtance run, &c.

* *Mr.* LANDEN (in the year following) ſays, Mr. *Powle*, in his ſolution to this queſtion, is right, by reckoning 20° 29′ from Aries into Piſces; but, by miſtake, makes day-break to follow ſun-riſe; *otherwiſe* his ſolution had been like Mr. *Sutton*'s.

*** $\odot$ is wanting at the center of the laſt figure.

VI. QUESTION 316 *anſwered by Mr.* W. Jepſon.

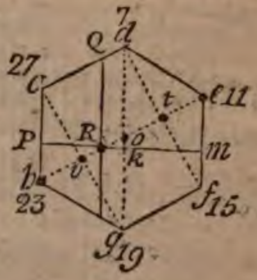
Let s. $\angle APC = s$, s. $\angle APB = n$, [*See the* △ *p.* 77] s. $\angle BPC = m$, $BP = a$, $AP = x$, $DC = y$; then, by a well-known theorem, $sxy = nax + may$, $\because sxy - may = nax$, and $y = \frac{nax}{sx - ma}$; whence $\frac{snaxx}{sx - ma}$ or $\frac{xx}{sx - ma}$ is a minimum by the queſtion; in fluxions $2x\dot{x} \times \overline{sx - ma} - sxx\dot{x} = 0$; $\because 2sx - 2ma - sx = 0$, or $sx = 2ma$, and $x = \frac{2ma}{s} = 19{\cdot}558$ &c. Hence $y = \frac{2na}{s} = 7{\cdot}012$ &c. and the area $= \frac{2mnaa}{s} = 6{\cdot}731$ acres = 6 a. 2 r. 36·96 p.

COROLLARY. The area APB = area $BPC = \frac{nmaa}{s}$, and $\because AB = BC$.

The Rev. Mr. *Baker*'s ſolution agrees with the above; as likewiſe does Mr. *John Turner*'s, Mr. *Charles Smith*'s, Mr. *Terey*'s, Mr. *Enſer*'s, Mr. *Spicer*'s, Mr. *Gibbon*'s, Mr. *Tho. Hare*'s, and others, which we omit inſerting to make room for other variety of ſubjects.

VII. QUESTION 317 *anſwered by Mr.* Turner, *of* Brumpton, Kent.

The ſolidity of the hexagon = 631·33 inches = 26·5 pounds $= w$; and let the weights be repreſented by 27, c; 7, d; 11 e; 15, f; 19, g; 23, b; whoſe ſum, with w, $= 128\frac{1}{2}$ pounds $= s$: Let R be the center of gravity, draw $QR \parallel cb$, and $Rm \perp QR$; put $2p$ = ſide hexagon $= cb$, $Pk = a = 15{\cdot}5884$, $PR = x$, $gk = y$. Then, by mechanics, $sx = \overline{d+g+w} \times a + \overline{e+f} \times 2a$, and $sy = \overline{b+f} \times p + w \times 2p + \overline{c+e} \times 3p + d \times 4p$. Theſe equations, reduced, are



x =

$$x = \frac{d+g+w+2e\,2f}{s}a = 12{\cdot}677 \text{ inches},$$

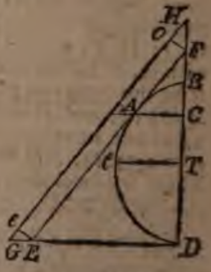
$$y = \frac{h+f+2w+3c+3e+4d}{s}p = 16{\cdot}319 \text{ inches}.$$

Whence the point R is determined by a general method, let the weights be what they will.

Anſwered by Mr. Heath.

With the given weights, the center of gravity R will fall on the axis eh at Ro diſt. from the table's center $= 3{\cdot}31618$ inches (which would be 4·23529, as anſwered by Mr. *Baker*, the weight of the table 26½lb. not being conſidered) for as the ſum of the weights ſuſpended on each ſide of the axis eh are equal, the center of gravity muſt needs fall on that line. The oppoſite weights 27 and 19 may be conſidered in one ſum $= 46$ pounds, ſuſpended from the point v; and the weights 7 and $15 = 22$ ſuſpended from t, and placing the weight of the table $= 26\frac{1}{2}$, in the middle at the point o, the queſtion will be reduced to find the center of gravity to five weights 23, 46, 26½, 22, and 11, placed at the equal diſt. of 9 inches on the line $he = 36$. Then ſubſtituting for the diſtance of the center of gravity from o, and multiplying the reſpective diſtances therefrom into the reſpective weights ſuſpended, making the products on each ſide the center of gravity, R, equal, the equation will give the value of oR as before.

VIII. QUESTION 318 *anſwered by the Rev. Mr.* Baker.



Seeing that on every variation of the viſto fence EF, the $\square\, eoFE$, increaſes or decreaſes, in magnitude, much faſter than the ſim. $\triangle$s GeE and oHF, therefore when the viſto $GHFE$ is a minimum, the fence EF will be alſo a minimum. Put $b = Tb = 50$, $c = Tt = 40$, $x = BC$; then $CA = \frac{c}{b}\sqrt{2ax-xx}$, $FC = \frac{2bx-xx}{b-x}$, $FD = \frac{2bb-bx}{b-x}$: Now by ſim. $\triangle$s $FC : CA :: FD : DE = \frac{c}{x}\sqrt{2bx-xx}$, and by 47 Euc. 1,

$\frac{cc}{x} \times \overline{2b-x} + bb \times \frac{\overline{2b-x}|^2}{\overline{b-x}|^2} = EF^2$, a minimum; or, by reduction, $\frac{2cc}{bx} + \frac{\overline{2b-x}|^2}{\overline{b-x}|^2}$ is a minimum; whoſe fluxion made $= 0$, and reduced, gives $x^3 + \frac{3cc - 2aa}{aa - cc}axx - \frac{3aacc}{aa-cc}x = \frac{-a^3cc}{aa-cc}$, where $x = 16{\cdot}785$ fere, whence $EF = 153{\cdot}702$ required.

Mr. *John Turner* ſolved this queſtion in the ſame elegant manner, as did Mr. *Terey* the propoſer: And therefore a certain gentleman is out in his calculation, who undertook to demonſtrate that this was not a queſtion *de maximis & minimis*, as he may perceive his miſtake by reviewing the breach of connexion in his fluxionary proceſs.

IX. QUESTION 319 *anſwered by Mr.* Turner, *of* Brumpton, *near* Rocheſter.

The longeſt cord to the ſhorteſt being as 2 to 1, which is as 24 to 12, conſequently the ſecond cord will be as 24 to 13, the third as 24 to 14 or 12 to 7, the fourth as 24 to 15 or 8 to 5, the fifth as 24 to 16 or 3 to 2, the ſixth as 24 to 17, the ſeventh as 24 to 18 or 4 to 3, the eighth as 24 to 19, the ninth as 24 to 20 or 6 to 5, the tenth as 24 to 21 or 8 to 7, the eleventh as 24 to 22 or 12 to 11, the 12th as 24 to 23, the thirteenth as 24 to 24 or 1 to 1. *Q.E.F.*

N. B. This queſtion, intending to ſhew the nature of harmonical proportion, was miſtakenly propoſed.

X. QUESTION 320 *anſwered by Mr.* T. Cowper.

The laſt mean oppoſition of the Sun, Mars, and Venus (in the ſuperior part of her orbit) was January 20, 1695, and I cannot find (by the proportion of the one conjunction of the ſun and another with thoſe planets) that they will be ſo conjoined again till the 11th of January 2942, though they happen very nearly in a right line about the 20th of October, anno 2006.

N. B. Thoſe who are more curious may calculate the time of their being conjoined, according to the true motions, as we expect to ſee performed by Mr. Gael Morris, whoſe numbers given us from the Royal Obſervatory, correct the places in Dr. Halley's tables, lately publiſhed by Mr. Innys, by exactly 7 min. leſs of ſun's anomaly, &c.

XI. QUESTION 321 *anſwered by Mr.* John Turner, *of* Heath, Yorkſhire.

Putting $b = 2310$ feet in $3\frac{1}{2}$ furlongs; $c = 16\frac{1}{12}$ feet, then $2\sqrt{bc} =$ the celerity of the muſquet ball when it falls to the ground, which being multiplied by its weight (viz. 1 ounce) produces 358 ounces, or 24 pounds, equal to its abſolute force.

2. Let $x =$ the time of its fall in ſeconds, when its abſolute force $= 9$ pounds, or 144 ounces. Say, $1'' \times 1'' : c :: x \times x : cxx$ the ſpace deſcended; whence $2cx =$ the celerity at that time, which multiplied by 1 ounce, is $144 = 2cx$; whence $x = \frac{144}{2c} = 4\frac{1}{2}$ ſeconds nearly, and the ſpace deſcended, when the ball's force $= 9$ pounds, is 322·321 feet.

The Rev. Mr. Baker's *Solution*.*

Putting $r = 21000000$ feet $=$ the earth's rad. $d = 2310$ feet $= 3\frac{1}{2}$ furlongs, $b = 9$ pounds, $c = \frac{1}{16}$ of a pound, the ball's weight at reſt, $s = 16\frac{1}{12}$ feet, $x =$ ſpace deſcended. The velocities of falling bodies being in the ſubduplicate ratio of the ſpaces deſcended through, we have $\sqrt{s} : 2s :: \sqrt{x} : 2\sqrt{sx} =$ the ball's velocity at x diſtance deſcended. By mechanics, and the condition of the queſtion, $2c\sqrt{sx} = b$, $\because x = \frac{bb}{4ccs} = 322{\cdot}321$ feet, exactly agreeing with the foregoing number by Mr. Turner. And when $x = d$, then $2c\sqrt{sd} = 24{\cdot}0937$ pounds, the ball's force coming to the earth's ſurface.

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* *Mr.* LANDEN (in the year following) ſays, Mr. *Turner* and Mr. *Baker* are both wrong in their ſolutions to this queſtion. This gentleman ſolves it, by aſſuming gratis, that 1100 grains, deſcending one-fourth of a foot, acquires a force $= 4660$ grains weight; and the reſt upon the ſame principles with others, ſuppoſing the force to be as the velocity and quantity of matter; and ſo it may be ſolved by as many different ſuppoſitions as any one pleaſes. But *Newtonienſis* ſays there can be no proper ſolution to this queſtion, for want of proper data, like the exhalation queſtion.

face. But accurately thus. From p. 369 of Mr. Emerſon's Flux. we get $2r\sqrt{\frac{sx}{aa-ax}}$ = the velocity, whence $2cr\sqrt{\frac{sx}{aa-ax}} = b$, and $x = \frac{aabb}{4sccrr+abb} = 322{\cdot}3873$ feet (where $a = d + r$). And when $x = d$, then $2c\sqrt{\frac{sdr}{a}} = 24{\cdot}0924$ pounds &c. *Q.E.F.*

Mr. *Powle* ſent his ſolution.

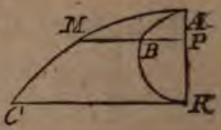
XII. QUESTION 322 *anſwered by Mr.* C. Bumpkin *only*.

If the ſtone were ſuſpended by two ropes hanging perpendicular, with one fixed at a point in the middle of its convex ſurface, and the other, at the center of percuſſion of that circular ſection in which the aforeſaid point is ſituated (which point in computing the place of the center of percuſſion is to be conſidered as the point of ſuſpenſion) and likewiſe the diam. in which the center of percuſſion is found, were parallel to the horizon, the tenſion of the rope fixed at the center of percuſſion would then be equal to the motive force bringing the ſtone down, if that rope was unfixed, and the ſtone left to deſcend in the manner deſcribed in the queſtion. Conſequently, putting W for the weight of the ſtone, a for the rad. thereof, then $\frac{3a}{2}$ will be the diſt. of the center of percuſſion from the center of ſuſpenſion; and $\frac{2W}{3}$ that motive force which would act continually upon the ſtone. Moreover, $W : 32{\cdot}2\ (=s) :: \frac{2W}{3} : \frac{2s}{3}$, the accelerating force, or velocity generated per ſecond, by the deſcending ſtone. Putting v for the velocity of the ſtone's center of gravity, and z the ſpace deſcended, then $\frac{\dot{z}}{v}$ being the fluxion of the time $\frac{2s}{3} \times \frac{\dot{z}}{v}$ will be $= \dot{v}$, or $\frac{2s}{3} \times \dot{z} = v\dot{v}$, whence $v = 2\sqrt{\frac{sz}{3}}$, and applying $\dot{z}$, we have $\frac{\dot{z}}{2}\sqrt{\frac{3}{sz}}$ = fluxion of the time, whoſe fluent $\sqrt{\frac{3z}{s}}$, when $z = 60$, is $= 2''\ 21'''\ 51''''$. *Q.E.F.*

XIII. QUES-

XIII. QUESTION 323 *anſwered by Mr.* Heath.

Say 14 d. 18 h. 22′ $1\frac{1}{2}''$ (the time of half a mean lunation) : $\frac{1}{2}$ (the moon's ſurface at moſt ſeen enlightened) : : 10 days (the time after conjunction with the ſun) : $\frac{10}{14{\cdot}765299} =$ ·3386319 &c. parts of the moon's ſurface then ſeen enlightened = 3245609437 acres; the moon's whole ſurface being 9584476353 acres.



Put $a =$ lunar mountain's height $= AR = 3$ furlongs, $x = AP$, any indefinite part of the cycloid's axis, then $BP = \sqrt{ax - xx}$, whoſe fluxion is $\frac{a - 2x}{2\sqrt{ax - xx}} \times \dot{x}$ and fluxion arch $AB = \text{flux. } MB = \frac{a\dot{x}}{2\sqrt{ax - xx}}$, whence flux. $\overline{MB + BP} =$ flux. $MP = \dot{y} = \dot{x}\sqrt{\frac{a - x}{x}}$, whoſe fluent $y = \sqrt{ax} \times : 2 - \frac{x}{3a} - \frac{x^2}{20a^2} - \frac{x^3}{56a^3} - \frac{5x^4}{576a^4} -$ &c. by ſeries; put $c = 2 \times 3{\cdot}1416$, then it being known that $\dot{x}\sqrt{\frac{a}{x}}$ is the flux. of the cycloid's arch, $cy\dot{x}\sqrt{\frac{a}{x}}$ will be the fluxion of a curved ſurface, by the rotation of the ſpace AMP about AP, whoſe fluent found is $2ax - \frac{x^2}{6} - \frac{x^3}{60a} - \frac{x^4}{224a^2} - \frac{5x^5}{2880a^3} -$ &c. $\times\, c =$ the curved ſuperficies; which when $x = a$, becomes $2aa - \frac{aa}{6} - \frac{aa}{60} - \frac{aa}{224} - \frac{5aa}{2880} -$ &c. $\times\, 6{\cdot}2832 = aa \times : 2 - \frac{1}{6} - \frac{1}{60} - \frac{1}{224} - \frac{5}{2880} -$ &c. $\times\, 6{\cdot}2832 = aa \times 11{\cdot}3617 = 1022{\cdot}553$ acres, or 1022 a. 2 r. 8·48 p. the quantity of the lunar mountain's ſurface required.

Solution by Curioſus.

Let $s =$ ſurface, $AP = x$, $PM = y$, $AM = z$, $AB = v$, $c = 2 \times 3{\cdot}1416$; then will $\dot{z} = \dot{x}\sqrt{\frac{a}{x}}$, and $\dot{s} = c \times PM \times \dot{x}\sqrt{\frac{a}{x}}$; but $PM = \sqrt{ax - xx} + v$, whence $\dot{s} = c\dot{x}\sqrt{aa - ax} + cv\dot{x}\sqrt{\frac{a}{x}}$. And $s = -\frac{2c\sqrt{a}}{3} \times \overline{a-x}|^{\frac{3}{2}} +$ flu. $cv\dot{x}\sqrt{\frac{a}{x}}$. But (by rule 8 p. 50 Emerſon's Fluxions) this fluent is $= 2cv\sqrt{ax} + 2ca\sqrt{aa - ax}$, becauſe $\dot{v} = \frac{a\dot{x}}{\sqrt{aa - ax}}$. Whence $s = -\frac{2c\sqrt{a}}{3} \times \overline{a-x}|^{\frac{3}{2}} + 2cv\sqrt{ax} + 2ca\sqrt{aa - ax}$, and corrected, $s = \frac{4ca + 2cx}{3}\sqrt{aa - ax} + 2cv\sqrt{ax} - \frac{4caa}{3}$; and when $x = a$, the whole ſurface $= \frac{3c - 8}{6}caa = 11{\cdot}36169aa$.

C. Bumpkin's ſolution, agreeing with the above, came too late to be inſerted; which are the only ſolutions that we received to this queſtion, except the propoſer's.

The PRIZE QUESTION *anſwered by his Excelleny* *Sir* Stately Stiff.

The equation of the curve deſcribed by the eagle, is $2x = \frac{a^{n}y^{1-n}}{1-n} - \frac{a^{-n}y^{n+1}}{1+n}$, computing x from the point where the purſuit ended, y being an ordinate at right angles, $n = \frac{1}{3}$, and a an invariable quantity to be determined. Moreover, if the variable diſtance of the eagle and kite be $= d$, and the diſtance of the kite from the point where x begins $= z$, and s and c be the ſine and coſine of 60°, we have, by the nature of the curve, $\frac{z}{n} - nx = d$, $xx - 2zx + zz + yy = dd$; and at the beginning of flight, when $z = 230{\cdot}9$ yards, $cy = sx$; at which time, as it is proved from theſe three laſt equations, $d = 582{\cdot}3$ yards, the eagle's diſtance from the kite at firſt ſetting out, $x = 331{\cdot}2$, $y = 573{\cdot}6$. Now by putting theſe values of x and y in the firſt equation, a is determined $= 967{\cdot}1$.

A

At the loweſt point of the curve, $s : y :: 1 : \frac{y}{s} = d$, and $s : y :: c : \frac{cy}{s} = x - z$; therefore $z = \frac{sx - cy}{s}$, by means of which, and the former equations, x and y are found correſponding to that point; and then, by a ſhort computation, the eagle's neareſt approach to the ground = 117·8. Her height above it when the chicken began to fall, is found = 119·7 (by means of the former equations, trigonometry, and ſolving a cubit equation) and the eagle's diſtance from the kite, at the ſame inſtant is found = 251·7 yards; and laſtly, the whole time of flight = 9·44 ſeconds.

N. B. The data ſhould be corrected by writing, *when after 5 ſeconds flying*, inſtead of, *when after ſome time flying.* The whole operation requires too much room to be inſerted at length. *Q.E.F.**

We wonder at ſome perſons for ſending criticiſms on the impropriety of this queſtion, who did not underſtand one ſtep of the proceſs in giving a ſolution: but like the author of the ſham doctrine of ultimators, and the Iriſh conjurer who raiſed the ghoſts of departed quantities, prove to be mere cyphers of mathematicians, whatever they may be in their own element.

The Prize of 12 Diaries was won by the above Anſwerer, and that of 8 by Mr. Baker.

The PARADOX *anſwered by* C. Bumpkin *only.*

Put m = maſs of each ball, a the velocity of the ſtriking ball before the ſtroke, w its velocity after the ſtroke, v the velocity of the balls impelled, and c = coſ. angle made by the path of the ſtriking ball with that of either of the impelled balls; then $mw + 2cmv$ will be the quantity of motion after impulſe $= ma$, the quantity of motion before the ſtroke given. Moreover, $maa = mww + 2mvv$, as is proved by Mr. Mac Laurin, in his Treatiſe of Fluxions, and alſo by others. By which equations (expunging w) we get $v = \frac{2ac}{1 + 2cc}$, and is a maximum when $c = \sqrt{\tfrac{1}{2}}$. From whence it appears that the balls muſt be laid about 2 inches aſunder, for the velocity, after the ſtroke of a third ball, to be the greateſt poſſible.

The

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* The above ſolution will be evident by reading prob. 15 *Simp. Flux. p.* 516.

The Eclipſes calculated for 1750, *by Mr*. John Smith.

There will happen five eclipſes, three of the ſun, and two of the moon, in the following order: The times and appearances according to the meridian of London.

1. Of the moon, on friday June 8th, at 3 minutes paſt 6 at night, viſible and total.[*]

2. Of the ſun, on friday June 22d, at 45 min. paſt 6 at night, inviſible.

3. Of the ſun, on ſunday the 18th of November, 56 min. in the morning, inviſible.

4. Of the moon, on ſunday Dec. 2d, at 31 min. paſt 6 in the morning, viſible and total.[†]

5. Of the ſun, on monday the 17th of December, at 17 min. paſt 6 at night, inviſible to any part of Europe.

The quantities of the viſible eclipſes are given by the above ingenious young artiſt as follows; which are very correct, as appears by the calculations ſent us by others.

A

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[*] The lunar Eclipſe of the 8th of June was obſerved in *Surry Street* in the *Strand, London*, by Mr. *John Catlin*, and Mr. *James Short*, F. R. S.

Emerſion, or end of total darkneſs	9h.	45m.	0s.
End of the eclipſe — —	10	51	30

[†] The total Eclipſe of the moon on the 2d of December, was obſerved in the *Strand, London*, (about 5″ of time W. of St. Paul's, and 27″ W. of *Greenwich* Obſervatory) by Dr. *Bevis* and Mr. *James Short*,

A ſenſible penumbra — —	4h.	32m.	0s.
The eclipſe judged to begin —	4	36	50
Total immerſion — — —	5	36	5
Emerſion — — — —	7	14	33

The end not obſerved.

Calculated by	A TABLE of the Moon eclipſed June 8th, at night, total.							Moon eclipſed Dec. 2d, morn. total.						
	Beg.	Beg. to.d.	Mid.	End to.d.	End	Dur.	Dig.	Beg.	Beg. to.d.	Mid.	End to.d.	End	Dur.	Dig.
	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	° ′	h. m.	h. m.	h. m.	h. m.	h. m.	h. m.	° ′
Mr. J. Smith, London	7 16	8 25	9 3	9 41	10 49	3 33	15 38	4 46	5 39	6 31	7 24	8 16	3 30	21
Mr. J. Smith, Coventry	7 10	8 19	8 57	9 35	10 43			4 40	5 33	6 25	7 18	8 10		
Mr. W. Sutton, Warwick	7 0	8 6	8 48	9 30	10 36	3 36	16 3	4 36	5 34	6 22	7 11	8 9	3 33	20 56
Moon riſes totally eclipſed 8h. 16m.														
Mr. Cowper, Wellingborough					10 41			4 32	5 32	6 20	7 8	8 7		20
Mr. Cowper, London								4 30	5 29	6 17	7 5	8 4		
Emerſon 9h. 34m.														
Mr. Ralph Hulſe, Ellworth-hall, Ch.	7 25		9 9	9 34	10 43	3 26	16 20	5 18		6 50		8 20	3 2	21 12
Mr. Al. Man, London	7 4	8 10	8 52	9 54	10 40	3 36	16 4	4 34	5 36	6 26	7 15	8 17	3 41	21
Mr. Al. Man, Carbonear, Newf.								0 51	1 53	2 43	3 32	4 34		
Mr. Al. Man, Boſton, New Eng.								11 49	0 51	1 41	2 30	3 32		
Mr. Al. Man, Paris	7 14	8 20	9 2		10 50									
Mr. Al. Man, Rome	7 56	9 2	9 14		11 32									
Mr. Joſ. Walker, Kettering	7 4	8 11	8 52	9 33	10 40	3 35	16	4 41	5 42	6 31	7 20	8 22	3 40	20 31
Mr. J. Hampſon, Leigh, Lancaſhire	7 11	8 18	9 0	9 42	10 49	3 38	16 21	5 9	6 6	6 55	7 44	8 41	3 34	21 15

With ſeveral other calculations ſent, of which theſe are the moſt correct.

New Queſtions.

I. QUEST. 325, *by Mr.* T. Cowper *of* Wellingborough

Near Albion's center, in Northamptonſhire,
Where bleating flocks on every ſide appear,
Stands [*] Wellingborough, [†] known to ancient kings,
For mineral waters, and ſalubrious ſprings;
Here wholeſome air, and a rich ſoil is found,
With crops luxuriant here the fields abound.
The ſouth-eaſt ſide the river Nen glides through,
And ſpires beyond admiring travellers view:
Three neighbouring ones, which I ſhall [‡] here diſcloſe,
Shaded my cottage, as bright Phœbus roſe.
Strait from my houſe, next Higham, o'er the plains,
I meaſured twenty-four of Gunter's chains;
Where at ſun-riſe, on the ſolſtitial days,
Ircheſter ſpire obſcures the ſolar rays.
On February's 'leventh, the riſing ſun diſplay'd,
On the ſame ſpot, a view of Ruſhden's ſhade.
If lines from Ruſhden, and from hence be drawn,
They will at Higham a right angle form.
Each ſpire's true diſtance from my houſe explore,
Counting refraction minutes thirty-four.

[*] Lat. 52° 20′.

[†] For the red wells, whoſe famous ſenative waters it's ſaid were conducive to the conception of king Charles II. when king Charles I. and his royal conſort the queen viſited there; when there was great reſort of nobility to drink the waters, as there is of the country inhabitants at this day.

[‡] With Ircheſter, the neareſt; on Jan. the 6th, Ruſhden, the remoteſt, on Feb. the 14th, and with Higham Ferrars on March the 13th, the center of the ſun, at riſing, appeared in a right line at Wellingborough.

II. QUESTION 326, *by Mr.* Chriſt. Maſon, *Surveyor to the Right Hon. the Earl of* Northampton.

Suppoſe the radius of our earthly ſphere
To be four thouſand miles, or very near;
Then fit materials let us next prepare,
To build a pendant caſtle in the air;
Rais'd to ſuch height, a ball let go from thence,
In falling takes the time found flies from hence.

Ingenious artiſts, tell me what degree
The ball's velocity at ground will be?
The different gravities next make appear,
Betwixt the ball below, and in the air?
If at the caſtle we ſuppoſe an eye,
How far can that the diſtant ſurface ſpy?

III. QUESTION 327, *by Mr.* W. Jepſon *of* Lincoln.

Required two general theorems, with their inveſtigation, to determine the leaſt triangle, and leaſt cone, that will circumſcribe any ſegment of an ellipſis, and fruſtum of a ſpheroid, when the dividing ordinate is parallel, and in any given ratio to the conjugate axis?

IV. QUESTION 328, *by the Rev. Mr.* Baker *of* Stickney, Lincolnſhire.

To what height will an exhalation aſcend, whoſe ſpecific gravity is, at the earth's * ſurface, equal to half that of common air, but decreaſes in the ſubtriplicate ratio of the ſpaces aſcended?

* *Fluxonienſis* ſays it muſt be a mile, or ſome diſtance from the ſurface, to make it conſiſtent.

V. QUESTION 329, *by Mr.* J Powle *of* Salop.

Three ſpheres of braſs in contact, whoſe diameters are 8, 9, and 10 inches reſpectively, ſupport a fourth ſphere, weighing 12 pounds; what quantity of weight does each ſupporting ſphere ſuſtain?

VI. QUESTION 330, *by* Upnorenſis.

To determine the path which a philoſopher muſt deſcribe, paſſing between two fires, at *d* diſtance from each other, and one fire *n* times as big as the other, ſo as to feel the leaſt heat poſſible?

VII. QUESTION 331, *by Mr.* Chriſt. Maſon.

There are two bridges over two different channels, having flood-gates underneath them; one has four gates, each 4 feet 2 inches wide; the other has two, each 3 feet 9 inches wide; there is 100l. a year paid as water-ſcot by lands which theſe channels help to drain: A mean depth of 45 inches was taken at the greater bridge, and 24 inches at the leſſer; the beds of both channels are ſuppoſed to incline alike in their level, or declivity; what part of the 100l. muſt be allotted to each channel, according to the proportion of water which they reſpectively diſcharge at the aforeſaid depths?

VIII. QUESTION 332, *by Mr.* Powle.

An equation of a curve is expreſſed by $y = \frac{Xx}{\sqrt{rr+xx}}$ (where X is the hyper. log. of x): Required an expreſſion of its area in finite terms?

IX. QUESTION 333, *by the Rev. Mr.* Baker.

What is the content of a caſk, whoſe head and bung diameters are 30 and 40 inches reſpectively, ſuppoſed to be formed by the caſſinian ellipſe revolving on its principal axe, which is juſt four-thirds of the caſk's length?

X. QUESTION 334, *by* Dictator Roffenſis.

Three Iriſh evidences, namely, a pedant, a prieſt, and an alderman, offer their attendance to the plaintiff's attorney, on a trial at Weſtminſter-hall, for the reward of half a hogſhead of wine; the pedant can drink it out by himſelf in 12 days, the prieſt in 10, and the alderman in 15, when the days are 12 hours long: Quere, in what time can the pedant, prieſt, and alderman drink out the whole, drinking together, when the days are 10 hours long? And what will be each evidence's ſhare?

XI. QUESTION 335, *by Maſter* Dickey.

If 10 packs of cards and 3 packs of knaves are of equal value with 9 packs of knaves and 4 packs of cards, what will be the value of one pack of knaves?

The PRIZE QUESTION, *by Mr.* Turner *of* Brumpton, *near* Rocheſter.

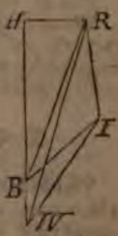
Three towns, *A*, *B*, *C*, at which make no wonder,
Seven, eight, and ten miles are exactly aſunder;
A thouſand good people in *A* live alert,
In *B* and *C* two and three thouſand expert;
Religiouſly bent, muſt on Whitfield attend,
And wou'd chuſe him a place, a la mode, for that end;
Where muſt he hold forth, that, in preaching to thoſe,
All walking to hear him ſhall wear out leaſt ſhoes?

1751.

Queſtions anſwered.

I. QUESTION 325 *anſwered by the Propoſer.*

LET *I*, *R*, and *H* repreſent the ſituation of the ſpires, Ircheſter, Ruſhden, and Higham; and *W*, *B*, the places of obſervation at Wellingborough, and next it and Higham. The declination of ☉ on 6th of Jan. laſt at riſing was 20° 42′ S. with which the complement of lat. of Wellingborough 37° 40′, and ☉'s zenith diſtance 90° 34′, I find (per ſpherics) the oppoſite angle, or ☉'s azimuth from *N.* when his center appears in the horizon = 124° 27′. In like manner, the ☉'s apparent azimuth from N. at riſing, Feb. 14th = 104° 29′, the difference of theſe 19° 58′ = $\angle IWR$. The ☉'s apparent amplitude March 13th was 3° 7′ N. Therefore the $\angle RWH =$ 17° 36′. Sun's apparent amplitude on the winter ſolſtice is 39° 44′, and on the 11th of Feb. 15° 53′ S. Conſequently, the $\angle IBH = 42° 51'$ and $\angle RBH = 19° 0'$; from whence, with the meaſured diſtance *BW* (by plain trigonometry) *WI* is found = 2 miles, 1 furl. 29 pol. *WR* = 3 m. 7 f. 39 p. and *WH* = 3 m. 6 f. 19 p. required.



Mr. William Sutton''*s Anſwer to the ſame.*

The viſible amplitudes of the ſun, at riſing, viz.

Dec. 10th	39°	45′	ſouth
Jan. 6th	34	25	
Feb. 11th	15	48	
Feb. 14th	13	57	
Mar. 13th	3	18	north.

From whence the diff. of amplitudes from Jan. 6th to Feb. 14th = 20° 28′ = $\angle RWI$. And the ſum of amplitudes from Feb. 14th to March 13th = $\angle RWH = 17° 15'$.

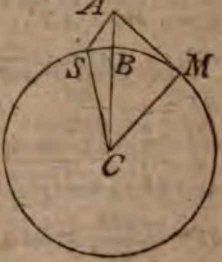
Alſo diff. amplitudes in the right line from Wellingborough to Higham, between Dec. 10th and Feb. 11th = 23° 57′ = $\angle IBR$.

The diff. between Feb. 11th and 14th is $\angle BRW = 1° 51'$, from whence the $\angle HBR = 19° 6'$, $\angle BIW = 5° 20'$, and conſequently *WI* = 2 m. 1 f. 25 p. *WR* = 3 m. 0 f. 13 p. *WH* = 2 m. 7 f. 9 p.

The ſame was curiouſly anſwered by Mr. *W. Devil*, Mr. *Roger Widger*, and others.

II. QUESTION 326 *anſwered by Mr.* Chr. Turner, *the propoſer.*

To find the height of the caſtle. Let $x=$ ſeconds required, either in the acceleration of a body or velocity of ſound propagated; $b=16\frac{1}{12}$ feet $=$ acceleration of a body the firſt ſecond, $c=1142$ feet the diſtance moved over by ſound in a ſecond.



Then, $bxx=cx=$ caſtle's height, per queſtion. Hence $x=\frac{c}{b}=71$ ſeconds, and the degree of velocity 141.

To find the proportion of gravity. Let $CB=r=4000$ miles $=$ radius given, $CA=s$, then $rr:1::ss:\frac{rr}{ss}$, being in a reciprocal proportion. If the weight upon the earth be unity, the ſame weight at the caſtle will be ·99 or as 100 to 99 nearly.

Laſtly, to find the viſual, or tangent line AM. By 36 E. 3, and 47 E. 1, $AC^2-CM^2=AM^2$. Hence $AM=350$ miles and 1745 yards.

Mr. *T. Cowper* elegantly ſolved the ſame.

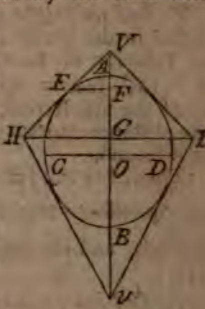
Anſwered by Mr. Roger Widger *of* Plymouth.

Put $a=1142$ feet ſound moves per ſecond, $b=16\frac{1}{12}$ feet deſcended by a heavy body in a ſecond, $x=$ diſt. of the caſtle from the earth's ſurface. Then, $b:1::x:\frac{x}{b}=$ ſquare time of the falling body. As $a:1::x:\frac{x}{a}=$ time of ſound returning, whence $\sqrt{\frac{x}{b}}=\frac{x}{a}$ per queſt. Whence $x=\frac{aa}{b}=81087{\cdot}917$ &c. feet; the reſt following as in the above anſwer.

Mr. *William Bevil* of *Harpſwell* has calculated the ſame in the like manner.

III. QUES-

III. QUESTION 327 *anſwered by the propoſer Mr.* Jepſon.



Let $AB = 2a$, $CD = 2b$, GO (which by the data will always be a known quantity) $= c$, $HI = 2y$, GV or $Gv = x$, OV or $Ov = x \pm c$. Then, by the properties of the figure, $VO : AO :: AO : FO$, viz. $x \pm c : a :: a : \frac{aa}{x \pm c} = FO$; $\because BF = a + \frac{aa}{x \pm c}$, $AF = a - \frac{aa}{x \pm c}$; and, by the properties of an ellipſis, $aa : bb :: \overline{a + \frac{aa}{x \pm c}} \times \overline{a - \frac{aa}{x \pm c}} : bb - \frac{aabb}{\overline{x + c}|^2} = EF^2$, therefore $EF = b\sqrt{1 - \frac{aa}{\overline{x \pm c}|^2}} = b\sqrt{\frac{\overline{x \pm c}|^2 - aa}{\overline{x \pm c}|^2}}$, and $VF = x \pm c - \frac{aa}{x \pm c} = \frac{\overline{x \pm c}|^2 - aa}{x \pm c}$: Now, by ſim. $\triangle$s, $\frac{\overline{x \pm c}|^2 - aa}{x \pm c} : x :: b\sqrt{\frac{\overline{x \pm c}|^2 - aa}{\overline{x \pm c}|^2}} : bx\sqrt{\frac{1}{\overline{x \pm c}|^2 - aa}} = y$; but xy is a minimum by the queſtion, $\because bxx\sqrt{\frac{1}{\overline{x \pm c}|^2 - aa}}$ or $\frac{x^4}{xx \pm 2cx + cc - aa}$ is alſo a minimum, which in fluxions is $2x^5\dot{x} \pm 6cx^4\dot{x} + 4c^2x^3\dot{x} - 4a^2x^3\dot{x} = 0$, $\because x^2 + 3cx + 2cc - aa = 0$, or $xx \pm 3cx = 2aa - 2cc$, whence $x = \sqrt{2aa + \frac{cc}{4}} \pm \frac{3c}{2}$, the theorem for finding the leaſt triangle.

Again, let $d = 3{\cdot}1416$, and $\frac{b^2dx^3}{3} \times \frac{1}{\overline{x \pm c}|^2 - a^2}$, or $\frac{x^3}{\overline{x \pm c}|^2 - a^2}$ is a minimum. In fluxions $x^4\dot{x} \pm 4cx^3\dot{x} + 3c^2x^2\dot{x} - 3a^2x^2\dot{x} = 0$, $\because x^2 \pm 4cx + 3c^2 - 3a^2 = 0$, or $x^2 \pm 4cx = 3a^2 - 3c^2$, whence $x = \sqrt{3a^2 + c^2} \pm 2c$, the theorem for finding the leaſt cone.

This gentleman informs us that ſeveral uſeful corollaries may be drawn from the above general theorems, which he promiſed to communicate for public benefit. And as we find him exceedingly well qualified, we ſhall diſtinguiſh his performances, with a due regard had to them. And we hope no ingenious contributor will take offence at our preferring what excels in this Diary, as it is the only means of improvement.

The ſame Queſtion *anſwered by Mr.* William Bevil.

The ratio of the dividing ordinate to the conjugate being given, their diſtance from each other is readily found, which diſtance call d, and let $a =$ ſemi-tranſverſe, $b =$ ſemi-conjugate of that ellipſis, and $x = OV$. Then, per conics, $x : a :: a : \frac{aa}{x} = OF$, $a - \frac{aa}{x} = GF$, $x - \frac{aa}{x} = FV$, $aa : bb :: aa - \frac{aa}{xx} : \frac{bb}{aa} \times \overline{aa - \frac{aa}{xx}} = EF^2$, $\frac{xx - aa}{x} : \frac{b}{x}\sqrt{xx - aa} :: x \pm d : \frac{x \pm d}{xx - aa} \times b\sqrt{xx - aa} = HG$, then $\frac{\overline{x \pm d}|^2}{xx - aa} \times b\sqrt{xx - aa} =$ area $\triangle\, HVI$ or HIv, which is a minimum; ſquared and put into fluxions, $4x^3\dot{x} + 12dx^2\dot{x} + 12d^2x\dot{x} + 4d^3\dot{x} \times \overline{xx - aa} - 2x\dot{x} \times \overline{x \pm d}|^4 =$ c. By reduction, $xx \pm dx = 2aa$, whence $x = \frac{1}{2}\sqrt{8aa + dd} \pm \frac{d}{2}$, a theorem for the leaſt triangle.

Now put $\cdot 2618 = c$, then $\frac{\overline{x \pm d}|^3}{\overline{xx - aa}|^2} \times \overline{xx - aa} \times 4bbc$ $=$ the cone's ſolidity, which, or $\frac{x \pm a}{\overline{xx - aa}|^{\frac{1}{2}}}$, is a minimum. In fluxions, and reduced, $xx \pm 2dx = 3aa$, whence $x = \sqrt{3aa + dd} \pm d$, a theorem for the leaſt cone.

IV. QUESTION 328 *anſwered by the propoſer, the Rev. Mr.* Baker, *only.*

Put $r = 4000$ miles $=$ earth's radius, $\frac{68444}{3 \times 1760} = c$, ſpace aſcended $= x$, air's denſity at the earth's ſurface $= d$. Then, per queſtion, that of the vapour there will be $= \frac{1}{2}d$. And $x^{\frac{1}{3}} : 1^{\frac{1}{3}} :: \frac{1}{2}d : \frac{d}{2x^{\frac{1}{3}}} =$ its denſity at x height. But from page 96th of Mr. Emerſon's Fluxions, we have $d \times$ number belonging to this log. $\frac{-rx}{cr+cx} = \frac{d}{2x^{\frac{1}{3}}}$, which reduced, according to the nature of logarithms, gives $\frac{3rx}{cr+cx} - l.8 = l.x$, whence, by a table of logarithms, $x = 7{\cdot}763$ miles, the height required.

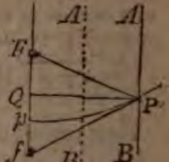
V. QUESTION 329 *anſwered by Mr.* Widger.

For want of room for the proceſs, we only inſert the numbers, viz. $\left\{\begin{matrix} 6{\cdot}0614 \\ 3{\cdot}2696 \\ 2{\cdot}669 \end{matrix}\right\}$ pounds ſupported by the globe of $\left\{\begin{matrix} 8 \\ 9 \\ 10 \end{matrix}\right\}$ inch. diameter.

Mr. *Powle*, the propoſer, did not ſend us his ſolution, as he propoſed.

VI. QUESTION 330 *anſwered by* Waltonienſis.

Let F be the greater fire, n times bigger than f the leſſer; $d = Ff$, their diſtance.



At any diſtance, draw AB parallel to Ff, then, ſince the philoſopher muſt paſs every line AB, we have only to find that point therein, at which if he ſtood ſtill he ſhould feel the leaſt heat poſſible from both the fires. Suppoſe P to be that point, and drawing $PQ \perp Ff$; call FQ, x; PQ, y; then $\frac{n}{xx+yy} + \frac{1}{\overline{d-x}|^2 + yy}$, expreſſing the heat of both fires, muſt be a minimum, y being given, or ſome conſtant quantity, while x is variable, the fluxion made $= 0$ will always ſhew the relation betwixt x and y.

The fluxion of this expreſſion, when $y = 0$, is $\frac{-2nx\dot{x}}{x^4} + \frac{2d\dot{x} - 2x\dot{x}}{\overline{d-x}|^4} = 0$, where $x = \frac{dn^{\frac{1}{3}}}{1+n^{\frac{1}{3}}}$; whence $\frac{d}{1+n^{\frac{1}{3}}}$ is the philoſopher's diſtance from the leaſt fire, directly betwixt both fires, moving along the curve Pp, to be roaſted on both ſides alike. Mr. *Powle*'s ſolution gives the ſame; the heat emitted being directly as the two fires, and inverſly as the ſquares of their diſtance.

Mr. *Sutton* and Mr. *Bevil* ſent us their ſolutions.

N.B. The point p being found for the vertex of the curve, and Fp and fp being in a given ratio to each other, if any other diſtances from the curve to the greater or leſſer fires, PF and Pf, be ſuppoſed in the ſame ratio, the path of the curve will be a circle, as obſerved by Fluxionenſis.

VII. QUESTION 331 *anſwered by* Britannicus.

200 inches, the breadth of the greater channel by 45 its depth = 9000 ſquare inches the area of the ſection; and 90 inches the breadth of the leſſer channel, by 24 its breadth = 2160 ſquare inches the area of its ſection; the velocity of water moving along each channel is as the ſquare roots of its depth, reſpectively, viz. as $\sqrt{45} = 6{\cdot}708$ and $\sqrt{24} = 4{\cdot}899$ fere; therefore $9000 \times 6{\cdot}708$ and $2160 \times 4{\cdot}899$, or 60372 and 10582 the momenta, are nearly as the water reſpectively diſcharged by each channel, in the ſame time; therefore the greater channel pays 85 l. 1 s. 8 d. $\frac{47310}{70934}$, and the leſſer 14 l. 18 s. 3 d. $\frac{23634}{70934}$, required.

N.B. Greater	Bridge	88 l.	13 s.	are the propoſer's numb.
Leſſer	Bridge	11	7	who ſent no proceſs.
		100	0	

Our ingenious friend Mr. *Hulſe*, correſponding with the propoſer, has ſent us 88 l. 14 s. and 11 l. 6 s. nearly, for each bridge to pay; correcting the propoſer's numbers.

VIII. QUESTION 332 *anſwered by* Newtonienſis.

Since $y = \frac{Xx}{\sqrt{rr+xx}}$, therefore $y\dot{x} = \frac{Xx\dot{x}}{\sqrt{rr+xx}}$, and fluent of $y\dot{x} = X\sqrt{rr+xx} - s$, whence (p. 58 Emerſon's Flux.) $\dot{s} = \dot{X}\sqrt{rr+xx} = \frac{\dot{x}}{x}\sqrt{rr+xx} = \frac{rr\dot{x}}{x\sqrt{rr+xx}} + \frac{x\dot{x}}{\sqrt{rr+xx}}$; and $s = \sqrt{rr+xx} + \text{fluent } \frac{rr\dot{x}}{x\sqrt{rr+xx}}$; and fluent

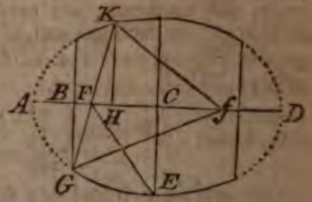
$rr\dot{x}$

$\frac{rr\dot{x}}{x\sqrt{rr+xx}}$ or $\frac{rrx^{-2}\dot{x}}{\sqrt{1+rrx^{-2}}}$ by the table is $=\frac{-L}{r}\times\log.\overline{\frac{r}{x}+\sqrt{\frac{rr+xx}{xx}}}$. Therefore the fluent of $\frac{X\dot{x}x}{\sqrt{rr+xx}}=\overline{X-1}\cdot\sqrt{rr+xx}+\frac{2{\cdot}3024}{r}\times\log.\overline{\frac{r}{x}+\sqrt{\frac{rr+xx}{xx}}}$.

N. B. There is no more difficulty where ſurds are concerned, the proceſs being the ſame as in ſimple quantities.

Mr. *Powle* ſends $\overline{X-1}\cdot\sqrt{rr+xx}+2{\cdot}3025r\times\log.\frac{r+\sqrt{rr+xx}}{x}$ = area.*

IX. QUESTION 333 *anſwered by the Propoſer only.*



Let F, f, be the foci of the gener. ellipſe, $BG=h$, $CE=b$, $\frac{3}{4}=n$, $3{\cdot}1416=p$, $AF\times DF=r$, $AC=v$, $Fc=z$, $BF=x$, $BC=y$. Then, by the figure, $v^2-z^2=EF^2=b^2+z^2$, $\because z=\sqrt{\tfrac{1}{2}v^2-\tfrac{1}{2}b^2}$, and $r=\frac{1}{2}v^2+\frac{1}{2}b^2$; and, becauſe $FG\times Gf=AF\times DF$, and $x^2+y^2=FG^2$, alſo $\overline{2z+x}^2+y^2=GF^2$, by elliptic property; therefore $\overline{x^2+y^2}\times\overline{\overline{2z+x}^2+y^2}=r^2$. Hence $y^2=\sqrt{r^2+4z^4+8z^3x+4z^2x^2}-2z^2-2zx-x^2$, is the equation of the curve. But when $y=h$, then, per queſt. $x+z=nv$, $\because x=nv-z$. And ſubſtituting the above values of r, x, y, and z, in the ſaid equation, we get, by reduction, $v^4+\frac{b^2-n^2b^2-2n^2h^2-h^2}{n^2-n^4}v^2=\frac{h^4-b^2h^2}{n^2-n^4}$, whence, in the preſent caſe, $v=12{\cdot}148$, or $32{\cdot}9267$, which laſt value is the true, becauſe v cannot be leſs than b. Hence we have $r=742{\cdot}08365$, $z=18{\cdot}4955$; and by ſubſtitut. for the known quantities $y^2 = \sqrt{a^2 + bx + c^2x^2} - d - cx - x^2$. Wherefore $py^2\dot{x} = p\dot{x}\sqrt{a^2 + bx + c^2x^2} - dp\dot{x} - cpx\dot{x} - px^2\dot{x} =$ the fluxion of ſolidity between *A* and *F*; whoſe fluent, when properly corrected, is $\frac{2c^2x + b}{4c^2} \times p\sqrt{a^2 + bx + c^2x^2} - \frac{abp}{4c^2} + \frac{4a^2c^2 - b^2}{8c^3} \times p \times$ hyp. log. $\frac{2c^2x + b + 2c\sqrt{a^2 + bx + c^2x^2}}{2ac + b} - dpx - \frac{1}{2}cpx^2 - \frac{1}{3}px^3$, which, when $x = BF$, will be 5445·9815 inches. Again, putting $x = FH$, $y = HK$, and proceeding as before, we get $py^2\dot{x} = p\dot{x}\sqrt{a^2 - bx + c^2x^2} - dp\dot{x} + cpx\dot{x} - px^2\dot{x} =$ the fluxion of the ſolidity between *F* and *C*, which differs from the other fluxion in nothing but the ſigns, conſequently its correct fluent is diſcovered to be $\frac{2c^2x - b}{4c^2} \times p\sqrt{a^2 - bx + c^2x^2} + \frac{abp}{4c^2} + \frac{4a^2c^2p - b^2p}{8c^3} \times$ hyp. log. $\frac{2c^2x - b + 2c\sqrt{a^2 - bx + c^2x^2}}{2ac - b} - dpx + \frac{1}{2}cpx^2 - \frac{1}{3}px^3 = 22116·2054$ when $x = CF$. Hence, the content of the whole caſk in $\left\{\begin{array}{l}\text{ale}\\ \text{wine}\end{array}\right\}$ gallons $\left\{\begin{array}{l}195·4758\\ 238·63278\end{array}\right.$ *Q. E. I.*

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* Mr. Powle's fluent is right, and Newtonienſis's would be the ſame if the latter part of it were drawn into r^2 as it ought.

X. QUESTION 334 *anſwered by Mr.* Steph. Hodges *of* Wellingborough.

The proportion of the quantities of wine drank by the pedant, prieſt, and alderman, in the ſame time, are as 1, 1·2, ·8, whoſe ſum = 3. Then ſay, inverſely, as 1 : 12 :: 3 : 4 days = 48 hours when the days are 12 hours long = 4 days 8 hours when the days are but 10 hours.

By direct proportion, ſay,

$$\begin{array}{llll} & \text{gall.} & & \text{gall.} \\ \text{As } 3 : 31·5 :: & \left\{\begin{array}{r}1·\\ 1·2\\ ·8\end{array}\right\} & : & \left\{\begin{array}{r}10·5\\ 12·6\\ \underline{8·4}\\ 31·5\end{array}\right\}\begin{array}{l}\text{pedant's}\\ \text{prieſt's}\\ \text{alderman's}\\ \end{array}\Bigg\}\ \text{allowance.}\end{array}$$

Mr. *T. Cowper* of *Wellingborough* anſwered this queſtion in the ſame conciſe and artificial manner, as did Mr. *Joſeph Orchard*, of *Goſport*, in *Hampſhire*.

The ſame anſwered by Mr. Will. Smith, *at* Churchdown, *near* Glouceſter.

Put $x =$ time they will all drink it in? $12 \times 12 = 144 = c$, $12 \times 10 = 120 = d$, $12 \times 15 = b$, $w =$ half the hogſhead.

Say, $c : w :: x : \frac{wx}{c}$; $d : w :: x : \frac{wx}{d}$; $b : w :: x : \frac{wx}{b}$, the ſeveral ſhares collected $= \frac{wx}{c} + \frac{wx}{d} + \frac{wx}{b} = w$. Whence $x = \frac{cbd}{bd + cd + cb} = 48$ hours when the day is 12 hours $= 4$ days 8 hours when the day is 10 hours.

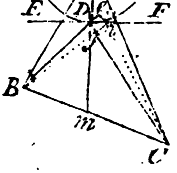
$$\text{Hence} \left\{\begin{array}{c} \text{gall.} \\ \frac{48}{144} \\ \frac{48}{120} \\ \frac{48}{180} \end{array}\right\} \begin{array}{l} \\ \text{pedant's} \\ \text{prieſt's} \\ \text{alderman's} \end{array} \Bigg\} \text{ Share } \left\{\begin{array}{c} \text{gall.} \\ 10{\cdot}5 \\ 12{\cdot}6 \\ 8{\cdot}4 \end{array}\right\} \text{ as above.}$$

Mr. *William Dod*, of *Brampton*, in *Cumberland*, ſolved the ſame, exactly to the truth, and ſo did ſeveral others.

XI. QUESTION 335 *anſwered by Maſter* Billy Branch, *of* Rocheſter.

If 10 p. cards + 3 p. knaves = 9 p. knaves + 4 p. cards. Then, by reduction, 6 packs of knaves is of equal value with 6 packs of cards; whence the value of a pack of knaves is only a pack of cards.

The PRIZE QUESTION *anſwered by* Newtonienſis.



Let A, B, C, be the three towns, D the place of meeting ſought; and ſuppoſe any of the diſtances, as AD to be given, and with the radius AD, deſcribe the circular arch $GDnH$, and let EDF be a tangent at D; draw ADm, and take Dn infinitely ſmall, and draw the lines BD, Bn, CD, Cn, and with the radii Bn, Cn, deſcribe the ſmall arcs ne, nf; then De is the increment of BD, and Df the decrement of CD.

Let $BD = x$, $CD = y$, $AD = z$, and a, b, c, three given numbers 2, 3, 4; in proportion as per queſtion, (4000 people living in the town A, as the author informs me, inſtead of 2000 the printed number) ſo that $ax + by + cz$ may be a minimum. Then, ſince z is ſuppoſed conſtant, we have $a\dot{x}+b\dot{y}=0$, and $a\dot{x}=-b\dot{y}$, or $\dot{x}:-\dot{y}::b:a$. In the two right-angled triangles Dne, Dnf, whoſe common hypothenuſe is Dn, it is as $De(\dot{x}):Df(-\dot{y})::\text{s.}\,Dne:\text{s.}\,Dnf::b:a$; but $\angle Dne=eDA=BDm$, and $\angle Dnf=mDf$ or mDC. Whence $\text{s.}\,BDm:\text{s.}\,mDC::b:a$, and $\frac{\text{s.}\,CDm}{a}=\frac{\text{s.}\,BDm}{b}$.

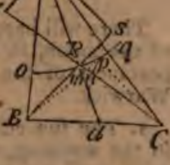
After the ſame manner it may be proved, that if y be ſuppoſed conſtant, $\frac{\text{s.}\,CDm}{a}=\frac{\text{s.}\,CDB}{c}$; $\because$ when $ax+by+cz$ is a min. $\frac{\text{s.}\,CDm}{a}=\frac{\text{s.}\,BDm}{b}=\frac{\text{s.}\,CDB}{c}$. Therefore, if $\text{s.}\,CDm=v$, then $\frac{b}{a}v=\text{s.}\,BDm$, and $\frac{c}{a}v=\text{s.}\,CDm+BDm$ their ſum. Therefore the problem comes to this, *To find the* $\angle$ CDm *whoſe ſine is* v, *and* $\angle$ BDm *whoſe ſine is* $\frac{b}{a}v$, *ſo that* $\frac{c}{a}v$ *may be the ſine of the ſum* CDm + BDm. And $\angle CDm$ is eaſily found by Mr. Heath's method in the Diary 1738.

N. B. $\angle CDm$ is nearly 29°.

Now, all the parts of $\triangle ABC$ being given, together with the angles at D, all the diſtances AD, BD, CD are eaſily found, viz. $AD=2{\cdot}6$, $BD=5{\cdot}02$, $CD=7{\cdot}64$. (See Ronayne, prob. 11 p. 363.)

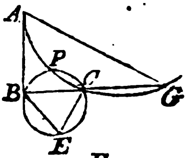
The ſame anſwered by Waltonienſis.

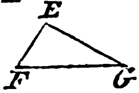
Let ABC repreſent the three towns, and let the number of people in A be $a=4000$ (correcting the printed number); in B, $b=2000$; in C, $c=3000$.



P the place of meeting ſought. Draw AP, BP, CP, and with the rad. AP, on the center A, deſcribe the arc oPq. A point being now ſuppoſed to move along the ſaid arc, lines drawn to it from B and C will be continually variable. Let the point have moved from P to p, over the indefinitely ſmall arc Pp, draw Bp, Cp, and with the radii Bp, Cp, deſcribe the ſmall arcs Pm, pn.

Then





Then b times BP plus c times CP being a minimum, b times mp, the increment of BP, will be equal to c times Pn, the decrement of CP; therefore, the right-angled triangles APr, APs, made by letting fall the perpendiculars Ar, As, being reſpectively ſimilar to the ſmall right-angled triangles Ppn, pPm, the ſine of the angle APr, or CPd, will be to that of $\angle APs$, or BPd, as b to c. In like manner it is proved, that the ſine of $\angle CPs$ is to that of $\angle CPd$, as a to b. Whence it follows, that if a $\triangle EFG$ be conſtructed, whoſe ſides FG, EF, and EG, are as a, b, and c reſpectively, and two circular arcs deſcribed without $\triangle ABC$, one upon the ſide BC, capable of the $\angle E$, and the other upon AC, capable of the $\angle G$, the ſupplement of thoſe arcs, when completed into circles, will interſect each other at the point P ſought: And AP will be found $= 2{\cdot}596$ miles, $BP = 5{\cdot}009$, $CP = 7{\cdot}623$. *Q.E.F.*

COROLLARY. If a, b, and c be equal, then two ſegments of circles deſcribed within the $\triangle ABC$, on any two ſcales, each capable of an angle of 120°, will interſect at the point required, according to Mr. Simpſon's new doctrine and application of Fluxions, p. 26 and 27. He has inſerted and ſolved this our queſtion at p. 505 of his doctrine aforeſaid.

Mr. *W. Jepſon* ſent his ſolution, which, with Dr. *Quibus*'s and two or three more were all the ſolutions we received.

The Prize of 12 *Diaries was won by* Newtonienſis, *and that of* 8 *by Dr.* Quibus.

The Eclipſes calculated for 1751, *by Mr.* William Sutton.

There will happen four eclipſes, two of the ſun, and two of the moon, in the following order: The times and appearances of the viſible ones according to the meridian of Warwick.

1. The ſun eclipſed on tueſday May 14th, at 43 minutes in the morning, apparent time of conjunction in the moon's orbit; will be central and total to the eaſtern parts of Aſia bounding on America; but inviſible at London, or to any *part of* Europe.

2. The moon eclipſed wedneſday May 29th, at one in the morning.

	h. m.	
Beginning 28th May	11 44 P. M.	apparent time.
Middle 29th —	1 26 A. M.	Digits eclipſed on the ſouth ſide 10° 47′.
Ecliptic opppoſit.	1 31	
End — — —	3 8	
Duration — —	3 24	

Calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m	Dig. ° ′
Mr. Randles,	London	12 21	2 3	3 44		
	Barbadoes	8 28	10 10	11 51	3 23	10 32
	Wem, Shropſh.	12 11	1 53	3 34		
Mr. Hulſe,	Cheſter	0 28	1 0	3 32	3 4	10 3
Mr. Cowper,	London	11 58	1 38	3 17	3 18	10 19
	Wellingboro.	11 56	1 35	3 14		

3. Sun eclipſed on thurſday November 7th, at 37 minutes in the morning, equal time of conjunction in the moon's orbit; inviſible at London, but will be a central eclipſe to ſome parts of the globe.

4. The moon eclipſed on thurſday November 21ſt, at 9 h. 45 m. equal time.

	h. m.	
Beginning —	8 26	apparent time.
Middle ——	9 48	Digits eclipſed on the north ſide 8° 27′.
Ecliptic oppoſit.	9 54	
End — —	11 10	
Duration —	2 24	

Calculated by		Beg. h. m.	Mid. h. m.	End h. m.	Dur. h. m.	Dig. ° ′
Mr. Randles,	London	8 30	9 52	11 14		
	Peterſburgh	11 6	12 28	1 5		
	Liſbon,	7 53	9 15	10 37	2 44	3 21
	Wem, Shropſh.	8 20	9 42	11 4		
Mr. Hulſe,	Cheſter	8 20	9 48	11 16	1 56	3 4
Mr. Cowper,	Wellingborough	8 13	9 36	10 59	2 46	8 38

With others, which for want of room we are obliged to omit.

New Queſtions.

I. QUESTION 337, *by Mr.* T. Cowper *of* Wellingborough, *Surveyor.*

The latitude explore,
And time laſt winter, when
Day broke exact at four,
And the ſun roſe at ten?

II. QUESTION 338, *by Mr.* William Leighton.

Two perſons, *A* and *B*, playing at hazard, *A* wins from *B* a certain number of guineas, conſiſting of three places, whoſe digits are in arithmetical progreſſion, in ſuch manner, that, if the number of guineas be divided by the ſum of their digits, the quotient will be 48; and, if from the ſaid number of guineas you take 198, the digits will be inverted: Quere the number of guineas won?

III. QUESTION 339, *by Mr.* William Bevil.

From what height muſt a ball of 4 ounces weight fall, to have $49\frac{67}{100}$ pounds force, on an inclining plane, whoſe angle of incidence is 40°?

IV. QUESTION 340, *by Mr.* Davis, *Teacher of the Mathematics, at* Painſwick, Glouceſterſhire.

In latitude of forty-eight,
A monument ſtood tall and ſtraight,
Which finiſh'd was o'th' firſt of June,
At five o'clock i'th' afternoon;
When, by repeated trials made,
The length of its extended ſhade
Was found in ratio to its height,
As ninety-two to twenty-eight;
And, where its baſe with earth did join,
An angle form'd of ninety-nine?
What was the Julian period then?
If 'twas erected ſince, and when?
Or, if before erected found,
How many years have ſince gone round?

QUES-

V. QUESTION 341, *by Mr.* Bevil.

Two men bought an equal number of ſheep, and it being demanded of them what they gave a-piece for each parcel; it was anſwered, that if the number of ſheep each of us bought be ſeverally multiplied by $\frac{24}{27}$ and $\frac{54}{27}$, 49 being reſpectively added to and ſubtracted from each product, both the ſum and remainder will be equal to the ſquare of the number of ſhillings given for each reſpective parcel: How many ſheep did each perſon buy? And what did each parcel coſt?

VI. QUESTION 342, *by Mr.* Steph. Hodges, *the younger.*

In an exciſeman's round,
An oblong ciſtern's found,
The ſum of one ſide and one end being given,
With diag'nal below,
The contents you're to ſhow,
Whoſe breadth's to the depth twenty-five is to ſeven.

84 *inches = the ſum of one ſide and one end.* 60 = *the diagonal.*

VII. QUESTION 343, *by Mr.* John Randle, *of* Wem, *in* Shropſhire.

A gentleman has a piece of ground in form of a geometrical ſquare, the difference between whoſe ſides and diagonal is 10 poles; he would convert two-thirds of the area into a garden of an octagonal form, but would have a fiſh-pond at the center of the garden, in the form of an equilateral triangle, whoſe area muſt be equal five poles Required the length of each ſide of the garden, and of each ſide of the pond?

VIII. QUESTION 344, *by* Upnorenſis.

To determine the ſides of the leaſt right-angled triangle in whole numbers, whoſe legs are in proportion as 7 to 11?

IX. QUESTION 345, *by* χρονονμονοντυβλικος.

If a bookſeller buys a copy for 21 l. pays 21 l. for paper, 21 l. for printing 500 impreſſions, and 10 l. for advertiſements

and

other contingent expences, amounting in all to 73l. and ſells 100 books yearly of the hiſtory, at 5s. each: What is his gain per cent. allowing compound intereſt, for the time he lies out of his money?

The PRIZE QUESTION, *by Mr.* T. Cowper, *of* Wellingborough, *Surveyor.*

Admit the moon, on the 17th of February, 1750, roſe four minutes ſooner in the latitude 51° 32′ north, than in the latitude 52° 20′ north, and was obſerved to come upon the meridian in the former latitude on the ſame morning 42 minutes after four, and the preceding morning 54 minutes after three; from whence her longitude and latitude at riſing, in the latitude of 51° 32′, are required?

1752.

Queſtions anſwered.

I. QUESTION 337 *anſwered by Mr.* T. Cowper, *the propoſer.*

PUT $a =$ ſine $\odot$'s aſcenſional difference, 60°; $b =$ ſine hour from 6, at day-break, or 30°; $d =$ ſine $\odot$'s depreſſion at 18°; and x and y the ſine and coſine of the latitude: alſo e and v the ſine and coſine of $\odot$'s declination: By ſpherics $bvy + ex = d$ and $ex = avy$, and ſubſtituting avy for ex in the firſt equation $vy = \frac{d}{a+b}$: And alſo in the other equation putting $\frac{d}{a+b}$ for vy, and we have $ex = \frac{ad}{a+b}$; therefore $\frac{1-a}{a+b}d = \cdot 0303074 =$ coſ. ſum of lat. and ſun's declin. 88° 15′ 48″. And $\frac{1+a}{1+b}d = \cdot 4221252$ the coſ. of their diff. or 65° 1′ 52″. Hence the lat. 76° 38′ 50″, and declin. 11° 36′ 58″; nearly agreeing with Mr. Gibbons's anſwer.

THEOREM. As the ſum of the ſines of the ſun's aſcenſional difference and arch of time from day-break to ſix o'clock, is to the ſine of the ſun's depreſſion at day-break, ſo is the verſed ſine of the arch of time from ſun-riſe to noon, to the ſine of the meridian altitude; and ſo is the verſed ſine of the time from midnight to ſun-riſe, to the ſine of the ſun's depreſſion at midnight.

Mr. *William Bevil* has curiouſly and conciſely ſolved the ſame, exactly agreeing with the above. We wiſh we had more room, to inſert all he ſends us.

The ſame ſolved by Mr. Charles Smith *of* Rugby.

Put r and n for the coſines of the hour angles from midnight till day-break, and from ſun-riſe till noon, reſpectively; $d =$ coſ. of 108° $= Z \odot$, [*ſee fig. p.* 136] x and $y =$ ſine and coſ. of the required latitude; u and $z =$ thoſe of declination. Then, in ſpheric triangles $\odot PZ$ and OZP, by common theorems, $rzy + ux = d$, and $nzy - ux = 0$, from whence $zy - ux = d \times \frac{1-n}{r+n} = {}^{\cdot}0303072 =$ coſ. 88° 15′ 48″ the ſum of lat. and declin. and $zy + ux = d \times \frac{1+n}{r+n} = {}^{\cdot}422125$ $=$ coſ. 65° 1′ 52″ the diff. Hence the lat. 76° 38′ 50″ N. and the declin. 11° 36′ 58″ S. (anſwering to 7th Feb.) required; proving the truth of Mr. *Cowper*'s anſwer.

Mr. *John Aſh*, Mr. *Sutton*, Mr. *William Spicer*, Mr. *James Terey*, Mr. *Charles Maſon*, *Obadiah Wittam* of *Whitby*, Mr. *William Cottam* at his Grace the Duke of Norfolk's, and ſeveral others, ſolved the ſame.

II. QUESTION 338 *anſwered by Mr.* Rich. Gibbons.

Let x, y, and z repreſent the three digits; then, by the queſtion, we have $x + z = 2y$, $\frac{100x + 10y + z}{x + y + z} = 48$, and $99x - 99z = 198$; whence $x = 4$, $y = 3$, and $z = 2$; alſo number of guineas 432, required.

N. B. This queſtion is the 21ſt of the Miſcellanea Curioſa Math. vol. I. inverted.

Philotheoros, putting $a = 198$, $c = 48$, makes $x = \frac{a}{198} \times \frac{c-4}{c-37} = \frac{44}{11} = 4$; whence the number $= 432$.

Mr. *Joſeph Orchard* ſolved the ſame; alſo Mr. *John Fiſh* of *Dartford*, and ſeveral others.

III. QUES-

III. QUESTION 339 *anſwered by Mr.* John Aſh.

Ecce homo!

As ſine 40° : rad. : : 49·67 pounds force : 77·2728 pounds, the momentum or force of the falling body $= m$. Put n for the given weight $=$ ·25 pounds, and x for the required height; then, by the laws of motion, $\frac{m}{n}$ will be the velocity of the ball arrived at the plane of the horizon; and (if Deſaguliers's experiment, Philoſ. Tranſactions, No. 375, p. 269, can be depended on) we have $\sqrt{x} = \frac{m}{n}$; whence $x = \frac{m^2}{n^2} = 9553{\cdot}7$ feet, required.

Mr. *Richard Gibbons* ſolves this queſtion in the ſame manner: Thus,

As the ſine of the angle of incidence 40° : 49·67 pounds force : : rad. : 77·273 pounds force on the plane of the horizon, being let fall from the ſame height. By Dr. Deſaguliers's experiments, an heavy body deſcending four feet will have twice the quantity of motion it had when it began to fall (i. e. *we obſerve at the end of one foot fallen*) the time of its falling half a ſecond. Now, the force is always as the velocity and quantity of matter, *i. e.* $\sqrt{\text{ſpace}} \times$ matter, perpendicularly deſcended; putting $m = 77{\cdot}273$ the momentum, perpendicularly deſcended; $q = 0{\cdot}25$ pounds the quantity of the ball; and $s =$ ſpace required to run through: Then $q\sqrt{s} = m$; whence $\sqrt{s} = \frac{m}{q}$, and $s = \frac{m^2}{q^2} = 9553{\cdot}7$ feet, as before.

We received numerous other learned ſolutions to the foregoing queſtion. Mr. *Harland Wild*, of *Whitby*, makes the diſtance to be deſcended by the ball no leſs than 386622·4536 feet, or 73·223 miles; and ſome about as far as from the moon's orbit.

A Diary-Critic, obſerving our remark in laſt year's Diary where Newtonienſis points out an impropriety in propoſing theſe ſort of queſtions, endeavours to make the diſcovery his own, and is very angry at our ignorance, that we ſhould ſuffer ſuch a queſtion to be printed (ſee London Gazetteer for Dec. 13, 1750). But he ſhould have firſt conſidered, that Mr. *John Turner* and the Rev. Mr. *Baker*, who ſolved a like queſtion (in Diary 1750), the propoſer Mr. *Bevil* (in Diary *1751*), and Mr. *Landen*, (in the ſame Diary, and in what is called Gentleman's Diary), are equally culpable: Though he frequently borrows inventions and obſervations, and borrowed our own remark from the place above-mentioned. See our prize queſtion for the Diary 1750, inſerted in a late book of fluxions, p. 505.

To a famous Doctor, *on his Diſcoveries.*

So modern 'pothecaries, taught the art
By doctors bills to play the doctor's part,
Bold in the practice of miſtaken rules,
Preſcribe, apply, and call their maſters fools!

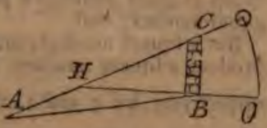
POPE.

Mr. *Landen* alſo, who refined upon Mr. *Turner*'s and Mr. *Baker*'s ſolution to a queſtion of this kind laſt year, now detects our modern philoſophers; but firſt ſaw the remark that we inſerted from *Newtonienſis*.

The relative forces of falling bodies being as the $\sqrt{\text{ſpace}}$ × quantity of matter, perpendicularly deſcended (*i. e.* as the rectangle of the velocity and matter) it will follow, that o and x, diſtances deſcended by two balls, whoſe weights are W and w, will have forces, as $o^{\frac{1}{2}} \times W$ to $x^{\frac{1}{2}} w$, that is, when W is at reſt, it will have no force in compariſon with the force of w at any diſtance deſcended: which is contrary to what is ſuppoſed in the 321ſt and 339th diary queſtions, where force compounded of weight of matter and velocity, is ſuppoſed equal to a degree of preſſure of matter unſuſpended, tho' at reſt.

IV. QUEST. 340 *anſwered by the propoſer, Mr.* Davis.

Let BC repreſent the monument, AB the ſhade's length, HO the horizon; then $\angle OH\odot$ = ſun's alt. The ſides AB to BC as 92 to 28, and $\angle ABC = 99°$. By trigon. $\angle OH\odot = 25°\ 0'\ 37''$ ſun's alt. and allowing 17′ 37″ for ſun's ſemi-diam. and refraction, 25° 0′ 37″ − 17′ 37″ = 24° 43′ = ſun's true alt.



Now, from comp. lat. 42° 0′, comp. ſun's alt. 65° 17′, and hour angle from noon of 5 h. = 75°, the complement of ſun's declin. will be found 69° 53′ 26″, and declin. 20° 6′ 34″, which anſwers to ♉ 29° 38′ the ſun's place in the ecliptic, or longitude from ♈ 59° 38′; and, by making proportion, I find, June 1ſt, 5 hours P.M. anno 965 ante chriſtum, the ſun's place is ♉ 29° 38′ 0″, as may be proved from Leadbetter's tables; being the time when the monument was erected.

Mr.

III. QUESTION 327 *anſw*[illegible]

Let $AB = 2a$, $CD = 2b$, G[illegible] always be a known quantity) $=$ [illegible] $= 2y$, GV or $Gv = x$, OV or $Ov =$ [illegible] $\pm c$. Then, by the properties of [illegible] figure, $VO : AO :: AO : FO$, [illegible]

$x \pm c : a :: a : \frac{aa}{x \pm c} = FO$ [illegible]

$BF = a + \frac{aa}{x \pm c}$, $AF = a -$ [illegible]

and, by the properties of an ell[illegible]

$aa : bb :: \overline{a + \frac{aa}{x \pm c}} \times \overline{a -}$ [illegible]

$bb - \frac{aabb}{\overline{x + c}|^2} = EF^2$, therefore EF [illegible]

$= b\sqrt{1 - \frac{aa}{\overline{x \pm c}|^2}} = b\sqrt{\frac{\overline{x \pm c}|^2 -}{x \pm}}$ [illegible]

$- \frac{aa}{x \pm c} = \frac{\overline{x \pm c}|^2 - aa}{x \pm c}$: Now, by [illegible]

$: x :: b\sqrt{\frac{\overline{x \pm c}|^2 - aa}{\overline{x \pm c}|^2}} : bx\sqrt{}$ [illegible]

xy is a minimum by the queſtion, [illegible]

or $\frac{x^4}{xx \pm 2cx + cc - aa}$ is alſo a minimu[illegible]
is $2x^5\dot{x} \pm 6cx^4\dot{x} + 4c^2x^3\dot{x} - 4a^2$[illegible]
$2cc - aa = 0$, or $xx \pm 3cx = 2aa -$ [illegible]
$\sqrt{2aa + \frac{cc}{4}} \pm \frac{3c}{2}$, the theorem for fi[illegible]
angle.

Again, let $d = 3{\cdot}1416$, and [illegible]

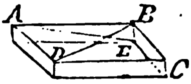
$\frac{x^3}{\overline{x \pm c}|^2 - a^2}$ is a minimum. In fluxions [illegible]
$3c^2x^2\dot{x} - 3|a^2x^2\dot{x} = 0$, $\therefore x^3 \pm$ [illegible]
or $x^2 \pm 4cx = 3a^2 - 3c^2$, whence $x =$ [illegible]
the theorem for finding the leaſt cone.

the ſides of the ſquares: Conſequently $\overline{\frac{24}{27}x+49}\times 9$ and $\frac{54}{27}-49\times 4=18^2$; from either of which equa- $=65$ the number of ſheep: And, conſequently, $\frac{24}{27}\times$ $49=\frac{31^2}{9}$, and $\frac{54}{27}\times 65-49=\frac{18^2}{4}$, whoſe ſquare are $\frac{31}{3}$ and $\frac{18}{2}$, or 10s. 4d. and 9s. the ſheep coſt a- in each parcel of 65; whence 65 × 10s. 4d. = 33l. d. one parcel coſt; and 65 × 9s. = 29l. 5s. the other lt: the true anſwer.

me method may be purſued with the factors 7 × 91 when the ſides of the ſquares will be 49 and 42, and mber of ſheep in each parcel 245; conſequently $\frac{49}{3}$ or 16s. 8d. and 21s. the ſheep in each parcel coſt and 257l. 5s. and 204l. 3s. 4d. the price of each being dearer, and therefore not the true anſwer.

UESTION 342 *anſwered by Mr.* Joſeph Orchard *of* Goſport.

$AB+AD=84=a$; $AE=60=d$; and $\frac{BE}{AD}$ $\cdot 28=r$. Let $AD=x$, then AB



x, and $Be=rx$: But AB^2+ $=AE^2$, i.e. $aa-2ax+xx+$ dd, $\because xx-\frac{2ax}{rr+1}=\frac{dd-aa}{rr+1}$,

$$=\frac{a-\sqrt{aa+dd-aa\times rr}}{rr+1}=24{\cdot}39$$ nearly, the h; and the length is 59·61; alſo depth 6·8292: whence tent $\left\{\begin{matrix}35{\cdot}208 \text{ ale}\\ 42{\cdot}982 \text{ wine}\end{matrix}\right\}$ gallons.

if by "*diagonal below*" is meant DB at the bottom of the ciſtern, then this is the ſolution:

$AB+AD=84=a$, $DB=60=d$; let $AD=x$. $B=a-x$: But $AB^2+DA^2=DB^2$ i.e. $aa-$ $xx=dd$; ſolved $x=\frac{a-\sqrt{2dd-aa}}{2}=36$ the

Mr. *Charles Maſon*, jun. of *Sapperton*, *Glouceſterſhire*, ſends us word, that the above ſolution was done by him, though ſent us in Mr. *Davis*'s name.

Some make the time long before creation, when there were no men to build; and others in the time of the firſt Chineſe emperors, who reigned before the European time of creation.

V. QUESTION 341 *anſwered by the propoſer, Mr.* Bevil, *of* Harpſwell, *near* Gainſborough, Lincolnſhire.

It happened, through haſte or inadvertency, that this queſtion wanted a word or two to make it properly underſtood, which are now ſupplied in contrary characters: *Two men bought an equal number of ſheep; and it being demanded of them what they gave a-piece for* their ſheep in *each parcel, it was anſwered, that, if the number of ſheep each of us bought be ſeverally multiplied by* $\frac{24}{27}$ *and* $\frac{54}{27}$, 49 *being reſpectively added to or ſubtracted from each product, both the ſum and remainder will be equal to the ſquare of the ſhillings given* a-piece *for* ſheep in each reſpective parcel. *How many ſheep did each perſon buy? And what did each parcel coſt,* at the cheapeſt price? for ſo every man would buy. Or, it had been better propoſed, *Two men bought an equal number of ſheep and hogs, &c* to diſtinguiſh one parcel, and the price of each hog and ſheep in each parcel, the better from one another.

Put $x =$ number of ſheep; then $\frac{24}{27}x + 49$ and $\frac{54}{27}x - 49$ are ſquare numbers, whoſe roots are the ſhillings a-piece the ſheep in each reſpective parcel (of different value, though equal number) coſt.

But a ſquare number, multiplied by a ſquare number, will produce a ſquare number. The expreſſions being multiplied reſpectively by 9 and 4, two ſquare numbers will be $\frac{216}{27}x + 441$, and $\frac{216}{27}x - 196$, whoſe difference is 637.

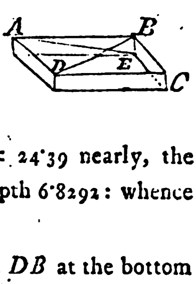
To find two ſquare numbers having that difference.

RULE. Reſolve the given difference into any two factors; then the half ſum and half difference of thoſe factors will be the ſides of the ſquares having the difference given: $637 = 13 \times 49 = 7 \times 91$. Therefore $\frac{49+13}{2} = 31$, and $\frac{49-13}{2} = 18$

will be the ſides of the ſquares: Conſequently $\overline{\frac{24}{27}x+49}\times 9 = 31^2$, and $\overline{\frac{54}{27}-49}\times 4 = 18^2$; from either of which equations $x = 65$ the number of ſheep: And, conſequently, $\frac{24}{27}\times 65 + 49 = \frac{31^2}{9}$, and $\frac{54}{27}\times 65 - 49 = \frac{18^2}{4}$, whoſe ſquare roots are $\frac{31}{3}$ and $\frac{18}{2}$, or 10s. 4d. and 9s. the ſheep coſt a-piece, in each parcel of 65; whence 65 × 10s. 4d. = 33l. 11s. 8d. one parcel coſt; and 65 × 9s. = 29l. 5s. the other parcel coſt: the true anſwer.

The ſame method may be purſued with the factors 7 × 91 = 637, when the ſides of the ſquares will be 49 and 42, and the number of ſheep in each parcel 245; conſequently $\frac{49}{3}$ and $\frac{42}{2}$, or 16s. 8d. and 21s. the ſheep in each parcel coſt a-piece, and 257l. 5s. and 204l. 3s. 4d. the price of each parcel; being dearer, and therefore not the true anſwer.

VI. QUESTION 342 *anſwered by Mr.* Joſeph Orchard *of* Goſport.



Given $AB + AD = 84 = a$; $AE = 60 = d$; and $\frac{BE}{AD} = \frac{7}{25} = {}^{\cdot}28 = r$. Let $AD = x$, then $AB = a - x$, and $Be = rx$: But $AB^2 + BE^2 = AE^2$, i.e. $aa - 2ax + xx + r^2x^2 = dd$, $\because xx - \frac{2ax}{rr+1} = \frac{dd-aa}{rr+1}$, and $x = \frac{a - \sqrt{aa+dd-aa\times rr}}{rr+1} = 24{\cdot}39$ nearly, the breadth; and the length is 59·61; alſo depth 6·8292: whence the content $\left\{\begin{matrix}35{\cdot}208 \text{ ale}\\ 42{\cdot}982 \text{ wine}\end{matrix}\right\}$ gallons.

But, if by "*diagonal below*" is meant DB at the bottom or top of the ciſtern, then this is the ſolution:

Given $AB + AD = 84 = a$, $DB = 60 = d$; let $AD = x$. Then $AB = a - x$: But $AB^2 + DA^2 = DB^2$ i. e. $aa - 2ax + 2xx = dd$; ſolved $x = \frac{a - \sqrt{2dd - aa}}{2} = 36$ the breadth; the length is 48; and depth 10·08; whence the content $\left\{\begin{matrix}65{\cdot}312 \text{ ale} \\ 75{\cdot}403 \text{ wine}\end{matrix}\right\}$ gallons.

Mr. *T. Cowper* ſolves it trigonometrically: As 60 : 84 :: coſ. 45° : coſ. 8° 8′ half the diff. of ∠s; conſequently the ∠s are 53° 8′ and 36° 52′: Whence the length 48; breadth 36; depth 10·08; and content 8·1 buſhels.

Mr. *Fiſh*, of *Dartford*, by a ſhort proceſs, ſolved this queſtion, and finds the length, breadth, and depth of the ciſtern, exactly as above.

Mr. *W. Bevil*, of *Harpſwell*; Mr. *Randle*, of *Wem*, *Shropſhire*; and Mr. *Samuel Smith*, of *Cambden*, *Glouceſterſhire*, alſo ſolved it: as did *Obadiah Wittam*, of *Whitby*; Mr. *Hopkinſon Farmer*; Mr. *William Cottam*, at his Grace the Duke of Norfolk's; and others.

VII. QUESTION 343 *anſwered by Mr.* Orchard.

Let $d = 10$ the difference between the ſide and diagonal of the ſquare; then $\overline{1 + \sqrt{2}} \times d =$ the ſide of the ſquare; two-thirds of the ſquare of which are 388·5618 &c. = the area of the octagonal garden: And, if x be the ſide thereof, then $xx \times 4{\cdot}8284$ &c. = 388·5618 &c. the ſaid area (*ſee Palladium for 51, p. 24*), ∵ $x = \sqrt{\dfrac{388{\cdot}5618 \text{ \&c.}}{4{\cdot}8284 \text{ \&c.}}} = 8{\cdot}9707$ poles, each ſide of the garden: And each ſide of the pond is $2\sqrt{\dfrac{5}{\sqrt{3}}} = 3{\cdot}398$ poles required.

Mr. *T. Cowper* anſwers it thus, very conciſely and elegantly: As $3 - 2\sqrt{2} : 10 :: 10 : \dfrac{100}{3 - 2\sqrt{2}} = 582{\cdot}842696$; two-thirds of which $= 388{\cdot}561797 =$ area of the octagonal garden; then $\sqrt{\dfrac{388{\cdot}561797}{4{\cdot}8284272}} = 8{\cdot}9707$ poles, the ſide thereof; and $3{\cdot}398 =$ ſide of the triangular pond.

Obadiah Wittam ſolved the ſame in an elegant manner; ſo did Mr. *Cottam*, at his Grace the Duke of Norfolk's, and ſeveral others.

VIII. QUES-

VIII. QUESTION 344 *anſwered by* Upnorenſis, *the propoſer, only.*

1. To find two ſuch ſquare numbers, whoſe roots may repreſent one leg and the hypothenuſe of a right-angled triangle; and the difference of thoſe ſquares to be a ſquare number, whoſe root may repreſent the other leg.

Put x for one leg, or ſide of the ſquare, $x+d$ for the hypothenuſe, or ſide of the other ſquare; then the ſquares will be denoted by xx and $xx+2xd+dd$, whoſe difference will be $2xd+dd=\overline{2x+d}\times d=yy$ for the ſquare of the other leg, by queſtion. It is evident, that $2x+d$ and d muſt be ſquare numbers. Let $2x+d=n$, then the leg $x=\frac{1}{2}\times\overline{n-d}$, and hypothenuſe $x+d=\frac{1}{2}\times\overline{n-d}$. Now, if $rr=n$, $ss=d$, then $\frac{1}{4}\times\overline{rr+ss}|^2-\frac{1}{4}\times\overline{rr-ss}|^2=rrss=yy$, per 47 E. 1. and, by tranſpoſition and reduction, $\overline{rr+ss}|^2=4rrss+\overline{rr-ss}|^2$. Whence we have this theorem: $2rs$ and $rr-ss$ will expreſs the legs of a right-angled triangle, and $rr+ss$ the hypothedufe, r and s being aſſumed any rational or whole numbers at pleaſure.

The ratio of the legs, as 7 to 11, being given ſo far as in whole numbers (for, exactly given, it would be no queſtion, and an impoſſible one, if the ſum of their ſquares were not a ſquare) by a trial or two, r will be found $=3\frac{1}{2}$, and $s=1$, by the theorem; when the complete ratio of the legs will be 7 to $11\frac{1}{4}$, the neareſt to the given numbers, and the correſponding hypothenuſe as $13\frac{1}{4}$, four times which values will be 28, 45, and 53, the ſides of the leaſt right-angled triangle, in whole numbers, required. See p. 186 of Dodſon's Mathematical Repoſitory, requiring two numbers in the complete ratio of 8 to 15, the ſum of whoſe ſquares ſhall be a ſquare number; where the required is given, and a ſuperfluous theorem that finds the numbers 576 to 1080, being 72 times 8 to 15; whereas 2, 3, 4, 5, &c. 8 to 15, had been a direct anſwer. And there was no way to propoſe this queſtion, but as it was propoſed, without giving what was required (or to the ſame effect) or elſe propoſing an impoſſibility.

N. B. The foregoing anſwers a ſcandalous and falſe advertiſement in the London Gazetteer of Dec. 13, 1750.

IX. QUESTION 345 *anſwered by* Upnorenſis.

The bookſeller buys an annuity of 25l. a year, to continue five years, for 73l. ready money—To find his gain per cent. according to the allowance of compound intereſt.

$$\text{Let}\left\{\begin{array}{c} a \\ r \\ t \\ z \end{array}\right\}\text{ſignify}\left\{\begin{array}{l}\text{the annuity.}\\ \text{1 l. and its intereſt for one time or year.}\\ \text{the number of times the annuity is to be paid.}\\ \text{the whole amount of the annuity's preſ. worth.}\end{array}\right.$$

Say,

$r : 1 :: a : \frac{a}{r}$ preſent worth of a payable at the end of 1ſt time.

$: 1 :: \frac{a}{r} : \frac{a}{r^2}$ preſent worth of a payable at the end of 2d time.

$r : 1 :: \frac{a}{r^2} : \frac{a}{r^3}$ preſent worth of a payable at the end of 3d time.

Confeq. $\frac{a}{r^t}$ preſent worth of a payable at the end of t^{th} time.

$$\left.\begin{array}{l}\text{The ſum of all which}\\ \text{progreſſionals } \frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3},\\ \text{\&c. } \frac{a}{r^t}\text{, multip. into } a,\end{array}\right\} \frac{\overline{r^t - 1} \times a}{\overline{r - 1} \times r^t} = z \left\{\begin{array}{l}\text{the whole preſent}\\ \text{worth of all the}\\ \text{payments of } a,\\ \text{from 1 to } t \text{ times.}\end{array}\right.$$

For, if z be the greateſt term, a the leaſt, and r the ratio, or any term decreed by the next leſſer, $\frac{rz - a}{r - 1} = s$ the ſum, univerſally.

The aforeſaid equation reduces to $r^t + 1 - \frac{a + z}{z} r^t + \frac{a}{z} = 0$; in which, according to Dr. Halley, if t the number of years be great, 40 or upwards, and the rate of intereſt be high, $1 + \frac{a}{z}$ will be nearly, or more accurately $\frac{z + a}{z} - \frac{z^t}{z + a} \times \frac{a}{z}$, the value of r, when $\frac{a}{r^t \times \overline{r - 1}}$ will be exceedingly near the value of the reverſion, which, if it be called x, then $1 + \frac{a}{z + x}$ will approach the value of r ſufficiently. *See Dr. Halley's method, at p.* 33 *and* 34 *appendix to Sherwin's Logarithms.* But t the years being ſmall, this rule fails; in which caſe, if $\overline{\frac{at}{z}}\Big|^{\frac{2}{t+1}} - 1 = y$, and $\frac{6}{t-1} = b$, then $1 + b - \sqrt{bb - 2by}$ is ſufficiently equal to r, and will ſtill be nearer the truth, as t the years be of the ſmaller value, the ſmall error being always in exceſs; viz. $r = 1 + b - \sqrt{b \times b - 2 \times \overline{\frac{at}{z}}\Big|^{\frac{2}{t+1}} - 1}$. And, putting capitals for the logarithms of quantities denoted by ſmall letters, then $2 \times \frac{A + T - Z}{t+1} = D$, and $\frac{B + L.\overline{b - 2 \times \overline{d - 1}}}{2} = E$, $\because\ r = 1 + b - e$; and conſequently $r - 1 = b - e = .21094$ &c. the rate of intereſt of 1 l. per year, and 21l. 1 s. 10½d. for 100l. a year, bookſeller's profit.

Mr. *Flitcon* elegantly conſiders z paid down as a principal put to intereſt, whoſe account at firſt year's end $= zr$; when, a becoming due, $zr - a$ is the principal running on; which, drawn into r, is $zr^2 - ar$ the amount at ſecond year's end; and, a being again paid, $zr^2 - ar - a$ will be the principal running on; which at the end of five years will be $zr^5 - ar^4 - ar^3 - ar^2 - ar - a = 0$ principal running on: the bookſeller then being repaid all his money at firſt laid out, with the intereſt thereof running on as a principal; conſequently the value of r in this equation ſhews the rate of intereſt as before.

N. B. The ſum of all the terms, except the firſt, $= a \times \frac{r^5 - 1}{r - 1}$, by the univerſal rule aforeſaid for ſumming geometrical progreſſions; and $\because\ zr^5 - a \times \frac{r^5 - 1}{r - 1} = 0$, which reduces to $zr^6 - \overline{z + a} \times r^5 + a = 0$ the ſame equation with the firſt.

Mr. *Terey* of *Portſmouth*, Mr. *Alexander Rowe*, Mr. *John Honey* of *Redruth*, *Cornwall*, and ſome others, ſolved this queſtion.

The PRIZE QUESTION *anſwered by the Propoſer.*

Put $t =$ co-tang. 51° 32′, $T =$ co-tang. 52° 20′, $x =$ ſine, and $y =$ coſ. of the ☽'s aſcenſional diff. in lat. 51° 32′; $s =$ ſine, and $c =$ coſ. of the diff. between the aſcen. diff. $= 1°$; then will $cx + sy =$ ſine of aſcenſ. diff. in lat. 52° 20′. Now, by ſpheric trigon. as 1 : t :: x : tx = tang. ☽'s declination. Again, 1 : T :: $cx+sy$: $Tcx+Tsy$; hence $tx=Tcx+Tsy$; i. e. $\frac{y}{x}=\frac{t-Tc}{Ts}$ = 1·6808489 the co-tang. of ☽'s aſcenſ. diff. (in lat. 51° 32') = 30° 45'. Hence her declin. is found = 22° 6½'. Then, to right aſcen. ☉ 341° 12' add 250° 30' (= time ☽'s ſouthing) the ſum, rejecting 360°, will be 231° 42' = right aſcen. ☽ at ſouthing. By the rule of proportion, the diff. of right aſcen. from ☽'s riſing to ſouthing = 1° 58½'; conſequently her right aſcen. at riſing, in lat. 51° 32', is = 229° 43½'. Thus, having her right aſcen. and declin. I find her true place to be ♏, 23° 7' 11", lat. 3° 38' 27" S.

Obadiah Wittam, of *Whitby*, (whoſe letter followed one we received from Mr. *Harland Wid* on the ſame ſubject) makes the moon's longitude ♏ 23° 44' 30", and her lat. 3° 29' 30" S. which is not the truth: Conſequently Mr. *T. Cowper*, the propoſer, claims the prize, he having no competitor.

We approve Mr. *William Cottam*'s method, at his Grace the Duke of Norfolk's, who makes ☽'s long. ♏ 22° 36' 25", and lat. 2° 55' 23" S. allowing for the moon's parallax and refraction at riſing—whoſe anſwer had been very near the propoſer's, had he not made a ſmall overſight in his tabular computation.

The Prize of 12 *Diaries was won by Mr.* T. Cowper, *and that of* 8 *by Mr.* C. Maſon.

The Eclipſes calculated for 1752, *by Mr.* Ralph Hulſe.

There will be but two eclipſes of the ſun for this preſent and both inviſible, as follows:

1. On Saturday May 2d, at 6h. at night. At the Bay of Honduras, in North America, the ſun will be totally eclipſed in 23° of ♉. The beginning 4h. 30m. Middle 6h. 0m. End 7h. 34m. Total duration 3h. 4m. Digits eclipſed 12° 8'.

2. On Friday November 17th, at 2 morn. At Carpentaria, in South America, at 6h. 6m. in the morning, the ſun is ſeen eclipſed 14° of ♏ lat. 16° N. long. 148°. E. 9h. 52m. where it will appear very formidable to its inhabitants. The begin. 0h. 28m. Middle 2h. 2m. End 3h. 36m. Total Dur. 3h. 6m. Digits eclipſed 12° 4'.

	Beg.	Mid.	End	Dur.	
Mr. Al. Man, London, May 2d		5 52			
Jamaica, Nov. 17th	11 44	1 20	2 48	3 4	10 45
Boſton ———	10 30	1 44	2 50	2 20	6 15

Mr. *John Child*, of *Barnet*, *Hertfordſhire*, ſends us his obſervations on the moon's eclipſe that happened on ſunday Dec. 2d, 1750, in the morning, by a clock exactly ſet.

		h.	m.	
Barnet, Hertford.	Beginning — —	4	36½	app. time.
	Beg. total darkneſs	5	36	
	End of tot. darkneſs	7	14¾	
	Dark — — —	1	38¾	

By which our aſtronomical tables may be proved.

Of the Alteration of the Style.

By 365 days, 6 hours, the mean Julian year, being long reckoned for 365 d. 5 h. 48 m. 54 s. 41 th. and 27 fourths, the year by the ſun, according to Dr. Halley, (ſee Palladium, 1750, p. 53.) The account of time has each year run a-head of time by the ſun 11 m. 5 s. 18 th. 33 fourths, or 44 m. 21 s. 14 th. 12 fourths, every 4 years, and conſequently 3 d. 1 h. 55 m. 23 s. 40 th. in 400: And ſo from the council of Nice, when the kalendar was ſettled, in the year 325, to this preſent year 1752, being 1427 years, the time by account is forward of that by the ſun 10 d. 23 h. 43 m. and therefore 11 days is left out of account in this month [*September the 3d being accounted the 14th day*] as the moſt convenient for reducing the kalendar or year to its firſt eſtabliſhed order. And for keeping the ſhorteſt and longeſt days (or the ſolſtices) and alſo the days of 12 hours long (or the equinoxes) on the ſame nominal days of the month for the future, it is ordained by act of parliament, that every fourth hundred year is to conſiſt of 366 days as uſual, but all other whole hundred years of 365 days only: The years between which whole hundreds to be common and biſſextile as formerly, and the date of the year henceforward to begin on the firſt of January [*inſtead of the 25th of March.*]

New Queſtions.

I. QUESTION 347, *by Mr.* T. Cowper *of* Wellingborough, *Surveyor.*

By a meand'ring limpid brook,
In the blithe month of May,
Early one morn a walk I took,
And did ſome land ſurvey:

Triangular its form I found,
The baſe, then meaſur'd o'er,
On horizontal verdant ground,
Made perches thirty-four.
From midſt the perpendicular,
The baſe's ends I view,
And find the angle forming there,
Degrees juſt ninety-two.
The vertex angle I behold
Juſt fifty-five degrees,
From whence the unknown ſides are told,
And acres, if you pleaſe.

II. QUESTION 348, *by Mr.* James Terey *of* Portſmouth.

The greateſt ſpheroid, and parab'lic conoid,
Inſcrib'd in a cone are by art,
From whence as below*, the contents you're to ſhow,
Of each ſeparate ſolid apart.

* *Diam. of cone's baſe* = 35 *inches, and its altitude* = 30 *inches.*

III. QUESTION 349, *by Mr.* Obadiah Wittam, *of* Whitby.

On what two days of the year 1752 will the ſun riſe at the ſame inſtant of time at Peterſburgh and Jeruſalem?

IV. QUESTION 350, *by Mr.* William Honnor.

Required a theorem for determining the length of a lever of the firſt kind (ſuppoſed of no weight) capable of being divided into two brachias, *y* the greater, and *x* the leſſer, ſo that $y^m - x^m = y^n \times x^n$; on whoſe ends two given weights being ſuſpended, *w* the greater, and *v* the leſſer, ſhall equipoize each other?

V. QUESTION 351, *by* Taptinos.

In a right-angled triangle, there is given the diſtance from the angle at the baſe to the center of an inſcribed circle 4 chains; and if it be prolonged 2 chains further it will touch the cathetus: To find the ſides?

VI. QUESTION 352, *by Mr.* Randles.

A gentleman has an orchard of fruit trees, one-half of the trees bearing apples, one-fourth pears, one-ſixth plumbs, and 50 of them bearing cherries: How many fruit trees in all grow in the ſaid orchard?

VII. QUESTION 353, *by* Taptinos.

In a plain triangle there is given the rectangle of the ſides 195, the rectangle of the ſegments of the baſe 45, and the perpendicular 12; to find the ſides.

VIII. QUESTION 354, *by* Philotheros.

Given the area of the greateſt trapezium that can be inſcribed in an apolonian parabola, whoſe abſciſſa and ſemiordinate are as 3 to 2, equal 256: Required the dimenſions of the parabola and trapezium by a ſimple equation.

IX. QUEST. 355, *by* Anagramenſis Holy in Heart, Ebor.

There are two cities in the ſame parallel of latitude, whoſe difference of longitude is 144° 15′, and their diſtance in the arch of a great circle 6797·4 ſtatute miles: Required their latitude, and what day of the year the ſun riſes to the one city exactly at the ſame time he ſets to the other?

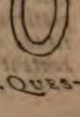
X. QUESTION 356, *by Mr.* John Williams, *of* Mold, *in* Flintſhire.

A gentleman would make a corn mill to be turned by a current of water that runs a tun in a minute, and has 16 feet fall or perpendicular deſcent: It is required to know the diameter of the water-wheel, ſo that the iſſuing water may give the wheel the greateſt power, or force poſſible?

XI. QUESTION 357, *by the Rev. Mr.* Baker, *of* Stickney, Lincolnſhire.

Let $ABCE$ repreſent a compound barometer, filled with mercury from B to C, and with water from C to E: How then muſt the bores of the two tubes $ABCF$, and FEK be adjuſted, or proportioned, ſo that the ſcale of variation in the leſſer tube of this barometer may be to the common ſcale, as 10 to 1? [*See the fig. to the ſolution.*]

XII. QUESTION 358, *by* Upnorenſis.



To determine the ſolidity and ſuperficies of an elliptical ring*, of any dimenſions (c the conjugate and t the tranſverſe inches) the ſubſtance filling whoſe periphery is circular of p inches diameter?

XIII. QUES-

XIII. QUESTION 359, *by* Honorius.

Miſs's apron grown ſhort, ſhe is full of complaint,
And to merit your pity ſhe looks like a ſaint!
On the floor falls her tea; then her ſcreams you may hear,
And fainting ſhe ſinks in a fit on the chair.
Mamma for the doctor immediately ſends,
Who, in honour to miſs, in his chariot attends;
He examines her pulſe, and appearing ſo wiſe,
Deſcants on the languiſhing looks of her eyes—
But alas! neither ſpirits, nor letting miſs blood,
Specifics, nor preaching, are found to do good:
For a ſurgeon came in, who the cauſe did declare,
And the doctor's fineſſe, and his art made appear.
Mamma now was told Miſs's hoop was too ſmall,
Therein lay her grievance, diſorder, and all;
The queſtion was aſk'd—Polly ſighing, reply'd,
A French hoop will cure me, and ſo will a bride.
A hoop of the faſhion to cure her diſeaſe,
Extends from her center quite round to her knees:
In the right and left wing a French placket * is made,
To her elbows advancing, and forms a parade.
Miſs Polly to church now, or play can repair,
And wherever ſhe goes is admir'd for her air!
At the ſight of a beau, how her heart beats alarms!
While the winds ſwell her pride, and her legs tell their charms:
Her hidden perfections ſhe knows will invite,
Or enſnare the beholder, ſhould chance give them ſight.
By the pow'r of her hoop Polly ſteps into fame,
By out-priding the reſt ſhe conceals her own ſhame;
In the country ſhe reigns o'er the 'ſquire and the clown,
O'er the lords and the fops ſhe's triumphant in town.
Her hoop is the ſecret—and if you would know
What it holds with her petticoat, ſeek from below †.

* *Opens and ſhuts, forms a pair of bellows, and riſes and falls by the means of ſtrings or bowlings.*

† *Form of the hoop is the lower fruſtum of an ellipſoid, with its vertex next the head.*

$$\left.\begin{array}{l}\textit{Tranſverſe}\\ \textit{Conjugate}\end{array}\right\}\textit{diams.}\left\{\begin{array}{l}42\ \textit{inches}\\ 26\end{array}\right\}\textit{above,}\left\{\begin{array}{l}48\ \textit{inch.}\\ 29\end{array}\right\}\textit{below.}$$

Altitude of the fruſtum 12 inches.

From the lower part of the hoop's circumference to the bottom of the petticoat, the form is an elliptic cylinder, by the petticoat hanging nearly perpendicular from thence; the altitude of which elliptical cylinder is 18 inches: Quere the content of the whole concavity in wine gallons?

The

The PRIZE QUESTION, *by* χρονονμονοντπύϐλικος.

Archimedes, the renowned mathematician of Sicily, once bathing himſelf, obſerved the water to riſe ſo much higher on his going into the bath. It was from thence he firſt took the hint for meaſuring the ſolidities of all irregular bodies, not meaſurable by the known rules of art; and alſo for determining the different ſpecific gravities of bodies. For, being tranſported with the diſcovery, he came out of the bath, forgetting he was naked, and ran home, crying out, Εὕρηκα, Εὕρηκα, ſignifying, *I have found it*; and, afterwards, diſcovered the quantity of ſilver mixed with the gold in King Hiero's crown, which the workman confeſſed.

It is propoſed to determine by the beſt method, the neareſt ſuperficial content in inches of a modern mathematician, of a middle age, weighing 160 pounds avoirdupois, being naked, all his parts middle-ſized, and meanly proportioned; and his muſcles not rigidly ſwelled, nor yet quite unbraced?

N. B. The ſame rule will hold good for male or female menſuration; and man and woman being microcoſms, expreſſions of many elegant and uſeful curves may thence be diſcovered; and ſeveral improvements made in the rectifications of curve lines, and quadrature of curvilinear ſpaces; beſides cubation of ſeveral important ſolids; whoſe forms of fluxions, with their fluents, we ſhall inſert in our new Harmonia Menſurarum.

New Paradoxes.

PARADOX I. *by Mr.* Honnor.

A truſs of hay weighing but half a hundred weight in a ſcale, weighed two hundred weight ſtuck upon the end of a fork carried on Hodge's ſhoulder: How could that be?

PARADOX II. *by Mr.* James Collingridge.

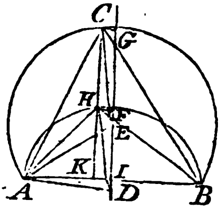
How can a mechanic file a ſquare hole with a round file? and fill up an oval hole with a round ſtopper?

1753.

Queſtions anſwered.

I. QUEST. 347 *anſwered by Mr.* Terey *of* Portſmouth.

LET ABC be the triangle; then on the baſe AB (per 33 Euc. 3) deſcribe two ſegments of a circle, ACB and AHB, containing the given angles; through the centers D, E draw DG; alſo draw DH, EC to the ⊥ CK; and let fall CG and HF ⊥s to GD. By trigonometry, $AD = DH = 17{\cdot}0104 = a$; $AE = EC = 20{\cdot}7531 = b$; $ID = {\cdot}593651 = c$; $IE = 11{\cdot}9035 = d$. Put $x = CK$, then (by 47 Euc. 1) $aa - cc - cx - \frac{1}{4}xx = bb - dd + 2dx - xx = CG = HF$; hence $x = \frac{2c + 4d \pm \sqrt{3 \times \overline{b-a} \times \overline{b+a} + \overline{2c+d}^2}}{3} = 32{\cdot}5340 = CK$; whence $KI = 2{\cdot}2524$; $AC = 35{\cdot}7204$; $CB = 37{\cdot}8036$; and the area $=$ 3 a. 1 r. 33·078 p. required.



Mr. *Widd*'s ſolution agrees. Mr. *F. Holden* ſolves it by the ſame method, and brings out the ſame correct numbers; with which Mr. *T. Cowper*'s ſolution, by another method, exactly agrees. *Philotheros* ſolved it.

The ſame anſwered by Nichol Dixon *of* Blackwell.

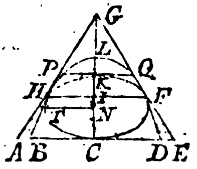
Put $t =$ tangent ∠ $ACB = 55°$, and $T =$ tangent ∠ $AHB = 92°$, and $2b = AB = 34$ poles, $y = HK = HC$, $x = KI$, the diſtance from the ⊥ CK to the middle of the baſe. Then, by trigonometry, as $y : 1 :: b + x : \frac{b+x}{y} =$ tangent ∠ KHB; and $y : 1 :: b - x : \frac{b-x}{y} =$ tangent angle AHK: Now, by prob. 8 p. 21 of Mr. Emerſon's excellent

Trigonometry, As $1-\frac{b^2-x^2}{y^2} : 1 :: \frac{2b}{y} : \frac{2by}{y^2-b^2+x^2} = T$; and, by the ſame reaſoning, $\frac{4by}{4y^2-b^2+x^2} = t$; from which equations $y = \frac{4b}{3t} - \frac{2b}{3T} = \overline{\frac{2}{t}-\frac{1}{T}} \times \frac{2}{3}b$: But rad. divided by the tangent = co-tangent: Therefore $y = \overline{2\times\text{co-tangent } 55^\circ - \text{co-tangent } 92^\circ} \times \frac{2}{3}b = 16{\cdot}2666$; hence $AC = 35{\cdot}718$, $CB = 37{\cdot}804$, and area = 3a. 1r. 33·06 p. &c.

Mr. *Joſeph Orchard*, of *Goſport*, putting b and x as above, tang. $\angle AHB = 92^\circ = -v$, tang. $\angle ACB = t$, rad. 1, makes the tangents of the reſpective angles $\frac{2by}{yy-bb+xx} = -v$, and $\frac{4by}{4yy-bb+xx} = t$, the equations brought out of fractions, the former multiplied by t and the latter by v; and, taking the ſum of both, we get $\overline{2v+t}\times 2b = 3vty$, whence $y = \frac{2v+t}{3vt}\times 2b = 16{\cdot}2665$, whence the area as before.

Mr. *Bevil* ſolves it by the ſame method, whoſe numbers exactly agree; as do the ſolutions of Mr. *Stephen Hartley*, Mr. *Cottam*, at his Grace the Duke of Norfolk's, Mr. *Cha. Smith*, Mr. *Richard Gibbons*, Mr. *John Wiggleſworth*, Mr. *John Croſs*, and ſome others.*

II. QUESTION 348 *anſwered by Mr.* J. Orchard, *Writing-maſter and Teacher of the Mathematics at* Goſport.



Let $GC = a = 30$ inches, $AE = 2b = 35$ inches, $m = {\cdot}7854$, and $KC = x$. Then $KG = a-x$; and per ſim. △s GCA, GKP, $a : b :: a-x : \frac{a-x}{a}\times b = PK$; per conics, $PK \times AC = TN^2 = bb \times \frac{a+x}{a}$. Then $8mbb \times \frac{ax-xx}{3a}$ = the ſolidity of the ſpheroid, which, or $ax - xx$, is a maximum. In fluxions $aa\dot{x} - 2x\dot{x} = 0$, whence $x = \frac{1}{2}a$. Again, let $GL = LI = x$; per ſim. $\triangle$s GAC and GHI, $a : b :: 2x : \frac{2bx}{a} = HI$; per conics $x : \frac{4bbxx}{aa} :: a - x\ (= LC) : 4bb \times \frac{ax - xx}{aa} = BC^2$; then $\frac{8mbbx \times \overline{a-x}|^2}{aa}$ is the conoid's ſolidity, which, or $aax - 2axx + x^3$, is a maximum. In fluxions $aa\dot{x} - 4ax\dot{x} + 3x^2\dot{x} = 0$; whence $x = \frac{1}{3}a$. Theſe values of x ſubſtituted in the maximums, and dimenſions, by proper theorems for the ſegments and fruſtums, the content of each ſolid, with the ratio they are in, are exhibited in the following table, by Mr. *Joſeph Orchard*.

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* A conſtruction is given in p. 313 Britiſh Oracle.

Generating lines from the cone's axis.	Ratio of each ſolid to the cone.	Content of each ſolid.
GHL	4	712·677
LHK	5	890·847
KHC ſpheroid.	27 } : 54	4810·575
CHBC	16	2850·712
AHB	2	356·338
ſum = whole cone.	54	9621·15

Philotheros ſolved it. Mr. *Terey*'s ſolution: He puts $AE = b = 35$, $CG = a = 30$, and $z = \cdot 78539$, &c. when the cone's ſolidity $= \frac{1}{3}bbaz = S = 9621\cdot 0274$.

1. Then put $CK = x$: As $a : b :: a - x : \frac{a-x}{a}b = PQ$; but $PQ \times AE = \square$ diam. ſpheroid; whence its content $= \frac{bbax - bbxx}{a} \times \frac{2}{3}z$, is a max. when in fluxions, $a\dot{x} - 2x\dot{x} = 0$, and $x = \frac{1}{2}a$, by writing which value in the above expreſſion, its content $= \frac{1}{2}$ of $S = \frac{1}{2}$ content of the cone.

2. *To find* CI. (N is the center of the ſpheroid) $NG : NK :: NK : NI$; i. e. $\frac{3}{4}a : \frac{1}{4}a :: \frac{1}{4}a : \frac{1}{12}a = NI$. Whence $CI = \frac{1}{3}a$, $IK = \frac{1}{6}a$. By fluxions, content of the ſpheroidal ſegment $= \frac{2ccxx}{t} - \frac{\frac{4}{3}ccx^3}{tt} \times z$; for t put $\frac{1}{2}a$; for cc, $\frac{1}{2}bb$; and for x, $\frac{1}{6}a$; then the ſegment $HKF = \frac{7}{54}$ of S, and ſegment $HCF = \frac{10}{27}$ of S.

3. *To find the greateſt parab. conoid* BLD. Let $LC = x$, then $LG = LI = a - x$; ſay, $a : b :: 2a - 2x : \frac{2ab - 2bx}{a} = HI$, and per known property, $a - x : 2b \times \frac{a - x}{a} \times 2b \times \frac{a - x}{a} :: x : \frac{4bbax - 4bbxx}{aa} = BD^2$; whence the content $BLD = \frac{2aabb - 2bbx^3}{aa} \times z$, a max. from whence, by fluxions, $x = \frac{2}{3}a = CL$, and $LG = \frac{1}{3}a$; conſequently, the contact, in this caſe, cuts off $\frac{1}{3}$ of the axis, viz. IC, the the ſame as of the ſpheroid: For x put $\frac{2}{3}a$, and the conoid $BHLFD = \frac{8}{9}$ of S.

4. Conoid $HLF = \frac{4}{9}bb \times \frac{1}{6}az = \frac{2}{9}$ of $S = 2139·006$.

5. (3 and 4) $BHLFD - HLF = BHFD = \frac{2}{3}$ of $S =$ 6414·0182.

6. Cone $HGF = \frac{8}{27}$ of S: Conſequently, fruſtum $AHFE = \frac{19}{27}$ of $S = 6770·3526$.

7. (4 and 6) cone HGF — conoid HLF = ſolid $HGFL = \frac{2}{27}$ of $S = 712·6687$.

8. (2 and 4) conoid HLF — ſegment HKF = ſolid $HKFL = \frac{5}{27}$ of $S = 890·8358$.

9. (2 and 5) fruſtum $BHFD$ — ſegment HCF = ſolid $HCFDB = \frac{8}{27}$ of $S = 2850·6748$.

10. (5 and 6) fruſtum $AHFE$ — fruſtum $BHFD$ = ſolid $HABDEF = \frac{1}{27}$ of $S = 356·3343$. *W. W. R.*

Mr. *Thomas Cowper*'s ſolution and numbers are very correct, as they always are: So is Mr. *James Hartley*'s ſolution, Mr. *Bevil*'s, Mr. *F. Holden*'s, Mr. *Charles Smith*'s, Mr. *William Cottam*'s, and thoſe by ſome others, who are every one complete artiſts.

III. QUEST. 349 *anſwered by Mr.* T. Cowper, *Teacher of the Mathematics, at* Wellingborough.

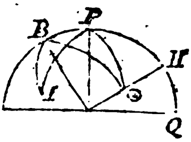
By Dr. Halley's aſtronomical tables, lat. of Peterſburgh = 60°, lat. of Jeruſalem = 31° 55′, their diff. of long. = 5°. Put a = tang. 60°, b = tang. 31° 55′, and s and c = ſine and coſ. 5°, alſo x = tang. ſun's declination. Then, by ſpherical trigonometry, $1 : a :: x : ax$ = ſine aſcenſional diff. at

Peterſburgh; coſ. $= \sqrt{1-a^2x^2}$; alſo $1 : b :: x : bx =$ ſine aſcenſional diff. at Jeruſalem. But $acx - \sqrt{s^2 - a^2s^2x^2} = bx$, or $\overline{ac-b} \times x = s\sqrt{1-a^2x^2}$: Therefore, $a^2c^2x^2 - 2abcx^2 + b^2x^2 = s^2 - a^2s^2x^2$, or (becauſe $1 = s^2 + c^2$) $a^2x^2 - 2abcx^2 + b^2x^2 = s^2$; whence $x^2 = \frac{s^2}{a^2+b^2-2abc}$; But writing $v =$ ſine ſun's declination, we have $\frac{v^2}{1-v^2} = \frac{s^2}{a^2+b^2-2abc}$, or $2v^2 = \frac{2s^2}{a^2+b^2-2bac} = \cdot 0121914$ = the verſed ſine of 8° 57′ 21″ = twice the ſun's declination; conſequently the ſun's declin. = 4° 28′ 40½″, correſponding to the 26th of February and the 20th of March, and the 31ſt of Auguſt and the 23d of September, O. S. *W. W. R.*

Mr. *John Peachy* found the ſame: Mr. *Thomas Allen*, of *Goſberton* ſchool, *Lincolnſhire*, ſolved it: Alſo *Philotheoros.*

N. Dixon's *Anſwer.*

Let *B* be Peterſburgh, *I* Jeruſalem, *P* the pole, by Gordon's Geographical Grammar, $BP =$ 38° 35′; $PI =$ 57° 16′; $\angle BPI =$ 5° 25′, the diff. long. Let $B\odot = 90°$ be perpendicular to BI, then $\odot$ is the place of the ſun at riſing. In the triangle BPI, is given BP, PI, and $\angle P$, to find the $\angle B =$ 169° 54′, from whence take 90°, and you have $\angle PB\odot$. Then in the quadrantal triangle $BP\odot$ there is given BP 30° 35′, $\angle B$ 79° 54′, to find $P\odot$ the comp. of the ſun's declin. = 84° 53′, whence the ſun's declin. = 5° 7′. And the days correſpondent thereto are the 22d of March and the 28th of Auguſt, O. S. on which the ſun will riſe at both places nearly at the ſame time. *W. W. R.*



Mr. *Widd*, the propoſer's ſolution, gives the ſame days.

Mr. *James Hartley*, of *Yarum*, ſolves it thus: Lat. Jeruſalem = 32° 30′, tang. = t; lat. Peterſburgh = 60° 4′, tang. = T; diff. of merid. = 3° 30′; let its ſine = s, rad. = 1, and x = tang. ſun's declin. Then $1 : t :: x : tx$. Again, $tx + s : x :: T : 1$, whence $\frac{s}{T-t} = x =$ 3° 10′ 40″, anſwering to the 1ſt and 17th of March, and the 3d and 19th of September, O. S. *W. W. R.*

Mr. *Cottam* brings out the ſame days, and ſo does Mr. *F. Holden*, by making the ſun's declination 3° 8′ 15″. For we did not give the latitudes of Jeruſalem and Peterſburgh that we might ſee the anſwer from different authorities of thoſe latitudes. A gentleman (without name) with a large figure ſtuck on with a wafer, alſo ſolved it 5° 6′ 44″ declination.

IV. QUESTION 350 *anſwered by Mr.* James Hartley *of* Yarum.

Take $R = \frac{v}{w} =$ the ratio of the given weights. Then, $Ry = x$; and the given equation will become $y^m - R^m y^m = R^n y^{2n}$, which being reduced, we have the following theor.

When
- m is greater than $2n$, $\overline{\frac{R^n}{1-R^m}}\Big|^{\frac{1}{m-2n}} = y$.
- $m = 2n$, $R^m + R^n = 1$. Here R is limited, and y indeterminate.
- m is leſs than $2n$, then $\overline{\frac{1-R^m}{R^n}}\Big|^{\frac{1}{2n-m}} = y$.

Mr. John Honey, *of* Redruth, *in* Cornwall, *ſolves it thus:*

Put $N =$ length of the lever, and $s = w + v$, then $x = \frac{Nv}{s}$, and $y = \frac{Nw}{s}$; alſo $\frac{N^m w^m - N^m v^m}{s^m} = \frac{N^{2n} w^n v^n}{s^{2n}}$, per queſtion; reduced $N = \overline{\frac{s^{2n-m} \times w^m - v^m}{w^n v^n}}\Big|^{\frac{1}{2n-m}} =$ length required.

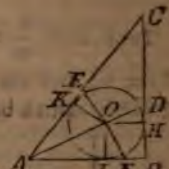
Mr. *Charles Smith*'s theorem is $\overline{\frac{w^n}{v^n} - \frac{v^{m-n}}{w^{m-n}}}\Big|^{\frac{1}{2n-m}} + \overline{\frac{w^{m-n}}{v^{m-n}} - \frac{v^n}{w^n}}\Big|^{\frac{1}{2n-m}} = y + x$. Who ſays, if $m = 3$, and $n = 2$, that $y + x = \frac{w}{v} \times \overline{1 + \frac{w}{v}} - \frac{v}{w} \times \overline{1 + \frac{v}{w}}$.

Mr. *F. Holden*, Mr. *Priſſon*, Mr. *Rob. Butler*, *Philotheoros*, and ſome others, conciſely ſolved the ſame.

V. QUES-

V. QUESTION 351 *anſwered by Mr.* Joſeph Orchard, *of* Goſport.

Geometrical conſtructions of problems being always valuable, where they are to be had, we inſert the conſtruction firſt, as follows: Draw $AD = 6$, making $AO = 4$, and $OD = 2$; through O draw EF, at right angles to AD, making $OE = OF = OD$; draw AB and AC through F and E, and BC through D perpendicular to AB. Then ABC will be the triangle required, as is evident; which is the greateſt of demonſtration.



Calculation: In $\triangle AOF$ we have $AO = 4$, $OF = 2$, whence $AF = \sqrt{20} = 2\sqrt{5}$. Per ſim. triangles, $AF : AO :: AD : AB = \frac{12}{\sqrt{5}}$. Per trigon. tang. $\angle FAO = \frac{1}{2}$, and (per ſchol. to prop. 2 of Mr. Emerſon's Elements of Trigonometry) tang. $\angle BAC =$ tang. $2\angle FAO = \frac{4}{3}$, $\because 1$ (rad.) : $\frac{12}{\sqrt{5}}$ (AB) :: $\frac{4}{3} : BC = \frac{16}{\sqrt{5}}$, and $AC = \frac{20}{\sqrt{5}}$; the ſides are in arithmetical progreſſion, and are $AB = 5{\cdot}366$, $BC = 7{\cdot}155$, and $AC = 8{\cdot}944$. *W. W. R.* With which Mr. *T. Allen*'s, of *Goſberton* ſchool, in *Lincolnſhire*, and Mr. *Widd*'s ſolutions agree.

Mr. *Thomas Cowper*, Teacher of the Mathematics, at *Wellingborough*, computes thus: Put $x =$ ſine, and $y =$ coſine $\angle DAB = \angle DOH$; then $1 : 4 :: x : 4x = OI = OH$. And $1 : 2 :: y : 2y = OH$, $\because \frac{x}{y} = \frac{2}{4} = {\cdot}5$, the tangent of $\angle OAI = 26^\circ\ 33'\ 54'2''$; whence $AB = 5{\cdot}3665626$, $BC = 7{\cdot}1554168$, and $AC = 8{\cdot}944271$. *W. W. R.*

Mr. *Cottam* anſwers it elegantly in the ſame numbers; as likewiſe does Mr. *John Williams*, Mr. *James Hartley* of *Yarum*, Mr. *William Bevil*, Mr. *Robert Butler*, Mr. *Hollingſworth*, Mr. *F. Holden*, Mr. *Richard Gibbons*, Mr. *Gha. Tate* of *Hull*, Mr. *John Adams*, (who conſtructs the ſolution) Mr. *William Honnor*, our old friend Mr. *John Ramſay*, (mathematician and enigmatiſt) Mr. *John Hampſon*, Mr. *Alex. Roe*, Mr. *Joſeph Hilditch*, Mr. *Brownbridge*, Mr. *J. Hirſt*, Mr. *Stephen Hartley*, Mr. *Honey*; and, in a beautiful and incomparable hand-writing, Mr. *Thomas Huntley* ſolved the ſame in latin diction; as he has done many other problems in our Diary, fit for poſterity to look it) putting at the bottom of his elegant latin letter, *Dabam Burfordiæ, in comitatu Oxonienſi, pridie kalendas 4ti menſis, anno* 1752.

Mr. *Joſeph Orchard*, Mr. *Thomas Allen*, Mr. *Cha. Smith*, Mr. *Tho. Cowper*, Mr. *James Terey*, Mr. *James Hartley*, Mr. *Wm. Bevil*, Mr. *Holden*, and ſome others, add to the value and correctneſs of their performances by propriety of diction, and hand-writing, being clear, full, and conciſe; from whoſe compoſitions we find pleaſure to collect; as we do to encourage all uſeful and correct correſpondents in general.

VI. QUESTION 352 *anſwered by Mr.* Henry Watſon, *of* Goſberton *School, in* Lincolnſhire.

Put $x =$ the number of fruit trees unknown; then $\frac{x}{2} + \frac{x}{4} + \frac{x}{6} + 50 = x$, and, by tranſpoſition, $x - \frac{x}{2} - \frac{x}{4} - \frac{x}{6} = 50$; by diviſion $x = \frac{50}{1-\frac{1}{2}-\frac{1}{4}-\frac{1}{6}} = \frac{50}{1-\frac{11}{12}} = \frac{50}{\frac{1}{12}} = 600$, the number of fruit trees required.

The ſame conciſely and elegantly anſwered by Mr. John Fiſh, *of* Dartford.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{24}{48} + \frac{12}{48} + \frac{8}{48} = \frac{44}{48} = \frac{11}{12}$ wanting $\frac{1}{12}$ of the whole. Therefore $\frac{1}{12} = 50$ trees; conſequently the whole $= 600$ trees. Now $\frac{1}{2} = 300$ apple trees, $\frac{1}{4} = 150$ pear trees, $\frac{1}{6} = 100$ plumb trees, whoſe ſum $= 550$, to which adding 50 trees, the ſum total $= 600$ trees, the proof.

Mr. *John Nicholſon* of *Rocheſter*, anſwers it in the ſame conciſe and eaſy manner; as did Mr. *James Hartley* of *Yarum*, (who ſolved all the problems) Mr. *F. Holden*, Mr. *Hollingſworth*, Mr. *Peter Brooke*, Mr. *Thomas Trimingham* of *Hull*, Mr. *Richard Gibbons*, Mr. *John Adams*, Mr. *John Williams*, Mr. *John Ramſay*, Mr. *Joſeph Hilditch*, Mr. *Robert Butler*, Mr. *William Cottam*, Mr. *John Hampſou*, (who alſo ſent the times of eclipſes for Leigh) Mr. *A. Brook*, Mr. *T. Cowper*, Mr. *Brownbridge*, Mr. *John Potter*, of *Duke-ſtreet*, in *Southwark*, Mr. *John Peachy*, Mr. *Honnor*, Mr. *Henry Watſon*, and Mr. *Thomas Allen*, of *Goſberton* ſchool, in *Lincolnſhire*, and others.

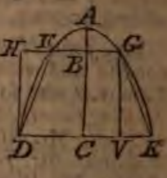
But Mr. *Thomas Huntley* of *Burford*, putting $12x =$ trees; then $6x + 3x + 2x + 50 = 12x$; whence $x = 50$, and $12x = 600$, required.

VII. QUESTION 353 *anſwered by Mr.* Henry Watſon, *of* Goſberton *School, in* Lincolnſhire.

Put $x = AD$ the greater, and $y = DB$ the leſſer ſegments; [*ſee the fig. p.* 223] $z = AC$, and $u = BC$, the ſides of the triangle; $a = 45$, $b = 195$, and $c = 12$. Then $xy = a$, $zu = b$, per queſt. and $xx + cc = zz$, and $yy + cc = uu$, by 47 Euc. 1. Whence $zz = \frac{bb}{uu} = xx + cc$, $uu = \frac{bb}{xx + cc} = yy + cc$, and $yy = \frac{bh}{xx + cc} - cc = \frac{aa}{xx}$; from which laſt $-x^4 + \frac{bbxx - aaxx}{cc} - ccxx = aa$: Put $d = \frac{bb - aa}{cc} - cc$, then $-x^4 + dxx = aa$, or $x^4 - dxx = -aa$; whence $xx = \frac{d}{2} \pm \sqrt{\frac{dd}{4} - aa} = 81$, and $x = 9$; whence $y = 5$, $z = 15$, and $u = 13$. *Q.E.F.*

Moſt of the gentlemen beforn-mentioned ſolved this queſtion, and particularly Mr. *Thomas Allen* and Mr. *John Williams*: all agreeing with *Taptinos* the propoſer's ſolution: And Mr. *Thomas Huntley* ſolved it in latin.

VIII. QUEST. 354 *anſwered by Mr.* Terey *of* Portſmouth.



Let $AC = a$, $DC = b$, $BC = x$. Per property of the curve, $a : bb :: a - x : \frac{a - x}{a} \times bb = FB^2$; and $FB = b\sqrt{\frac{a - x}{a}}$. But $bx + bx\sqrt{\frac{a - x}{a}}$ = area of the trapezium $DFGE$ to be a maximum. In fluxions and reduced, $x = \frac{8}{9}a$ (let b be what it will). Now, ſubſtituting this value of x in FB above, $FB = \frac{1}{3}b$, and alſo $\frac{2}{3}a$ for b (per queſt.), $FB = \frac{2}{9}a$: But $\overline{DC + FB} \times BC = \overline{\frac{2}{3}a + \frac{2}{9}a} \times \frac{8}{9}a = \frac{8}{9}a \times \frac{8}{9}a = 256$; whence, by extraction, $\frac{8}{9}a = 16$. And $a = 18 = AC$, and $DC = 12$, $BC = 16$, $FB = 4$; the area of the parabola $= \frac{4}{3}ab$; and of the trapezium $\frac{4}{3}b \times \frac{8}{9}a = \frac{32}{27}ab$. Hence every parabola is to the greateſt inſcribed trapezium, as $\frac{4}{3}$ to $\frac{32}{27}$, *i. e.* 9 to 8. *W.W.R.*

Mr. James Hartley *ſolves it thus*:

The ratio of the abſciſſa and ſemi-ordinate being as 3 to 2, the ſhorteſt ſide of the greateſt inſcribed trapezium will be the parameter, and its area will be equal to the ſquare, whoſe ſide HG will be double the parameter. Put $2x = DC$, then $3x = AC$, and from the nature of the curve $\frac{DC^2}{AC} = FG = \frac{4x}{3}$. And $AB = \frac{x}{3}$; but $AC - AB = BC = \frac{8x}{3}$. And $DC + BG = DV = \frac{8x}{3} = \sqrt{256}$; whence $x = 6$: Conſequently $DE = 24$, $FG = 8$, and $FD = GE = \sqrt{256 + 64} = 17{\cdot}8885$; and laſtly, $AC = 18$. *W.W.R.*

The above ſolutions are ſhort and elegant, as the propoſer's ſolution; as is likewiſe the ſolution by Mr. *Cottam*, and thoſe by Mr. *Charles Smith*, Mr. *Joſeph Orchard*, Mr. *F. Holden*, and ſeveral others, which 'tis needleſs to publiſh, being of a ſpecies with the two ſolutions above, ſufficient to ſatisfy the curious.

IX. QUEST. 355 *anſwered by Mr.* T. Cowper, *Teacher of the Mathematics, at* Wellingborough.

Firſt, $\frac{6797{\cdot}4}{69{\cdot}5} = 97^\circ\, 48'$, the diſtances of the two places, in the arch of a great circle, half of which $= 48^\circ\, 54'$. By ſpherics, As ſine $\frac{1}{2}$ diff. long. $= 72^\circ\, 7\frac{1}{2}'$: ſine $\frac{1}{2}$ the diſt. $= 48^\circ\, 54'$ :: rad. : ſine $52^\circ\, 24' =$ comp. lat. Hence the lat. $= 37^\circ\, 39'$. Again, as radius : co-tang. lat. $(37^\circ\, 39')$ :: coſ. $\frac{1}{2}$ diff. long. $(72^\circ\, 7\frac{1}{2}') =$ ſun's ſemi-diurnal arch : tang. ſun's declination $= 21^\circ\, 42'$ almoſt; anſwering to the 29th of November and the 12th of January, alſo the 29th of May and the 14th of July, N.S. *W.W.R.*

Mr. *Peter Brooke* found the ſame, as did Mr. *John Williams*.

Mr. *F. Holden* ſolves it thus: $\frac{6797{\cdot}4 \text{ miles}}{69{\cdot}5} = 97^\circ\, 48'\, 15''$, the diſt. of the cities in degrees. If a perpendicular be let fall from the pole, it will biſect that diſt. and alſo the diff. of long. Therefore, by oppoſite ſides and angles, as ſine $\frac{1}{2}$ diff. long. $72^\circ\, 7\frac{1}{2}'$: ſine $\frac{1}{2}$ diſt. $48^\circ\, 54\frac{1}{8}'$ :: rad. : coſ. lat. $37^\circ\, 38'\, 38''$. Now, the ſun's ſemi-diurnal arch $= \frac{1}{2}$ diff. long. $= 72^\circ\, 7\frac{1}{2}'$. Therefore by right-angled ſpheric triangles, as tang. lat. : rad. :: coſ. $72^\circ\, 7\frac{1}{2}'$: tang. ſun's declin. $= 21^\circ\, 42'$.

But if the apparent time of riſing to one city and ſetting to the other be required, Say, as ſine $\frac{1}{2}$ diſt. cities 48° $54\frac{1}{3}'$: ſine ſun's refraction 33′ :: rad. : 43′ 48″, which being added to the above-found declin. gives 22° 25′ 48″ declin. at the time of riſing and ſetting required.

Mr. *James Hartley*, of *Yarum*, ſolves it in the very ſame manner, and determines the ſituation of the cities upon the globe thus:

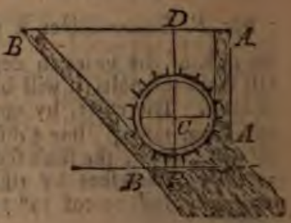
1753 { 29th of May and 14th of July, the ſun riſes at Japan when he ſets in Spain.
12th of January and 30th of November, the ſun ſets at Japan when he riſes in Spain.

And vice verſâ. With which Mr. *Richard Gibbons* agrees, and Mr. *Thomas Allen* nearly.

Mr. *Cottam* determines the ſame latitude, and days of the year, very near; and properly obſerves, that, in the queſtion, there ſhould have been expreſſed, *on what days*, inſtead of *on what day* of the year, &c. But, we obſerve, as the declination can ſeldom, if ever, correſpond with the true time of ſun-riſing or ſetting in the required latitude, there is no propriety in this ſort of queſtions, which only admit of anſwer near the truth. And, for the ſame reaſon, the 349th queſtion (where the days of ſun-riſing at Peterſburgh and Jeruſalem, at the ſame inſtant, are required) contains the ſame geometrical abſurdities; ſince the required declination is hardly ever poſſible to hit the true time of ſun-riſing in both thoſe different latitudes; the declination being ſtill variable every moment of time. And, if the ſun's declination be ſuppoſed the ſame for the ſpace of twenty-four hours, in this ſort of queſtions, ſtill the declination and time of ſun-riſing, on a particular day of the month, cannot exactly correſpond, according to computation of the ſun's place for that day, at noon, or time of riſing, by aſtronomical tables.

X. QUESTION 356 *anſwered by Mr.* William Cottam, *at his Grace the Duke of Norfolk's.*

When the water is conveyed by the trough $AA = DC$, then 'tis called an over-ſhot mill, and the diameter of the wheel is thus found: Let $DP = 16$ feet $= a$, the diſtance of the fall; $CP = CA = x$. By mechanics $nx = n\sqrt{a-x}$, where $n =$ force of water at A; therefore



$x = \sqrt{a + \frac{1}{4}} - \frac{1}{2} = 3{\cdot}5313$, whence $7{\cdot}06226 =$ diam. of the wheel required.

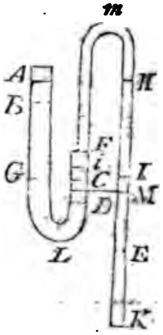
If the water be conveyed by the trough *BB*, then 'tis called an under-ſhot mill; and the greater the diameter of the wheel, the greater will be its force, and conſequently will have more force than the over-ſhot wheel.——And, on ſecond conſideration, I make the force of the water drawn into the radius of the over-ſhot mill a max. i. e. $nx\sqrt{a-x}$, a maximum; whoſe fluxion $\frac{2ax - 3xx}{2\sqrt{a-x}} = 0$, being reduced $x = \frac{2}{3}a = 10\frac{2}{3}$, and therefore $21\frac{1}{3}$ feet = wheel's diameter.

But Mr. *James Hartley*, of *Yarum*, ſolves it thus: Let $DP = 16$ feet $= a$; $16\frac{1}{12}$ feet $= c$; $x'' =$ time of deſcent of the water from *D* to *C*; then $cx^2 = DC$, and $2cx =$ velocity at *C*. But $\frac{a - cx^2}{2} =$ ſemi-diam. of the wheel, which multiplied into the velocity is $= acx - c^2x^3$, and its fluxion made $= 0$, and reduced gives $\frac{a}{3} = cx^2 = DC = 5\frac{2}{3}$ feet, whence $CP = 10\frac{2}{3}$; though I don't ſee the neceſſity of the queriſt's mentioning "*a tun a minute.*"

Mr. *John Rickerby*, of *Wooburn*, *Bucks*, ſays, he has ſpent great part of his life among the beſt paper mills in the nation; and obſerves, that a ſwing wheel, which receives its force of water eight inches above its breaſt center, exceeds an over-ſhot wheel; provided the current of water and fall are alike: And ſays, though Mill-wrights differ in their opinions concerning the true pitch of the water-wheel, that this opinion of his own is true. He ſpeaks of a pen, to give the diſcharge of water the greater force, at that part of the fall where the water-wheel receives its impetus, or depreſſing force; and computes the diameter of ſuch a wheel to be 26 feet 8 inches; but on principles a little doubtful.

XI. QUESTION 357 *anſwered by Mr.* James Hartley, *of* Yarum.

Let *AB* be the length of the whole ſcale in the common barometer $= s$; while the mercury deſcends from *A* towards *B*, it will equally aſcend from *D* towards *C*; ſo that $DC = \frac{s}{2}$. Let *M* be the place of the water and mercury at a middle ſtate of the atmoſphere; then per queſt. $MH = MK = 5s$. Put $d =$ diam. greater tube, $y =$ diam. leſſer, and let $Ci = x$; then $\overline{\frac{s}{4} + x} \times d^2 = \overline{\frac{21s}{4} + x} \times y^2$; and, as mercury is about fourteen times as heavy as water, $14x \times d^2 = \overline{\frac{21s}{4} + x} \times y^2$; whence $\frac{s}{52} = x$. Now, if inſtead of *x*, in the firſt equation, we ſubſtitute its value, and aſſume $d = 1$, we get, by reduction, $y^2 = \frac{7}{137}$, whence $y = \cdot 243$; if therefore *d* be taken at pleaſure, it will be as $1 : \cdot 243 :: d : y$. *W.W.R.*



The Rev. Mr. *Baker*'s ſolution is thus: s to 1 the ſpecific gravity of mercury to that of any fluid in a leſſer tube; r to 1 the ratio of the tubes; v a given variation in the common barometer, and x the correſpondent variation in the leſſer tube. Then $r^2 : x :: 1^2 : \frac{x}{r^2} =$ variation at the upper ſurface *C* of the greater tube (being reciprocally as ſquares of the tubes diameters); the whole variation of the leſſer tube $= \frac{r^2x + x}{r^2}$; the variation of the mercury's ſurface at *C*, in the greater tube, the ſame with that of the water in the ſame place, is $\frac{x}{r^2} =$ variation at *B*, of the ſame diameter. The whole variation of the greater tube $BG = \frac{2x}{r^2}$. The preſſure of the mercury and water together upon the air at *K* is from the length of the tubes, the contained fluids in *GLC* and *CmI* being ſuſpended in equilibrio; the variation in the preſſure of the different columns depend on their weights $\frac{r^2x+x}{r^2}$ and $\frac{2x}{r^2}$: Say, $s:1::\frac{r^2x+x}{r^2}:\frac{r^2x+x}{r^2s}=$ weight of that column, in reſpect of a column of mercury of the ſame length; $\frac{2x}{r^2}-\frac{r^2x+x}{sr^2}=v$, the variation in the common barometer; whence $x=\frac{vr^2s}{2s-r^2-1}$, i. e. $x:v::r^2s:2s-r^2-1$; and per queſt. $10:1::14r^2:28-r^2-1$, (here $s=14$ nearly) whence $r=3{\cdot}3541$, and the diameters of the tubes are as 3·3541 to 1. *W. W. R.*

COR. I. If $s=1, 2, 3, 4, 5, 6$, &c. and $r=\sqrt{\frac{1}{2}}, \sqrt{\frac{3}{3}}, \sqrt{\frac{5}{4}}, \sqrt{\frac{7}{5}}, \sqrt{\frac{9}{6}}, \sqrt{\frac{11}{7}}$, &c. correſpondent; or if $s=\frac{r^2+1}{2-r^2}$, or $r=\sqrt{\frac{2s-1}{s+1}}$, the variations in this barometer will equal thoſe in the common ſort.

COR. II. If $s=1, 2, 3, 4$, &c. and $r=\sqrt{1}, \sqrt{3}, \sqrt{5}, \sqrt{7}$, &c. or if $s=\frac{r^2+1}{2}$, or $r=\sqrt{2s-1}$, the variation will be infinite. Hence,

COR. III. The ſcale of variation in this barometer may have any aſſignable ratio to the variation in the common barometer.

Mr. *J. Williams* ſays, that Mr. Rowning (in his Compendious Syſtem, p. 112) determines the ratio of the variation of x, in the leſſer tube, to the common ſcale by $x=\frac{vmd^2}{2m-s^2-1}$, whence $d=\sqrt{\frac{2mx-x}{vm+x}}$.

XII. QUESTION 358 *anſwered by* ΦΙΛΟ-משקה.

t being the tranſverſe, and c the conjugate diameters of the inner ellipſis of a ſolid elliptical ring, whoſe circumference is circular, of p inches diameter; then $t+p$ and $c+p$ will be the tranſverſe and conjugate diameters of the ellipſis paſſing through the middle of the ring, whoſe circumference is in the center of gravity; which circumference put $=a$; then, ſince the ſolid or ſurface generated is equal to the product of the generating plane, or circular line, reſpectively multiplied into the way made by the center of gravity, therefore $\cdot7854\,ppa$ is the ſolidity, and $3{\cdot}1416\,pa$ the ſurface, of the elliptical ring, required.

The ſame anſwered by Mr. John Hartley, *of* Yarum.

If d be taken $= \frac{tt - cc}{tt}$, in the queſt. and $m = 3{\cdot}1416$, alſo A, B, C, &c. = the next preceding term in the following ſeries, then $1 - \frac{d}{2.2} - \frac{3d}{4.4}A - \frac{3.5d}{6.6}B - \frac{5.7d}{8.8}C - \frac{7.9d}{10.10}D$, &c. $\times \begin{Bmatrix} m^2pt = \text{ſurface} \\ \frac{1}{4}m^2p^2t = \text{ſolidity} \end{Bmatrix}$ of the ring.

N.B. The above ſeries are taken from Mr. Emerſon's excellent Doctrine of Fluxions, p. 174.

XIII. QUESTION 359 *anſwered by Mr.* John Honey, *of* Redruth, Cornwall.

Put a and $b = 42$ and 26 inches, the tranſverſe and conjugate diameters of the hoop above; and c and $d = 48$ and 29, the dimenſions of thoſe below; alſo $m = 12$, the fruſtum of the ellipſoid's altitude; and $n = 18$ inches, the elliptical cylinder's altitude: Then, by a known theorem, $\frac{ab + cd + \sqrt{abcd}}{882{\cdot}36} \times m = 50{\cdot}54$ wine gallons, the content of the hoop's concavity; and $\frac{cdn}{294{\cdot}12} = 85{\cdot}18$ wine gallons, the content of the cylindrical concavity; whence the concavity of both $= 135{\cdot}72$ wine gallons.

Mr. *John Wiggleſworth* anſwers it thus: Let $a = 42$ inches, $b = 26$ = tranſverſe and conjugate diameters above; $t = 48$, $c = 29$ = tranſverſe and conjugate diameters below; $h = 12$, $p = 18$, and $m = {\cdot}2618$, then the content of the whole concavity $= \frac{mh \times \overline{ct + \frac{1}{2}bt + ab + \frac{1}{2}ac} + 3mtcp}{231} = 135{\cdot}7416$ wine gallons.

The ſolutions by Mr. *T. Cowper*, Mr. *J. Hartley*, Mr. *J. Adams*, and Mr. *Cottam*, agree the ſame; but they miſſed the ſecret of half Miſs Polly's ſolidity, which being taken from the laſt-found concavity, leaves the concavity ſought. Mr. *Adams* obſerves, the fruſtum of the ellipſoid may be reduced, with advantage, to the fruſtum of a cone, and the elliptic cylinder to a round cylinder.

The PRIZE QUESTION *anſwered by Mr.* F. Holden, *at* Weſthouſe, *near* Settle, Yorkſhire.

Take a piece of wood or a ſtone, of a known ſuperficies, and, dipping it into a veſſel full of melted tallow, you may, by trying the weight of the tallow and dipping-veſſel in a ſcale before and after dipping, know the quantity of tallow, in weight, taken up by the piece of wood or ſtone. Then take about 18 or 20 modern mathematicians, (the more the better) ſtrip them ſtark-naked, and ſuſpend them (like Abſaloms) by the hair of their heads, as chandlers hang their candles, or elſe by ſoft bandages under the chin and behind the nape of the neck, ſo that they may be raiſed or let down by pullies without hurting; dip them alſo, one by one, in the ſame veſſel where the wood or ſtone was lately dipped; and mark the tallow they all take up, by weighing the veſſel and tallow, before and after they are all dipped, (keeping the tallow juſt melted and of an equal warmth): Then ſay, as the quantity of tallow, in weight, taken up by the wood or ſtone, is to the known ſuperficies of either, ſo is the weight of tallow taken up by all the mathematicians to the ſuperficies of all the mathematicians. But, by all means, take care that they are kept naked till they are ſhivering, and almoſt as cold as the wood or ſtone itſelf, before they are dipped, elſe this proportion will not hold good.

When they are all dipped, well ſcoured with ſoap, and cleanſed from the tallow, let them be weighed, (or they may be all weighed before dipping) and ſay, as the weight of them all in pounds is to the late-found ſuperficies of them all in ſquare inches or ſquare feet, ſo is 160 pounds weight to the ſuperficies of the modern mathematician required to be known, (= 14½ ſquare feet, nearly, as we find by another method). *W.W.R.*

Paradoxes anſwered.

I. PARADOX *anſwered by Mr.* Richard Gibbons, *of* Plymouth.

The fork was as the ſteelyard, Roger's ſhoulder as the fulcrum, ſuſtaining the burthen, between the two powers, acting at both ends of the fork.

II. PARADOX *anſwered by Mr.* Edward Griffiths, *of* Elleſmere, Shropſhire, *and others.*

Caſe 1. A piece of pliable metal being doubled, by applying a round file to the doubled edge, and filing a half-ſquare gap, on opening the metal, a ſquare hole will appear.

Or, the ingenious Mr. *Cato* informs us, that, if two corners and an edge, at the end of a miſer's iron cheſt, be filed away, with a round or any other file, there may be an exact ſquare hole left.

Caſe 2. A cylindrical body being cut obliquely, the plane of the ſection will be an oval; and, conſequently, a round body, ſituated obliquely in an oval hole, will completely fill it. *Archimedes.*

The Eclipſes calculated for 1753, *by* *Mr*. Cowper, *of* Wellingborough.

1. Moon riſes eclipſed 4¼ digits, on Tueſday, April 17th, N. S. at 7h. 3m. apparent time in the evening, viſible at London. End 7h. 47m.

2. Sun eclipſed, Thurſday, May 3d, 7h. 40m. N. S. in the morning; inviſible at London. But (according to Mr. *Hulſe*) will be near 8 digits eclipſed on the ſouth ſide ♉ in 13°; being vertical to the Arabian ſea, lat. 16° ſouth, long. 62° eaſt; and viſible to the inhabitants of Madagaſcar.

3. Moon eclipſed, Friday the 12th of October, 9h. 40m. morning, N. S. inviſible at London. But (according to Mr. Hulſe) eclipſed on the ſouth ſide in ♈ 19° vertical to the great ocean, weſt of America, where it is partly viſible to that quarter of the earth.

4. Sun eclipſed, Friday the 26th of October, N. S. viſible. Beginning 8h. 35m. 5s. Middle 9h. 43m. 36s. Viſible conjunction 9h. 44m. 34s. End 10h. 56m. 39s. Duration 2h. 21m. 34s. Digits on the ſun's lower limb 8° 20'.

1. Eclipſe calculated by		Beg.	Mid.	End	Dur.	Dig.
		h. m.	h. m.	h. m.	h. m.	° '
Mr. Cowper, by Dr. Halley's corrected Tab.	London	5 27	6 37	7 47	2 20	5 2
	Wellingbor.	5 25	6 35	7 44		
		M.r.Lo.4°4' dig. at 7h. 3m. Ev.				
Mr. Man,	London	4 59	6 17	7 30	2 31	2 30
	Plymouth	4 43	6 1	7 14	2 31	2 27
	Conſtantinople	6 55	8 13	9 26	2 31	5 8
Mr. Hulſe,	London	7 58	8 22	9 42	2 52	5 44

4. Eclipſe calculated by		Beg.	Mid.	End	Dur.	Dig.
		h. m.	h. m.	h. m.	h. m.	° '
Mr. Cowper, by Dr. Halley's corrected Tab.	London	8 35	9 43½	10 56½	2 21½	8 22
	Wellingb.				2 26	
Mr. Man,	London	8 42	9 57	11 8		8 15
	Plymouth,	8 30	9 47	10 59	2 29	8 29
	Liſbon	7 50	9 0	11 20	3 30	11 49
	Conſtantinople	11 24	12 46	2 8	2 44	7 45
Mr. Hulſe,	London	8 30	9 48	10 30	2 26	8 36

There is a Tranſit of ☿ on May 6th, N. S. (Mr. *Hulſe* ſays 5th). Beginning 2 h. 47 m. 24 s. morn. Ending 10 h. 43 m. 20 s. according to Mr. *Cowper*, from Dr. Halley's tables, being 2 hours ſooner than by Leadbetter.

For proving Aſtronomical Tables.

The moon's eclipſe, June 8th, O. S. appeared thus at Rome. Beginning 8 h. 2 m. 2 s. Immerſion 9 h. 6 m. 10 s. Emerſ. 10 h. 35 m. 19 s. End 11 h. 39 m. 49 s. *Alexander Man.*

New Queſtions.

I. QUESTION 361, *by a Perſon of Honour.*

A water mill is to be built where there is a fall of water of 24 feet.—It is required to determine whether a wheel of 18 feet with 6 feet fall, or a wheel of 16 feet with 8 feet fall, will grind the moſt corn with leaſt water?

II. QUESTION 362, *by Mr.* John Fiſh, *of* Dartford.

A ball weighing 4 pounds upon the ſurface of the earth, to what height, in the air, muſt it be carried to weigh but 3 pounds, and how long would it be in falling to the ground?

III. QUESTION 363, *by* Nichol Dixon, *of* Blackwell.

In Craven as I walk'd alone*,
Three objects once appear'd in ſtone,
I do proteſt I ne'er ſaw bigger.
And ſtood in right triang'lar figure.
Theſe ſtones (as being told to me)
Go by the names of *A*, *B*, *C*;
From *C* to *A* I meaſur'd, then,
In Engliſh miles exactly ten.
From *A*, for *B*, due north I ſtride,
Till I the riſing ſun eſpy'd
Appearing in a line with *C*,
Directly, as I ſtopt at *D*:
And there old Bob (who came in ſight)
Told me "the angle *C* was right;
"That three miles further on ſtood *B*,"
And ſaid "that courſe was true for me:"

* Lat. 54°.

The time this happen'd, I may ſay,
Was on the 28th of May.
Now, without meas'ring, I don't doubt
But you'll the miles to *C* find out.
Ye, who to cards or dice pretend!
This problem ſolv'd, the game will end:
Tho. Simpſon ſent it firſt for fun,
Now, ſolve it you, ſome ſon of a gun!
For cards nor dice can Simpſon charm,
Like old Sir John, that keeps him warm.
[*See the fig. to the ſolution.*]

IV. QUESTION 364, *by Mr.* T. Cowper, *Teacher of Mathematics, at* Wellingborough.

On the 14th of laſt March, at half an hour after 11 in the forenoon, being in latitude 52° 22′, I obſerved 10 beats of my pulſe between the time of a ſmall cloud ſhading me, and that when the ſhadow thereof reached a tree, at the diſtance of 88 yards, eaſterly from me; immediately before, I likewiſe obſerved that the angle formed by the ſhadow of a ſtick perpendicular to the horizon, and a line drawn from the tree to the place of obſervation was 120°. Now, admitting 70 pulſations in a minute, the hourly velocity of the cloud, its direction, and what point of the compaſs the wind blew, are thence required.

V. QUESTION 365, *by Mr.* John Williams.

What pounds principal, being put out at its equal value per cent. at ſimple intereſt, for an equal number of years, will raiſe an intereſt equal to half the principal?

VI. QUESTION 366, *by the Rev. Mr.* Baker, *of* Stickney, Lincolnſhire.

If the thickneſs of two microſcopic glaſſes be three-eighths of their reſpective radii of convexity, and theſe be in the ratio of 10 to 3, how muſt thoſe glaſſes be diſpoſed, in a compound microſcope, ſo that an object, eight inches diſtant from the eye, ſhall be thereby magnified a thouſand times?

VII. QUESTION 367, *by Mr.* T. Cowper, *of* Wellingborough.

On the 19th of September, 1751, at night, the vertical angle between Jupiter and the ſtar Caſtor was obſerved to be

35°

35° 48′, and that between Jupiter and the bright ſtar in the whale's tail, 78° 29′; it is required to determine the latitude of the place and hour of the night, where and when thoſe obſervations were made?

VIII. QUESTION 368, *by Mr.* Chriſtopher Maſon, *of* Eaſtburn, *near* Petworth, *in* Suſſex.

A conſtant quantity being put for a factor, in Monſ. Ozanam's Mathematical Recreations, to be multiplied by variable factors, in order to produce 3 ones, 3 twos, 3 threes, &c. through all the digits, I deſire to know both the conſtant and variable factors, that will produce 6 ones, 6 twos, 6 threes, &c. alſo 9 ones, 9 twos, 9 threes, &c. through all the digits?

IX. QUESTION 369, *by Mr.* Terey, *of* Portſmouth.

Required the ſuperficial content of a ſcalenous cone, whoſe longeſt ſide is 12 inches, ſhorteſt 9, and baſe diameter 6 inches.

X. QUESTION 370, *by Mr.* Joſ. Hilditch, *at* Handal, *near* Shrewſbury.

The three diſtances from an oak, growing in an open plain, to the three viſible corners of a ſquare field, lying at ſome diſtance, are known to be 78; 59, 161, &c. and 78 poles, in ſucceſſive order: Quere the field's dimenſions, and the acres it contains?

XI. QUESTION 371, *by Mr.* Terey, *of* Portſmouth.

What is the content of a caſk, in ale gallons, whoſe ſtaves are exactly circular, and dimenſions of the head and bung diameters, and alſo length 24, 36, and 48 inches?

XII. QUESTION 372, *by Mr.* W. Bevil, *of* Harpſwell, Lincolnſhire.

Suppoſe the moon's diameter to be 2170, the earth's diameter 7970, and the diſtance of each other's ſurface 240000 miles, where muſt I view them on a line drawn betwixt their centers, to ſee the greateſt quantity of ſurface of both bodies poſſible?

XIII. QUESTION 373, *by Mr.* John Williams, *of* Mold, *in* Flintſhire.

Mathematicians take pains to deſcribe curves and ſolids that never exiſted, yet ſay little or nothing to the properties of thoſe things that are in nature; eſpecially the ſections, ſolidities, and curve ſuperficies of the egg; which is one of nature's principal productions. If any of the problematic problemiſts would be pleaſed to give the ſolution of the quantity of curve ſuperficies and ſolidity of the egg, when its axis is 2½ inches, greateſt ordinate 1½, and the diſtance from that ordinate to the neareſt end 1 inch, they would be intitled to a maximum of applauſe, inſtead of the minimum acquired, by puffing and cavilling about their ſuperior dignity, who are odd fiſhes at foot-ball.

XIV. QUESTION 274, *by* Philotheoros.

My Lord Mayor's gold chain being 50 inches in meaſuring round it, at what diſtance muſt it be hung over two pins, horizontally fixed in a wall, covered with crimſon damaſk, for a ſpectator to behold the moſt damaſk poſſible, within the circumference of the chain?

PRIZE QUESTION, *by Mr.* James Hartley, *of* Yarum.

Suppoſe *DAFE* to be a veſſel in the form of a conical fruſtum, [*ſee the fig. to the ſolution*] whoſe top diameter $DA = 20$, bottom diameter $= 10$, and depth $= 15$ inches; ſuſpended by an inflexible line $Ca = 100$ inches (the line and veſſel being ſuppoſed of no weight) and the veſſel full of water to weigh 100 pounds weight, when the veſſel is perpendicularly ſuſpended at *B*. Let the veſſel be drawn aſide by a cord faſtened at (*a*) the bottom of the veſſel, while at the other end of the cord is a weight *W* ſuſpended, the cord freely ſliding over the pulley at *P*, placed at ſuch a diſtance from *C*, in the horizontal line *PC*, as to require the leaſt weight, *W*, poſſible, to equipoize the veſſel when the tenſion of the cord *Pa* is a maximum: Required the weight, *W*, and diſtance *PC*?

New Paradoxes.

PARADOX I. *by Mr.* Cottam, *at his Grace the Duke of Norfolk's.*

Whene'er my Lord a journey takes,
Or to a friend a viſit makes,
His neareſt road ſouth always lies,
(Directly ſouth) which all ſurprize!
Remote the place, adjacent, either,
This ſide or that, no matter whether.
Hereof the reaſon make appear
In your fam'd Diary for next year.

PARADOX II. *by* John a Stiles, *Clerk of* H——n.

A man's lands, which he's ſupppoſed to have purchaſed, devolved to his only daughter, and then to her eldeſt ſon (according to common deſcents) and the only ſon of the purchaſer, who was begotten in lawful wedloc, as well as the daughter; yet he had nevertheleſs no manner of right to the inheritance after his deceaſe: How could this be?

1754.

Queſtions anſwered.

I. QUESTION 361 *anſwered by Mr.* J. M.

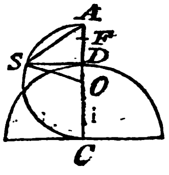
THE ſpaces paſſed through by falling bodies being as the ſquares of the acquired velocities, the velocity of the water falling 8 feet is to its velocity acquired in falling 6 feet, as $\sqrt{8}$ to $\sqrt{6}$. Therefore by the property of the lever, the force of the ſame water to turn a wheel of 16 feet, with a fall of 8 feet, is to its force to turn a wheel of 18 feet, with a fall of 6 feet, as $16\sqrt{8}$ to $18\sqrt{6}$, or as $8\sqrt{4}$ to $9\sqrt{3}$; that is, as $\sqrt{256}$ to $\sqrt{243}$. Whence, it is evident that the wheel of 16 feet diameter has the greateſt advantage.

It was anſwered in like manner, and on the very ſame principles, by ſeveral other contributors. But theſe principles, though true in themſelves, do not appear to us ſufficient to give a right and full determination of the problem under conſideration. To have the true quantity of the effect, not only the height of the fall, and the diameter of the wheel, but alſo the weight and force of the water in the wheel ought to be regarded; and conſequently the different poſitions of the buckets with reſpect to the horizon.——As this is a ſubject of much importance, it is hoped our ingenious correſpondents will think it worthy of a farther conſideration, and communicate their thoughts thereon, for the benefit of the public, in our next Diary; to which we ſhall refer for a fuller diſcuſſion of this matter.

II. QUESTION 362 *anſwered by* Timothy Doodle.

Let CD be the earth's ſemi-diameter, and DA the required height from whence the ball muſt fall: Then $3 : 4 :: CD^2 : CA^2$; and conſequently $CA = CD \times \sqrt{\frac{4}{3}} = 3980 \sqrt{\frac{4}{3}} = 4596$ miles. Whence DA is given $= 616$ miles, or 3252480 feet.

Now the diſtance deſcended in the firſt ſecond of time being always as the force, it will here be $= \frac{3}{4}$ of $16\frac{1}{12}$ feet $= 12\frac{1}{16}$ feet: And conſequently the time of deſcent through AD, with the ſame force uniformly continued, $= \sqrt{\frac{3252480}{12\frac{1}{16}}} = 519$ ſeconds. But, ſuppoſing a ſemi-circle ASC to be deſcribed upon AC, and DS perpendicular to AC, the true time of deſcent through AD will be in proportion to 519 ſeconds, the time juſt now found, as half the ſum of the ſine DS and the arch AS is to the chord AS (as is proved by the writers on fluxions). Now $AC : CD$ $(:: \sqrt{4} : \sqrt{3}) :: 2$ (twice the radius of the tables) : $\sqrt{3} = 1{\cdot}732 =$ the verſed ſine of the angle $SOC = 137^\circ\ 4'$. Whoſe ſupplement AOS is therefore given $= 42^\circ\ 56'$: The natural ſine of which will be ·6811, and the meaſure of the angle itſelf $=$ ·7494; the half ſum of theſe is ·7152: But the chord of $42^\circ\ 56'$ $(2 \times \text{ſine } 21^\circ\ 28')$ is ·7319. Hence it will be $·7319 : ·7152 :: 519 : 507$ ſeconds, or 8m. 27s. the true time required.



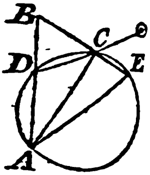
The

The ſame anſwer'd by Anthony Shallow, *Eſq.*

If the earth's ſemi-diameter (CD) be denoted by a, it is plain that $a\sqrt{\frac{4}{3}}-a$ will expreſs the required height AD from which the ball muſt fall. To determine therefore the time of the deſcent through AD, let the velocity of the ball, per ſecond, acquired in falling through any diſtance AF ($=x$), be denoted by v; putting $c=AC$; and d ($=\frac{3}{4}\times 16\frac{1}{12}$) = the diſtance deſcended in the firſt ſecond of time from A: Then, $2d$ being the meaſure of the velocity acquired in one ſecond, with the force at A, it will be as $\overline{c-x}|^2 : c^2 :: 2d : \frac{2dcc}{\overline{c-x}|^2}$, the velocity generated, per ſecond, by the force at F: Therefore $v : \frac{2dcc}{\overline{c-x}|^2} :: \dot{x} : \dot{v}$; and conſequently $\frac{v\dot{v}}{2d} = \frac{cc\dot{x}}{\overline{c-x}|^2}$. Hence, by taking the fluent, $\frac{vv}{4d} = \frac{cc}{c-x} - c = \frac{cx}{c-x}$. Therefore $v = 2\sqrt{dc} \times \frac{\sqrt{x}}{\sqrt{c-x}}$, and conſequently $\frac{\dot{x}}{v} = \frac{\dot{x}\sqrt{c-x}}{2\sqrt{cdx}} = \frac{1}{2\sqrt{cd}} \times \frac{c\dot{x}-x\dot{x}}{\sqrt{cx-xx}}$ = the fluxion of the required time. Whoſe fluent is $= \frac{1}{4}\sqrt{\frac{c}{d}}$ multiplied by the ſum of the ſine and arch whereof the correſponding verſed ſine is $\frac{2x}{c}$ (unity being the radius). But $\sqrt{\frac{c}{d}}$ (expreſſing the time of deſcent through AC, with an uniform force equal to that at D) is given = 1418 ſeconds. And $\frac{2x}{c}$ is = 0·26795; anſwering to an arch of 42° 56′; whoſe length is = 0·7494; and that of its ſine = 0·6811. Hence we have $\frac{0\cdot6811+0\cdot7494}{4}\times 1418 = 507$ ſeconds = 8 m. 27 s. *W. W. R.*

III. QUES-

III. QUESTION 363 *anſwered by Mr.* Rich. Gibbons.



Conſtruction. From the latitude of the place and the ſun's declination, the ſun's horizontal azimuth BDC is given $= 48° 35'$. Having therefore made the angle $BD\odot = 48° 35'$, and $DB = 3$; let a ſquare, whoſe ſide AC is 10, be formed, and let the angle thereof ACB be ſo moved along the line $D\odot$, that the end A, of the ſide CA, may, at the ſame time, paſs over the line BDA, till the other ſide of the ſquare paſſes through the given point B. In which poſition draw BC and AC, and the thing is done, as is evident by inſpection.

Algebraic ſolution. Put $b = AC = 10$, $c = BD = 3$, and $s =$ ſine of $BDC =$ ſine ADC; alſo put $d = \frac{s}{b} = 0{\cdot}074992$, and $x =$ ſine of ACD: Then is $\sqrt{1 - xx} =$ ſine BCD; and we have $b : s :: x : dx = AD$; alſo $\sqrt{1-xx} : c :: s : \frac{cs}{\sqrt{1-xx}} = BC$. And, by 47 Euc. 1, $b^2 + \frac{c^2 s^2}{1-xx} = c^2 + 2cdx + d^2x^2$. Reduced, $x = 0{\cdot}5517 =$ the ſine of 33° 29′. From whence $CD = 3{\cdot}4738$ miles.

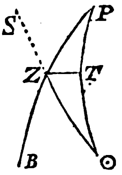
The ſame anſwered by Anthony Shallow, *Eſq.*

Suppoſe the circumference of a circle to paſs through A, D, and C, cutting BC produced in E. Then, AE being drawn, in the triangle ACE will be given (beſides the right angle) the ſide AC and the angle $E = BDC = 48° 34' =$ the ſun's horizontal azimuth from the north: Whence CE is given $= 8{\cdot}8265$; which put $= a$, making $AC = b$, $BD = c$, and $BC = x$: Therefore $AB = \sqrt{bb + xx}$; and, by the property of the circle, $c\sqrt{bb+xx} = x \times \overline{x+a}$. Hence $x = 2{\cdot}6966$; and conſequently $DC = 3{\cdot}473$ miles.

It was alſo anſwered, in a curious manner, by Mr. *J. Aſh*, Mr. *Sam Bamfield*, *Bathonius*, Mr. *W. Bevil*, Mr. *T. Coates*, Mr. *J. Hollingworth*, Mr. *J. Milbourn*, Mr. *B. Talbot*, and Mr. *J. Wiggleſworth*.

IV. QUES-

IV. QUESTION 364 *anſwered by Mr.* John Wiggleſworth.



Let P repreſent the north pole, Z the zenith, ZS the ſhadow of the ſtick, and ZT the direction of the cloud; then the angle $TZ\odot = 60°$; and, in the oblique ſpheric triangle $\odot ZP$, is given PZ the complement of latitude, $P\odot$ the complement of the ſun's declination, with the contained angle P; whence the angle $\odot ZR$ is found $= 9° \; 42' \; 30''$; which added to $TZ\odot$, gives $TZR = 69° \; 42' \; 30''$. Therefore the wind blew W. b. N. $\frac{3}{4}$ N. nearly. Again, by allowing 70 pulſations to a minute, the ſhade of the cloud will move over a ſpace = 88 yards in the time of 10 pulſations, or $8\frac{4}{7}$ ſeconds; which is at the rate of 21 miles per hour.

The ſame anſwered by Mr. James Robinſon.

During 10 pulſations, we may ſuppoſe the ſun does not ſenſibly change its place; and, conſidering the immenſe diſtance of the ſun, when compared with that of a cloud, we may take the rays proceeding from the ſun to the cloud, at both obſervations, to be parallel: Then, the ſun's azimuth being (per ſpherics) 9° 26′ from the ſouth, the direction of the cloud was E. b. S. $\frac{1}{4}$ S. fere; and the hourly velocity of the wind, blowing from the oppoſite point, 21 miles.

W. W. R.

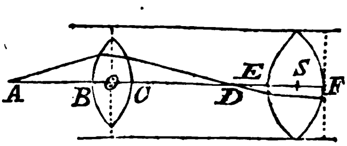
Mr. *Aſh*, *Bathonius*, Mr. *W. Bevil*, Mr. *T. Coates*, Mr. *Richard Gibbons*, and ſome others, alſo anſwered this problem, and agree, exactly, in the velocity of the cloud; but make ſome little difference in its direction, ariſing from having taken the ſun's declination from different authors.

V. QUESTION 365 *anſwered by Mr.* J. Milbourn.

Let 100l. principal $= a$, and for the required principal, &c. put x; then $a : x :: x : \frac{xx}{a}$, the intereſt for one year; and as 1 (year) is to x (years) ſo is $\frac{x^2}{a}$ to $\frac{x^3}{a}$, the intereſt for x years: Whence (per queſt.) $\frac{x^3}{a} = \frac{x}{2}$; reduced, $x = \sqrt{\frac{a}{2}}$ $= 7 \cdot 07106$ pounds = 7l. 1s. 5d.

In the very ſame manner it was anſwered by Mr. *J. Aſh*, Mr. *S. Bamfield*, Mr. *G. Dickinſon*, Mr. *J. Colinge*, Mr. *Gibbons*, Mr. *Holingworth*, Mr. *Jonah Milford*, Mr. *W. Newman*, Mr. *Joſ. Peil*, Mr. *D. Roberts*, Mr. *James Robinſon*, and Mr. *Ch. Tate*.

VI. QUESTION 366 *anſwered by* Κυβερνήτης.



Let the radius of convexity of the lens O, next the object A, be put $= a$, thickneſs $BC = b$, the radius of the eye-glaſs $S = d$, and its thickneſs $EF = e$; and let the ſine of incidence be to that of refraction, out of air into glaſs, as 1 to r; putting $q = 1 - r$, $t =$ the given linear amplification, and $x =$ the diſtance CD of the place of the image from the lens O.

Then, A being the place of the image of an object at D, by a known theorem in optics, AB will be $= \frac{rA\mathcal{Q}}{q\mathcal{Q} - a}$; $\mathcal{Q}$ being put $= b + \frac{x}{r - \frac{qx}{a}}$. And, by another known theorem (which is a corol. to the former) the principal focal diſtance ED of the lens S, will be $\frac{rd}{q} \times \frac{d - qe}{2d - qe}$.

But, by the queſtion, $b = \frac{3a}{8}$, $e = \frac{3d}{8}$, and $d = \frac{10a}{3}$. Therefore, by the ſubſtitution of equal values, $\mathcal{Q} = \frac{3a}{8} + \frac{x}{r - \frac{qx}{a}}$, and $DE = \frac{10ra}{3q} \times \frac{1 - \frac{3}{8}q}{2 - \frac{3}{8}q} = a \times \frac{10r}{3q} \times \frac{8 - 3q}{16 - 3q}$. Whence DS $(= DE + ES) = a \times \frac{10r}{3q} \times \frac{8 - 3q}{16 - 3q} + \frac{5a}{8}$ $= ca$, by putting $c = \frac{10r}{3q} \times \frac{8 - 3q}{16 - 3q} + \frac{5}{8}$. Again, by the queſt. $\frac{OD}{OA} \times \frac{FA}{SD} = t$; that is, in ſpecies, $\frac{x + \frac{3a}{16}}{\frac{ra\mathcal{Q}}{q\mathcal{Q} - 1} + \frac{3a}{16}} \times$

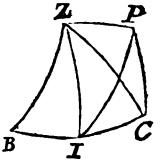
$ca +$

$\dfrac{ca+x+\frac{3a}{8}+\frac{ra\mathcal{Q}}{q\mathcal{Q}-a}}{ca}=t$. Put $\frac{x}{a}=z$, and $\frac{3}{8}+\frac{z}{r-qz}$ $=y$; then will $\mathcal{Q}$ $(=\frac{3a}{8}+\frac{az}{r-qz})=ay$; and our equation by ſubſtitution, &c. will be reduced to $\dfrac{z+\frac{3}{16}}{\frac{ry}{qy-1}+\frac{3}{16}}$ $\times\dfrac{c+\frac{3}{8}+z+\frac{ry}{qy-1}}{c}=t$; or, $\overline{z+\frac{3}{16}}\times\overline{c+\frac{3}{8}+z+\frac{ry}{qy-1}}$ $=ct\times\overline{\frac{3}{16}+\frac{ry}{qy-1}}$. From whence, and $\frac{3}{8}+\frac{z}{r-qz}=y$, the value of z, will be found by an equation of three dimenſions: And then, the value of FA being given in terms of a, by putting that value $=8$ inches, a itſelf will be found, and from thence every thing elſe, required.

But as the finding of z this way (the terms being numerous) will be ſomewhat troubleſome, the known method of approximation, by trial-and-error, may be here uſed with advantage. According to which, having aſſumed for the value of z, that of y $(=\frac{3}{8}+\frac{z}{r-qz})$ will be immediately found; and then, by ſubſtituting theſe two values in the other equation, the error will be determined, &c. &c.

VII. QUESTION 367 *anſwered by Mr.* W. T—t.

Let P be the pole of the world, Z the zenith of the place, and B, I, C, the three ſtars: From the given longitudes and latitudes of which, or from their right aſcenſions and declinations, the diſtances BI and IC, and the angle BIC, may be found, by common trigonometry.



Aſſume the value of ZI as near as you can to its true value: Then, having two ſides and one angle, in each of the triangles BIZ, CIZ, the angles BIZ and CIZ may be found, and conſequently their ſum BIC. Mark how much this value of BIC differs from the given value of the ſame angle: Then make a ſecond aſſumption for ZI; and find, again, the value of the angle BIC, marking the error, as before.

From theſe two errors a new value of IZ, by the known methods of approximation, may be found; and ſo on, till you arrive to what degree of exactneſs you pleaſe. Having thus determined ZI and ZIC, from the latter of theſe deduct PIC, the remainder gives the angle ZIP: From which, and the two given ſides including it, both ZP and ZPI will become known.

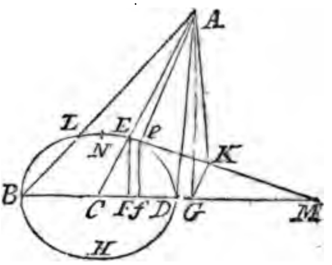
VIII. QUESTION 368 *anſwered by Mr.* J. Robinſon.

Let $999999 = a$ be the conſtant factor, in order to produce 6 ones, 6 twos, &c. The other factor call x, and the firſt product p; then $xa = p$, and conſequently $x = \frac{p}{a}$.

Univerſally, putting the conſtant factor (which is arbitrary) for the denominator, and the given product for the numerator, the fraction, or fractions, thence ariſing, will be the variable factor, or factors, required.

If any number of nines be taken for the denominator of a fraction, and the ſame number of any of the digits for a numerator, the fraction, when reduced to a decimal one, will have the very ſame figures as the numerator, repeated to infinity. Thus, for example, $\frac{1234}{9999}$ is $=$ 1234123412341234, &c. ad infinitum. Thus alſo, $\frac{444444}{999999} = {}'44444$, &c. and conſequently $999999 \times {}'4444$, &c. $= 444444$; and ſo of others. The variable factors, derived by this general method, are fractions; but there are particular anſwers to be had in whole numbers. Thus, becauſe $\frac{111111}{3} = 37037$, and $\frac{111111111}{3} = 37037037$, it is evident that, if 3 be taken for the conſtant factor, the reſpective variable factors, to produce 6 and 9 ones, will be 37037 and 37037037. The multiples of which by 2, 3, 4, 5, &c. will conſequently be the other variable factors required. In like manner, 37 being aſſumed for the conſtant factor, the variable ones will be 3003 and 3003003, together with their multiples.

IX. QUESTION 369 *anſwered by* Κυβερνήτης.



Let *ABHDEC* be the given cone, and *AG* its perpendicular height: Let *EM* be a tangent to the circular baſe *BEDH* at any point *E*; and, ſuppoſing *e* to be another point in the curve indefinitely near to *E*, let *EA* and *eA* be drawn in the ſurface of the cone: And from the ſame points *E*, *e*, upon the diameter *BD*, paſſing through *G*, let fall the perpendiculars *EF* and *ef*: Draw the radius *CE*, and the line *GK* parallel thereto, meeting the tangent *EM*, at right angles, in *K*; to which point from the vertex of the cone draw *KA*, which will be perpendicular to the tangent *EM*; becauſe (being equal to $\sqrt{AG^2 + GK^2}$) it will be the leaſt poſſible, in this poſition, where *GK* is the leaſt poſſible.

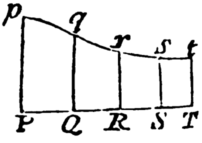
Put now $CE = a$, $CG = b$, $AG = c$, and $CF = x$, $BE = z$, and $Ee = \dot{z}$. Then, by the property of the circle, $CM = \frac{aa}{x}$: And, by ſimilar triangles, $CM : CE :: MG$ $(\frac{aa}{x} - b) : GK = a - \frac{bx}{a}$. Whence $AK = \sqrt{AG^2 + GK^2}$ $= \sqrt{c^2 + a^2 - 2bx + \frac{bbxx}{aa}}$: Which, multiplied by $\frac{1}{2}\dot{z}$, gives $\frac{1}{2}\dot{z}\sqrt{cc + aa - 2bx + \frac{bbxx}{aa}}$ for the area of the triangle *EAe*, or the fluxion of the required ſuperficies. Which fluxion, becauſe $\dot{z} = \frac{a\dot{x}}{\sqrt{aa - xx}}$, is alſo $=$

$$\frac{\frac{1}{2}a\dot{x}\sqrt{cc + aa - 2bx + \frac{bbxx}{aa}}}{\sqrt{aa - xx}}$$

: Whoſe fluent will expreſs the required ſuperficies of the cone.

But as the finding of this fluent is extremely troubleſome, and, when it is found, converges ſlowly (except where the cone is but little inclining) I ſhall therefore give the ſolution by a different, and more general method.

Let PT be a right line equal in length to the ſemi-circumference $BNED$; upon which, as a baſe, (or abſciſſa) ſuppoſe a curve $pqrst$ to be deſcribed, ſuch that, taking PS always $= BE$, the ordinate Ss ſhall be every-where equal to half the correſponding perpendicular AK: Then it is plain, that the area $PTtp$ of this curve, will be exactly equal to half the convex ſuperficies of the cone. To approximate which, conceive the axis PT of the curve and the ſemi-circumference $BNED$ of the circle, to be divided, each into four equal parts; and let the ſucceſſive values of AK ($= \sqrt{c^2 + \overline{a - \frac{bx}{a}}^2}$) anſwering to the points of diviſion B, L, N, E, D (or P, Q, R, S, T) be computed, and repreſented by d, e, f, g, and h, reſpectively. Then, theſe values being the doubles of the correſponding ordinates Pp, Qq, Rr, Ss, Tt, it is evident, by the method of equidiſtant ordinates, that $\frac{7\times\overline{d+h} + 32\times\overline{e+g} + 12f}{90} \times \frac{1}{2}PT$, will expreſs the area of the curve, or half the ſuperficies of the cone, very nearly.

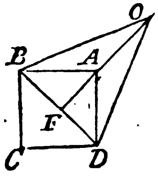


Now, in the caſe propoſed, AB being $= 12$, $AD = 9$, and $BD = 6$, we have $CE = 3 = a$; DG ($= \frac{AB^2 - AD^2 - BD^2}{2BD}$ $= \frac{9}{4}$; $CG = 5\frac{1}{4} = b$; and $AG^2 = 75{\cdot}9375 = c^2$. Here, therefore d ($= AB$) $= 12$; e ($= \sqrt{cc + aa + \frac{1}{2}bb + ab\sqrt{2}}$) $= 10{\cdot}9996$; f ($= \sqrt{cc + aa}$) $= 9{\cdot}2162$; g ($= \sqrt{cc + aa + \frac{1}{2}bb - ab\sqrt{2}}$) $= 8{\cdot}7432$; and h ($= AD$) $= 9$.

From whence ($\frac{7\times\overline{d+h} + 32\times\overline{e+g} + 12f}{90} \times NED$) the content of the whole, required, ſuperficies comes out $93{\cdot}13$ ſquare inches. By taking a greater number of ordinates, the anſwer may be brought out to any degree of exactneſs deſired, however great the inclination may be.

Mr. *Bamfield*, by an eaſy approximation, brings out nearly the ſame numbers with the above.

X. QUESTION 370 *anſwered by Mr.* J. Robinſon.



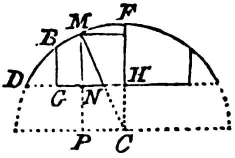
Let $a = OD = OB = 78$, $b = OA = 59{\cdot}161$, and $x = AF$. Then (per Eu. 47, 1) $bb + 2bx + 2xx = aa$. Reduced, $x = \sqrt{\frac{aa}{2} - \frac{bb}{4}} - \frac{b}{2}$: From whence the ſide of the ſquare will be found = 24 poles, and the area of the field = 3 acres, 2 roods, and 16 perches.

The ſame anſwered by Maſter John Birks, *a Tyro at* Goſberton *School.*

The three viſible corners of the field being repreſented by A, B, and D, it is evident, becauſe the given diſtances of the two laſt from the oak, at O, are equal, that the line OAF biſects the angle BAD, and conſequently that the angle $BAO = 135°$. From which, and the given ſides AO and BO, the angle ABO is found (by trigonometry) = 32° 26′; $AOB = 12° 34'$; and the ſide of the ſquare AB = 24 poles: Whence the area of the field = 3 acres, 2 roods, and 16 perches, which was required.

According to this laſt method it was alſo anſwered by Mr. *Richard Gibbons*, and Mr. *J. Milbourn*. Mr. *C. Tate*, Mr. *J. Wiggleſworth*, Mr. *J. Aſh*, and ſome others, ſolved it by different methods.

XI. QUESTION 371 *anſwered by Mr.* T. Coates, *Writing-maſter, at* Briſtol.



Put r = the radius CF of the generating circle, which is found (by common properties) = 51; alſo put $c = CH = 33$, $DH = b$, $HN\ (= CP) = x$, and $MN = y$.

Then, univerſally, $y = \sqrt{rr - xx} - c$: Therefore $yy = rr - xx + cc - 2c\sqrt{rr - xx} = bb - xx - 2cy$; and $py^2\dot{x} = p \times bb\dot{x} - xx\dot{x} - 2yc\dot{x} = \dot{S}$; whoſe fluent is $p \times \overline{bbx - \tfrac{1}{3}x^3 - 2c \times \text{area } MNHF} = S$: Which, when $x = 24$, gives the content 139·42 ale gallons.

The

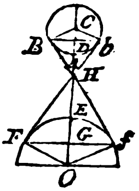
The ſame anſwered by Mr. John Wiggleſworth.

Let GH = half the length of the caſk $= 24 = a$, $BG = 12 = b$, $HF = 18 = c$; alſo put r = rad. $CM = CF = \frac{aa + \overline{c-b}^2}{2c - 2b}$ (by the nature of the circle) $m = PN = CH$, $p = 3.14159$ &c. $x = HN = CP$, and $y = MN$: Then $y = \sqrt{rr - xx} - m$; and therefore $p\dot{y}^2 x = p\dot{x} \times \overline{r^2 - x^2 + m^2} - 2mp\dot{x}\sqrt{rr - xx}$ = the fluxion of the ſolidity: Whoſe fluent $px \times \overline{r^2 - \frac{1}{3}x^2 + m^2} - 2mp \times$ area $CFMP$ = the ſolid deſcribed by the rotation of $HFMN$ about the line HN: And, if L be put for the length of the arc whoſe radius is r, and right ſine x, then $\frac{rL + {}^{\cdot\cdot}x + xy}{2}$ = area $CFMP$. Whence the ſolidity of the whole caſk, when $x = a$, and $y = b$, is $2p \times \overline{ar^2 - \frac{1}{3}a^3 - mrL - abm} = 39316.99$ cubic inches, or 139.4219 ale gallons.

Mr. *Richard Gibbons*, putting $x = 12$, makes the content, by Shirtcliff's Gauging, (p. 201) $= 0.080685x^3 = 139.4237$ ale gallons. Mr. *J. Milbourn*, by a different method, makes it 139.44; and Mr. *Charles Tate*, of Hull, 139.45.

XII. QUESTION 372 *anſwered by* Timothy Doodle.

Let O and C be the centers of the earth and moon, and H the place required: Suppoſe HF and HB to touch the two ſurfaces in F and B, and let FGf and BDb be perpendicular to OC.



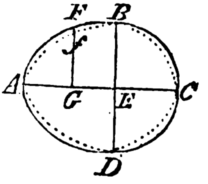
Put $a = OE = 3985$, $b = CA = 1085$, $c = OC = 245070$, and $x = OH$; and let $p = 2 \times 3.14159$ &c. So ſhall the circumference FEf, &c. $= pa$, and the circumference BAb, &c. $= pb$: And therefore the parts FEf, BAb of the two ſurfaces viſible to an eye at H, are equal to $pa \times EG$ and $pb \times AD$, reſpectively.

But, by ſimilar triangles, $OH\ (x) : OF\ (a) :: OF\ (a) : OG = \frac{aa}{x}$: Whence $EG = a - \frac{aa}{x}$: And, in the very ſame manner, $AD = b - \frac{bb}{c - x}$. Therefore, by ſubſtitution, $FEf + BAb = pa \times \overline{a - \frac{aa}{x}} + pb \times \overline{b - \frac{bb}{c - x}}$: Which being a maximum, $\frac{a^3}{x} + \frac{b^3}{c-x}$ muſt be a minimum; and its fluxion $-\frac{a^3\dot{x}}{xx} + \frac{b^3\dot{x}}{\overline{c-x}|^2} = 0$. Hence $\frac{x^2}{a^3} = \frac{\overline{c-x}|^2}{b^3}$; or $\overline{\frac{b}{a}}|^{\frac{3}{2}} \times x = c - x$, and conſequently $x = \frac{c}{1 + \overline{\frac{b}{a}}|^{\frac{3}{2}}} = 214585$. Therefore the place where friend *Bevil* muſt take his view is 210600 miles above the ſurface of the earth, if he can find his way up ſo high.

Mr. *Aſh*, Mr. *Wiggleſworth*, and Mr. *Bevil* the propoſer, (proceeding upon the ſame principles) bring out the ſame concluſion.

XIII. QUESTION 373 *anſwered by* Anth. Shallow, *Eſq.*

This queſtion, in the form it is propoſed, is indeterminate: The figure of the egg, as well as its principal dimenſions, ought to have been given; ſince, of an infinity of curves that may be deſcribed through the ſame, given, points, experience is not ſufficient to direct us which to chooſe; it not being known that ever two eggs were exactly of the ſame figure.



Let *AFBCD* be a ſection of the egg through its axis *AC*, and let *BD* be the given poſition of the greateſt ordinate. It is viſible that innumerable curves, *AFBCD*, *AfBCD*, &c. may be deſcribed thro' the given points *A*, *B*, *C*, and *D*, to cut *AC* and *BD* at right angles, conformable to the nature of the problem. But, the greateſt ordinate *BD* dividing the axis *AC* unequally, no curve of a lower order than the ſecond can poſſibly anſwer theſe conditions.

Let, therefore, *AFBC* be a curve of this order, whoſe equation is $yy = bx + cx^2 + dx^3$ (being the moſt ſimple the data will admit of): Alſo let $AC = p$, $AE = q$, $BE = r$: Then, by making $x = p$, and $y = 0$, our general equation becomes $bp + cp^2 + dp^3 = 0$, or $b + cp + dp^2 = 0$.

Alſo, by making $x = q$, and $y = r$, we have $bq + cq^2 + dq^3 = r^2$, or $b + cq + dq^2 = \frac{r^2}{q}$.

Laſtly, by making $b\dot{x} + 2cx\dot{x} + 3dx^2\dot{x}$ (the fluxion of $bx + cx^2 + dx^3$) $= 0$, and writing q in the room of x, we have $b + 2cq + 3dq^2 = 0$.

Now, from the three equations thus derived, d is found $= -\frac{rr \times \overline{2q-p}}{q^2 \times \overline{p-q}|^2}$; $c = \frac{rr \times \overline{3qq-pp}}{q^2 \times \overline{p-q}|^2}$; and $b = \frac{rrpq \times \overline{2p-3q}}{q^2 \times \overline{p-q}|^2}$. Therefore the general equation, in known terms, is $yy = \frac{rr}{q^2 \times \overline{p-q}|^2} \times \overline{\overline{2p-3q} \times pqx + \overline{3qq-pp} \times xx - \overline{2q-p} \times x^3}$.

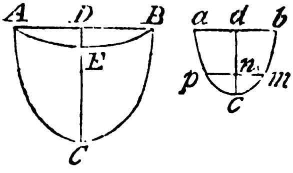
Whence, if a be put $= 3{\cdot}14159$ &c. we get $ayy\dot{x} = \frac{arr}{q^2 \times \overline{p-q}|^2} \times \overline{\overline{2p-3q} \times pqx\dot{x} + \overline{3qq-pp} \times x^2\dot{x} - \overline{2q-p} \times x^3\dot{x}}$ for the fluxion of the ſolidity. Whoſe fluent, when $x = p$, will be found $= ar^2p^3 \times \frac{6pq - 6qq - pp}{12q^2 \times \overline{p-q}|^2}$, expreſſing the true content of the whole ſolid. Which therefore is to ($arrp$) that of the circumſcribing cylinder, in the proportion of $p^2 \times \frac{6pq - 6qq - pp}{12q^2 \times \overline{p-q}|^2}$ to unity. This proportion, in the caſe propoſed, (where $p = 2\frac{1}{2}$, $q = 1\frac{1}{4}$, and $r = \frac{3}{4}$) becomes as $\frac{275}{432}$ to unity. Therefore the ſolidity (according to the above aſſumption) comes out 2·8124 cubic inches.

As to the ſuperficies, or ſhell of the egg, it may be alſo found from the ſame general equation; but it is hoped the facetious propoſer will himſelf determine that, and accept it as a proper reward for his trouble and induſtry in promoting uſeful ſcience.

Some correſpondents conſider the egg as formed of two unequal ſemi-ſpheroids; but this does not ſeem to agree well with the true figure, it being hard to conceive that the curvature ſhall be immediately changed by more than one-half, in paſſing from one ſide of the greateſt ordinate to the other.

XIV. QUES-

XIV. QUESTION 374 *anſwered by Mr.* W. Bevil.

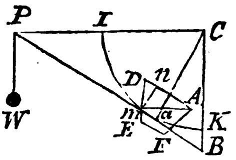


Let *acb* be a curve ſimilar to that (*ACB*) formed by the chain, ſuch that its ray of curvature (*a*) at the loweſt point *c* may $= 1$. Then, the area of the ſemi-curvilineal ſpace, *acd*, will be truly defined by $y\sqrt{1+zz}-z$, (as is proved by the writers on fluxions) z being $= ca$, and y $(= ad) =$ hyp. log. $\overline{z+\sqrt{1+zz}}$.

Hence, putting the length of the chain $= c$, we have (by the general property of ſimilar ſigures) as $\overline{z+y}|^2 : (\frac{1}{4}cc\ (\overline{AC+AD}|^2) :: y\sqrt{1+zz}-z : \frac{1}{4}cc \times \frac{y\sqrt{1+zz}-z}{\overline{z+y}|^2}$ $=$ area *ACD*: Which being a maximum, let its fluxion be therefore taken and made $= 0$; whence, after proper reduction, there will come out $\frac{1}{2}y\dot{z} \times \frac{z+y}{1+\sqrt{1+zz}} = y\sqrt{1+zz}-z$. From which equation (by the known methods of approximation) the values of z and y may be found. For, having aſſumed for z, y will be given from the equation $y =$ hyp. log. $\overline{z+\sqrt{1+zz}}$; and then, by ſubſtituting theſe values of z and y in the above equation, the error will be known; and from thence, by repeating the operation, &c. the true value of z; which comes out $= 5{\cdot}462$; and $y = 2{\cdot}399$. Then $7{\cdot}861\ (ac+ad) : 2{\cdot}399\ (ad) :: \frac{1}{2}c\ (AC+AD) : AD = 0{\cdot}15259 \times c$: Whence $AB = 0{\cdot}30518 \times c = 15{\cdot}259$ inches.

Anthony Shallow, Eſq; ſolves this problem exactly in the ſame manner. But Mr. *Timothy Doodle*, and Mr. *O'Cavanah*, taking the meaning of the queſtion in a different ſenſe [ſuppoſing the arch *ACB*, and not *ACB* + *AB*, to be given $= 50$] bring out $AB = 33{\cdot}575$ for the anſwer. In which caſe it appears that *AD*, *CD*, and *AC*, will be in the ratio of 0·6715, 0·6656, and 1, reſpectively; and that the area *ACB* is to the ſquare of the arch *ACB*, as 0·1549 to unity.—— But there is yet another way in which the queſtion may be taken, as it is not ſpecified whether the chain is to be faſtened to the pins, (in which caſe the area will be the greateſt poſſible) or whether it is ſuffered to ſlide freely over them, till the two parts thereof (here repreſented by ACB and AEB) acquire an equilibrium. In this laſt ſenſe the ſolution will be ſtill more complex; but the beſt method will be to aſſume two arcs, acb ($= 2Z$) and pcm ($= 2z$), of the ſame given catenaria, ſimilar to the two parts ACB and AEB of the chain. From whence (by the properties of the catenaria, and the conſideration of ſimilar figures, and of the equal action of the two branches of the chain at B) there will be had $\frac{\sqrt{1+ZZ}}{\sqrt{1+zz}} = \frac{\text{hyp. log. } \overline{Z+\sqrt{1+ZZ}}}{\text{hyp. log. } \overline{z+\sqrt{1+zz}}}$; and $\frac{z\times\overline{1+ZZ} - Z\times\overline{1+zz}}{\overline{z\sqrt{1+ZZ}+Z\sqrt{1+zz}}\,^{2}}$, a maximum. From which the values of z and Z may be determined.

The PRIZE QUESTION *anſwered by Mr.* J. Aſh.



'Tis plain that the leaſt weight will equipoize the veſſel in any degree of elevation when the cord Pa is perpendicular to Ca: Then, putting $s =$ ſolidity of the whole fruſtum, $c = \cdot 7854$, $w =$ tenſion of the cord, $n =$ the ratio of Dn to mn, (Am being the horizontal ſurface of the water) and $x^2 =$ diameter of the fruſtum at m, we have $\frac{a-x^2}{2} = Dn$, $\frac{na+nx^2}{2} = mn$, $\frac{a+x^2}{2} = An$, and, per theorem Diary 1744, Queſt. 240, $\frac{ca^3 - ca^{\frac{3}{2}}x^3}{2} \times \frac{n}{3} =$ (becauſe $n = 3$) $\frac{ca^3 - ca^{\frac{3}{2}}x^3}{2}$, the ſolidity of the hoof $DnAm$; therefore $\frac{2s - ca^3}{2} + \frac{ca^{\frac{3}{2}}x^3}{2} =$ the ſolidity of the remaining water, which let $= qx^3 - p$: Then (per ſtone) Ca : ſine of Ka :: $qx^3 - p : w$; or, which is the ſame thing, (becauſe the triangles are ſimilar) Am $\left(\sqrt{\frac{10a^2 - 16ax^2 + 10x^4}{4}}\right)$: mn $\left(\frac{na - nx^2}{2}\right)$ :: $qx^3 - p : w = \frac{\overline{na - nx^2} \times \overline{qx^3 - p}}{\sqrt{10a^2 - 16ax^2 + 10x^4}}$, the tenſion, a max. and conſequently $\frac{\overline{a-x}^2 \times \overline{ax^3-p}}{\sqrt{a^2 - 1{\cdot}6ax^2 + x^4}}$, thrown into fluxions and properly reduced, gives $x^2 = 13{\cdot}6$ fere. Hence the weight required $= 24{\cdot}7054$, and $PC = 201{\cdot}57$.

The ſame anſwered by Mr. Patrick O'Cavanah.

Put $a = 15 =$ the depth of the fruſtum, $b = 20$ the greateſt diameter AD, $c = 10 =$ the leaſt diameter EF, $p = {\cdot}7854$, and $(2am) = x$. Then, ſuppoſing mn perpendicular to DA, it will be $b-c : b-x :: a : a \times \frac{b-x}{b-c} = mn$. Whence, by a well-known theorem for the content of a conical ungula, the ſolidity of ADm is $= \frac{1}{3}pab \times \frac{bb - x\sqrt{bx}}{b-c}$. Which, ſubtracted from $\frac{1}{3}pa \times \overline{bb+bc+cc}$, the content of the whole fruſtum $AFED$, leaves $\frac{1}{3}pa \times \frac{bx\sqrt{bx} - c^3}{b-c}$ for the part $AFEm$, in which the water is contained. Now the tenſion of the rope, or the force of the weight W, acting at right angles to Ca, ſo as to ſuſtain the water in this poſition, is known to be in proportion to the weight of the water in the veſſel, as the ſine of the angle CPa (or DAm) to the radius; that is, as mn to Am, or, in ſpecies, as $a \times \frac{b-x}{b-c}$ to $\sqrt{\frac{a^2 \times \overline{b-x}|^2}{\overline{b-c}|^2} + \frac{\overline{b+x}|^2}{4}}$. Whence, by the queſtion, $\frac{\overline{b-x} \times \overline{bx\sqrt{bx} - c^3}}{\sqrt{a^2 \times \overline{b-x}|^2 + \frac{1}{4} \times \overline{b-c}|^2 \times \overline{b+x}|^2}}$ (which is in a given ratio to $AFEm \times \frac{mn}{Am}$) muſt be a minimum. This expreſſion, by putting $yy = bx$, $d = \frac{b-c}{2}$, and $f = \frac{aa-dd}{aa+dd} \times 2bb$, and dividing the denominator by $\sqrt{aa+dd}$, is reduced to $\frac{\overline{bb-yy} \times \overline{y^3-c^3}}{\sqrt{b^4 - f^2y^2 + y^4}}$. Which, in fluxions, &c. gives $\frac{3y}{y^3-c^3} - \frac{2}{bb-yy} + \frac{ff-2yy}{b^4-f^2y^2+y^4} = 0$. Hence $y = 16{\cdot}516$, $x = 13{\cdot}7578$, $CP = 202{\cdot}66$, and $W = 24{\cdot}709$ pounds.

Remark. As the taking of the fluxions of expreſſions compounded like that in the preceding ſolution, is ſomewhat troubleſome, the following method may, in ſuch caſes, be of uſe.

Seeing $\frac{\overline{y^3-c^3}\times\overline{b^2-y^2}}{\sqrt{b^4-f^2y^2+y^4}}$ is to be a maximum, by the queſtion, the logarithm thereof, or its equal, log. $\overline{y^3-c^3}$ + log. $\overline{b^2-y^2}$ − ½ log. $\overline{b^4-f^2y^2+y^4}$, muſt alſo be a maximum, and conſequently its fluxion $\frac{3y^2\dot{y}}{y^3-c^3}-\frac{2y\dot{y}}{bb-yy}+\frac{ff y\dot{y}-2y^3\dot{y}}{b^4-f^2y^2+y^4}=0$: Whence, dividing by $y\dot{y}$, we have $\frac{3y}{y^3-c^3}-\frac{2}{bb-yy}+\frac{ff-2y^2}{b^4-f^2y^2+y^4}=0$, the ſame as before. Which equation ſtands in a much better form, for a ſolution, than that immediately reſulting from the common method. In like ſort, ſuppoſing $\frac{\overline{a^2-x^2}|^{\frac{1}{2}}\times\overline{b^3+x^3}|^{\frac{1}{3}}\times\overline{c^4-x^4}|^{\frac{1}{4}}}{\overline{a-x}|^2\times\overline{a^2+2dx+x^2}}$ was to be a maximum, or minimum, we ſhould have ½ log. $\overline{a^2-x^2}$ + ⅓ log. $\overline{b^3+x^3}$ + ¼ log. $\overline{c^4-x^4}$ − 2 log. $\overline{a-x}$ − log. $\overline{a^2+2dx+x^2}$ a max. or min. And conſequently $-\frac{x}{a^2-x^2}+\frac{x^2}{b^3+x^3}-\frac{x^3}{c^4-x^4}+\frac{2}{a-x}-\frac{2d+2x}{a^2+2dx+x^2}=0$. And ſo of the others.

No ſolution came in time this year to obtain the prize.

Paradoxes anſwered.

PARADOX I. *anſwered by Mr.* R Pearſon.

Let Cottam tell his lord, he muſt
Dwell underneath the north pole juſt:
Then, let him viſit age, or youth,
His courſe he'll ſurely ſteer full ſouth.

PARADOX II. *anſwered by* Hodge, *the Miller.*

The man had his ſon and daughter by two ſeveral women; and the eſtate was ſettled on the daughter's mother and her heirs.——Mr. *J. Moreland* anſwers them both the ſame way.

The Eclipſes calculated for 1754, *by Mr.* Ralph Hulſe.

There will happen ſix eclipſes this year; but, what is remarkable, not one of them will be viſible to any part of Great Britain, or Ireland: At other places they will be ſeen according to the following order.

1. March 23d, at 6 afternoon, the ☉ is 2 digits eclipſed on the north ſide, vertical to a little ſea, weſt of Terra Firma, lat. 8° north, long. 90° weſt. Viſible in North America.

2. April 7th, at 4 in the morning, the ☽ is eclipſed totally 21 digits, vertical to the eaſtern borders of Peru, lat 6° ſouth, long. 70° weſt. Viſible to all America.

3. April 22d, at ten in the morning, the ſun is 2 digits eclipſed on the ſouth ſide, vertical to the eaſtern parts of Nigritia, in Africa, lat. 12° north, long. 30° eaſt. Viſible to the ſouthern ſeas.

4. Sept. 16th, at 1 afternoon, the ☉ is eclipſed in ♍ 23°, towards the eaſtern ocean, beyond the Phillipine Iſlands, lat. 3° N. long. 163° E. This eclipſe is ſtill leſs than the former.

5. Oct. 1ſt, at 6 in the morning, the moon is totally eclipſed 21 digits, in ♈ 8°, viſible to all America, vertical to the ſea weſt of Panama, lat. 3° north, long. 90° weſt.

6. Oct. 16th, at 1 in the morning, the ſun is eclipſed 2 digits on the north ſide in ♎ 22°, vertical to Medelzar, lat. 8° ſouth, long. 130° weſt. Viſible within the arctic circle.

As we do not know what tables Mr. *Hulſe* made uſe of in theſe calculations, we cannot ſatisfy the public in that particular, nor take upon us to judge of their exactneſs, not having made any calculations of theſe eclipſes ourſelves, as they will be all inviſible to us.

New Queſtions.

I. QUESTION 376, *by* Ruſticus.

An honeſt man a horſe did buy,
That was both lame and poor:
A golden guinea was the price,
And five good ſhillings more.
This horſe he fed with corn and hay,
Till he ſeem'd wond'rous ſound:
When, meeting with another chap,
He ſold him for three pound.

By which he loſt half the prime coſt,
One-fourth o'th' keeping too.
What did the keeping ſtand him in?
What did he loſe, ſay you?

II. QUESTION 377, *by Miſs* Maria A—t—ſ—n.

There are three cities, *A*, *B*, and *C*, lying in the ſame road; whereof the firſt is 136 miles diſtant from the ſecond, and the ſecond 104 miles diſtant from the third: From *A* to *B* a courier travelled in two days; and from *B* to *C* in two days more, diminiſhing his diſtance every day alike, from the firſt to the laſt. What number of miles did he travel each particular day.

III. QUESTION 378, *by Mr.* Charles Tate.

My wife's a ſcold, a niggard, and a ſlut,
And ev'ry day ſhe's ſure to pay my ſcott;
And yet for what, no mortal e'er can tell,
Unleſs her courage riſe from living well:
The which to tame, that I may live in quiet,
I am reſolv'd henceforth to ſtint her diet,
In quantity, to what it was before,
As *e* to *a*; which, gentlemen, explore,
From the equations * that you ſee ſubjoin'd:
Elſe come and take my place—if you've a mind.

$$\text{* Given}\begin{cases}\overline{aa+ee}\times\dfrac{e}{a}=b=83{\cdot}2.\\[1ex]\overline{aa-ee}\times\dfrac{a}{e}=c=1920.\end{cases}$$

IV. QUESTION 379, *by Mr.* John Morland.

Two perſons, *A* and *B*, having an equal claim to an annuity of 100l. to continue for 30 years, agree to ſhare it between them in this manner, viz. *A* for his part is to enjoy the whole annuity for the firſt 10 years; *B* and his heirs being to have the entire reverſion thereof for the remaining 20 years. The queſtion is, To find the rate of intereſt allowed in this contract, with the preſent value of the annuity correſponding.

V. QUESTION 380, *by* W. T—t.

The ſum of the ſquares of the two diagonals, of any trapezium, together with the ſquare of twice the line joining *their* middle points, is equal to the ſum of the ſquares of all the four ſides of the trapezium. A demonſtration of this is required.

VI. QUESTION 381, *by* Bathonius.

Two ſhips ſail, at the ſame time, from two ports under the ſame meridian, whoſe difference of latitude is 1° 25′. That from the ſouthermoſt port runs due eaſt at the rate of 4½ miles per hour; and that from the northermoſt E.S.E. at the rate of 7 miles per hour: I demand the diſtance ſailed by each ſhip, when they are at their neareſt diſtance from each other, and alſo what the diſtance will be.

VII. QUESTION 382, *by Mr.* Thomas Moſs.

To determine the leaſt triangle that can be circumſcribed about a given triangle, whereof the three ſides are 8, 10, and 12 inches.

VIII. QUESTION 383, *by* Anthony Shallow, *Eſq.*

To draw a right line parallel to a given line, which may cut three other lines given by poſition, in ſuch ſort, that the rectangle under the two parts thereof, intercepted by thoſe lines, may be given in magnitude.

IX. QUESTION 384, *by Mr.* Tho. Moſs.

Sailing due north, at the rate of 4 knots, in a current, a certain ſmall iſland bore E.N.E. from us, at the diſtance of 40 miles: After running 12 miles (by the log.) it bore due eaſt; and having run 16 miles more, upon the ſame courſe, its bearing was then found to be S.E. To determine, from theſe obſervations, the direction and velocity of the current.

X. QUESTION 385, *by* W T—t.

The vertical angle of a triangle being = 70°, and the ſum of the two including ſides = 100 feet; to determine the triangle itſelf, when the perpendicular is a mean proportional between the whole baſe and one of its two ſegments.

XI. QUESTION 386, *by Mr.* Timothy Doodle.

Within a rectangular garden, containing juſt an acre of ground, I have a circular fountain, whoſe circumference is 28, 40, 52, and 60 yards diſtant from the four angles of the garden. From theſe dimenſions the length and breadth of the garden, and likewiſe the diameter of the fountain, are required.

XII. QUESTION 387, *by Mr.* Patrick O'Cavanah, *of* Dublin.

In the latitude of 51° 32′ north ſtand two pillars S. W. and N. E. of one another, at the diſtance of 200 feet: The height of the ſouthermoſt pillar is 60 feet, and that of the northermoſt 40 feet. At what time of the day, on June 20, do the ſhadows of their ſummits approach the neareſt to each other?

XIII. QUESTION 388, *by Mr.* Timothy Doodle.

Suppoſing p, q, r, s, t, &c. to repreſent the tangents of any number of arcs P, Q, R, S, T, &c. equal, or unequal: To determine a general expreſſion for the tangent of the ſum $(P+Q+R+S+T+\&c.)$ of all thoſe arcs; the common radius being unity.

XIV. QUESTION 389 *by Mr.* E. R——n.

To determine the ratio of the denſities of the ſun and earth, independent of the ſun's parallax.

XV. QUESTION 390, *by* Anthony Shallow, *Eſq.*

Having given any three computed viſible latitudes of the moon, in a ſolar eclipſe, together with the correſponding differences of longitude of the ſun and moon: To ſhew the manner of finding, from thence, the true time of the greateſt obſcuration, and likewiſe the neareſt approach of the two centers.

The PRIZE QUESTION, *by* Anthony Shallow, *Eſq.*

To determine the figure which the piers (or the ſtarlings) of a bridge ought to have, ſo that the length, and greateſt breadth of each, and their diſtances from one another, being given, the water in its paſſage through the bridge ſhall ſuffer the leaſt reſiſtance poſſible.

N. B. The perſon who gives the beſt ſolution to this queſtion will be intitled to a prize of ſix Diaries: And whoever truly anſwers it before Candlemas-day, will have a chance, by lot, to win the ſame number of Diaries.

Queſtions

1755.

Queſtions anſwered.

I. QUESTION 376 *anſwered by Mr.* W. Liſton.

THREE pounds, two ſhillings, and eight-pence, the keeping it did coſt;
One pound, eight ſhillings, and eight-pence, was what the poor man loſt.

The ſame anſwered by Mr. Edward Gallyatt.

Put $4x =$ keeping, $2a = 26$, and $b = 60$: Then, per queſt. $4x + 2a - b = a + x$; whence $x = \frac{b-a}{3} = 15$ s. 8 d. Conſequently $4x = 3$ l. 2 s. 8 d. = the charge of keeping, and $a + x = 1$ l. 8 s. 8 d. = the money loſt.

The ſame anſwered by Mr. Phil. Williams.

Let $a = 26$, $b = 60$, and $x =$ the keeping; then $a + x - b = \frac{a}{2} + \frac{x}{4}$ (per queſt.) whence $x = \frac{4b - 2a}{3} = 62\frac{2}{3}$.

The corn and hay appear from hence
To coſt three pounds, two and eight-pence;
The money that the good man loſt,
Was eight groats more than the prime coſt.

Anſwers to this queſtion were likewiſe received from Mr. *G. Brownbridge*, Mr. *T. Barker*, Mr. *W. Peer*, Mr. *T. Boſton*, Mr. *R. Butler*, Mr. *T. Eſtob*, Mr. *W. Gawthorpe*, Mr. *F. Griffiths*, Mr. *G. Hicks*, Mr. *Samuel Koit*, Mr. *T. Lover*, Mr. *T. Padſon*, Mr. *T. Prichard*, Mr. *William Richardſon*, Mr. *T. Schalar*, Mr. *Benj. Thearle*, Mr. *Ja. Vacary*, Mr. *R. Younge*, and ſeveral others.

II. QUESTION 377 *anſwered by Mr.* Samuel Koit.

Let $2a = 136$, $2b = 104$, and $2x =$ the common difference of each day's journey; then $a + x$, $a - x$, $b + x$, and $b - x$ will be the reſpective diſtances travelled each day: But the firſt + the third = twice the ſecond, that is, $a + b + 2x = 2a - 2x$; whence $4x = a - b$, and $x = \frac{a-b}{4} = 4$. Therefore 72, 64, 56, and 48, are the four diſtances required.

The ſame anſwered by Mr. W. Gawthorpe.

Put $x =$ the firſt day's journey, and $y =$ the common difference: Then $2x - y = 136$, and $2x - 5y = 104$ (per queſt.); and, by ſubtracting the latter equation from the former $4y = 32$: Whence $y = 8$, and $x = \frac{136 + 8}{2} = 72$.

According to the one, or the other, of the above methods, it was likewiſe anſwered by Mr. *R. Butler*, Mr. *G. Brownbridge*, Mr. *T. Barker*, Mr. *T. Boſton*, Mr. *W. Beer*, Mr. *T. Drury*, Mr. *T. Eadon*, Mr. *T. Eſtob*, Mr. *E. Gillyatt*, Mr. *E. Griffiths*, Mr. *Ja. Giles*, Mr. *E. Johnſon*, Mr. *Alex. Rowe*, Mr. *J. Richardſon*, Mr. *G. Reed*, Mr. *T. Scholar*, Mr. *Ja. Vicary*. Mr. *Phil. Williams*, Mr. *R. Younge*, and many others.

III. QUESTION 378 *anſwered by* J. Milbourn.

Multiplying the given equations croſswiſe into each other, we have $\overline{ab - ee} \times \frac{ba}{e} = \overline{aa + ee} \times \frac{ce}{a}$; whence $a^4 - \frac{b+c}{b} ee \times aa = \frac{ce^4}{b}$; and, by compleating the ſquare, &c. $aa = ee \times \frac{b + c + \sqrt{bb + 6bc + cc}}{2b} = ee \times 25$; conſequently, $a = 5e$: Whence, by ſubſtitution, &c. $e = 4$, and $a = 20$.

The ſame anſwered by Mr. Tho. Todd.

Put $\sqrt{xy} = a$, and $\sqrt{\frac{x}{y}} = e$; then $\frac{1}{y} = \frac{e}{a}$, and $y = \frac{a}{e}$; whence, by ſubſtitution, $\overline{xy + \frac{x}{y}} \times \frac{1}{y} = b$, and $\overline{xy - \frac{x}{y}} \times y = c$: From the firſt of which we get $yy = \frac{x}{b - x} = \frac{c + x}{x}$, by the ſecond. Hence $x = \sqrt{\frac{bc}{2} + \frac{\overline{c - b}|^2}{16}} + \frac{b - c}{4} = 4$. Therefore $y = 5$, $a = 20$, and $e = 4$.

The ſame anſwered by Mr. W. Enefer.

Let $re = a$; then, by ſubſtitution, $\overline{rree + ee} \times \frac{1}{r} = b$, and $\overline{rree - ee} \times r = c$: By multiplying theſe equations croſswiſe, we have $br^4 - br^2 - cr^2 = c$, or (putting $1 + \frac{c}{b} = 2m$, and $\frac{c}{b} = n$) $r^4 - 2mr^2 = n$; whence $r = \sqrt{m + \sqrt{n + mm}} = 5$; and from thence e $(= \sqrt{\frac{br}{rr+1}})$ $= 4$, and a $(= re) = 20$.

The ſame anſwered by Sylvius.

Let $\frac{a}{e} = x$; then will $\frac{e}{a} = \frac{1}{x}$, and the given equations will become $\overline{eexx + ee} \times \frac{1}{x} = b$, and $\overline{eexx - ee} \times x = c$; therefore $ee = \frac{bx}{xx+1} = \frac{c}{x^3 - x}$; whence $bx^4 - bx^2 = cx^2 + c$; and, by reduction, $x = \overline{\sqrt{\frac{c}{b} + \frac{\overline{b+c}|^2}{4bb}} + \frac{b+c}{2b}}|^{\frac{1}{2}} = 5$; hence $a = 20$, and $e = 4$.

Mr. *J. Aſh*, Mr. *T. Barker*, Mr. *W. Bevil*, Mr. *T. Boſton*, Mr. *A. Brooke*, Mr. *Hugh Brown*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *W. Cottam*, Mr. *J. Eadon*, Mr. *E. Gillyatt*, Mr. *Ja. Hemingway*, Mr. *E. Johnſon*, Mr. *Wm. Kingſton*, Mr. *T. Moſs*, Mr. *J. Nichols*, Mr. *G. Reed*, Mr. *W. Richardſon*, Mr. *Alex. Rowe*, Mr. *W. Trott*, Mr. *Harland Widd*, and Mr. *R. Younge*, likewiſe anſwered the ſame in a neat, conciſe manner, by equations not exceeding a quadratic.

IV. QUESTION 379 *anſwered by Mr.* W. Kingſton.

Let x = the rate of intereſt, and $u = 100$: Then per Ward's theorem, we have $\frac{u}{x-1} - \frac{u}{x^{10} \times \overline{x-1}}$ = preſent worth for 10 years, and $\frac{u}{x-1} - \frac{u}{x^{30} \times \overline{x-1}}$ = preſent worth for 30 years; hence $\frac{x^{30}-1}{x^{30}} = 2 \times \frac{x^{10}-1}{x^{10}}$, or $2x^{20} - x^{30} = 1$; which, ſolved, gives $x = 1{\cdot}049298$; from which the required value of the annuity comes out 1594l. 13s.

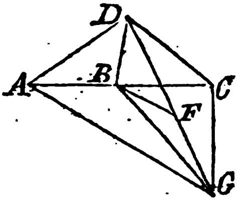
The ſame anſwered by Mr. Hugh Brown.

Let R be the amount of 1l. in one year; then, by the queſtion and the doctrine of annuities, we have $2 \times \frac{1 - R^{-10}}{R - 1}$ =

$= \frac{1-R^{30}}{R-1}$; $\because R^{30} - 2R^{20} + 1 = 0$; which, divided by $R^{10} - 1$, gives $R^{20} - R^{10} - 1 = 0$: Whence $R = \sqrt[10]{\frac{1+\sqrt{5}}{2}}$ $= 1{\cdot}049297$, &c. and the preſent value ſought = 1594 l. 13 s. $0\frac{1}{2}$ d.

Anſwers to this queſtion were likewiſe received from Mr. *J. Aſh*, Mr. *A. Brooke*, Mr. *S. Bamfield*, Mr. *W. Bevil*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *W. Cottam*, Mr. *R. Gibbons*, Mr. *T. Moſs*, Mr. *Ja. Robinſon*, *Sylvius*, and Mr. *Harland Widd*.

V. QUESTION 380 *anſwered by Mr.* J. Randles, *Teacher of Mathematics at* Wem, *in* Shropſhire.



Let the two diagonals AC and DG be biſected in B and F: Then, by the 12th of 2d of Simpſon's Geometry, $AD^2 + CD^2 = 2AB^2 + 2BD^2$, and $AG^2 + GC^2 = 2AB^2 + 2BG^2$, and conſequently $AD^2 + CD^2 + AG^2 + GC^2 = 4AB^2 + 2BD^2 + 2BG^2$; but DG by hyp. is alſo biſected by BF, and ſo $BD^2 + BG^2 = 2DF^2 + 2BF^2$ or $2BD^2 + 2BG^2 = 4DF^2 + 4BF^2$; whence, by equal ſubſtitution, $AD^2 + CD^2 + AG^2 + GC^2$ $(= 4AB^2 + 4DF^2 + 4BF^2) = AC^2 + DG^2 + 2BF^2$. *Q. E. D.*

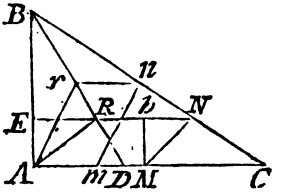
In the very ſame manner it was demonſtrated by Mr. *Brown*, Mr. *Moſs*, Mr. *Watſon*, and Mr. *Younge*.——Mr. *Cottam* and Mr. *Enefer* proved it by different methods.

VI. QUESTION 381 *anſwered by Mr.* J. Aſh.

Let A and B repreſent the two ports, a the diſtance between them = 85 miles, m and n the ſine and coſine of the angle EBN, q = the given ratio, and $x = BN$ = the diſtance ſailed by one of the ſhips; then $mx = EN$, $nx = EB$, and $qx = AM$; whence $a - nx = AE = hM$, and $mx - qx$ (which let $= px$) $= hN$, and then $ppxx + aa - 2nax + nnxx = MN^2$, a minimum; which, put in fluxions and reduced, gives $x = \frac{an}{pp + nn} = 144{\cdot}3$; whence $AM = 92{\cdot}764$, and MN (the neareſt diſtance) $= 50{\cdot}311$.

The ſame anſwered by Mr. R. Younge, *Teacher of Mathematics in* Cheſter.

Construction. Suppoſing the two courſes to interſect in C, take CD to CB in the given ratio of the celerity in AC to that in BC; and having drawn BD, make AR perpendicular thereto; make alſo RN parallel to AC, and NM parallel to AR, ſo ſhall M and N repreſent the required places of the two ſhips when they are the neareſt poſſible to each other.



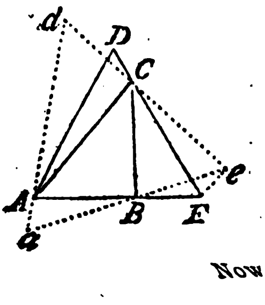
Demonstration. From any point r in BD, draw rA and rn, the latter parallel to AC; alſo draw nm parallel to Ar. By conſtruction and ſimilar triangles, $Bn : nr \ (Am) :: BC : CD ::$ the velocity in BC : velocity in AC; whence it is evident, that n and m are always contemporary poſitions: But Ar, or its equal mn, is, evidently, the leaſt poſſible when Ar coincides with AR, or when $mn = MN = AR$. *Q.E.D.*

Calculation. As $7 + 4\frac{1}{2} : 7 - 4\frac{1}{2} ::$ co-tang. $\frac{1}{2}C$ ($11^\circ\ 15'$) : tang. $\frac{CDB - CBD}{2} = 47^\circ\ 32'$; whence $CBD = 31^\circ\ 13'$, $ABR = 36^\circ\ 17'$, $AR\ (= MN) = 50{\cdot}302$, $RN\ (= AM) = 92{\cdot}79$, and $BN = 144{\cdot}34$. *W.W.R.*

Anſwers to the ſame were likewiſe received from Mr. *S. Bamfield*, Mr. *W. Bevil*, Mr. *G. Brownbridge*, Mr. *R. Butler*, Mr. *T. Cottam*, Mr. *W. Eneſer*, Mr. *R. Gibbons*, Mr. *E. Gallyatt*, Mr. *E. Johnſon*, Mr. *W. Kingſton*, Mr. *J. Nichols*, Mr. *Ben. Thearle*, and ſeveral others.

VII. **Quest.** 382 *anſwered by the Propoſer, Mr.* Moſs.

It is evident, that one ſide AE of the required triangle AED muſt fall upon, or coincide with, one ſide AB of the given triangle ABC; for, if another equilateral triangle aed be deſcribed about ABC, near to the former, the ſide thereof (ae) will be greater than the ſide AE of the former, becauſe both the angles aAB and BEe being obtuſe, Ba will be greater than BA, and Be greater than BE, and conſequently ae ($Ba + Be$) greater than AE ($BA + BE$).



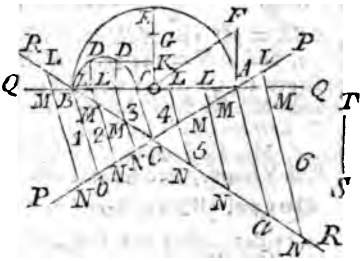
Now, all the three ſides of the triangle ABC being given, the angle BAC may be found: Then it will be as the s. E (60°) : AC (12) :: s. ACE (120° — BAC) : AE; which quantity (as the two firſt terms are given) will be the leaſt when 120° — BAC is the leaſt, that is, when BAC is that angle of the given triangle which is neareſt (below) the angle of the equilateral one. In the preſent caſe, the angle BAC (oppoſite to the mean ſide) being 56° 46′, AE will be found 12·48, and the area of the triangle 67·43 ſquare inches. *W.W.R.*

Note, An equilateral triangle, conſtituted on the longeſt of the three given lines as a baſe, will be leſs than that above determined; but cannot be ſaid to be deſcribed about the given triangle (as the queſtion requires) ſince all the angles of the latter are not ſituate in the ſides of the former.

This queſtion being wrong printed (by omiſſion of the word *equilateral*) no anſwer to it, beſides the propoſer's, has been received.

VIII. QUESTION 383 *anſwered by Mr.* H. Watſon.

Let B, A, and C be the points of interſection of the three given lines QBQ, RCR, and PAP; and let Cc be the line to which the required line LMN is to be parallel; moreover, let ST be the ſide of a ſquare, equal in area to the rectangle which is to be contained under the parts LM, MN, of the ſaid line.



It is evident, that the problem (unleſs it be reſtrained by giving ST too large) will admit of ſix anſwers, or different poſitions of the line LN: But the determination of all theſe depends upon two caſes only.

Conſtruction of Caſe 1. (No. 1.) Upon AB let a ſemicircle be deſcribed, and at c erect the perpendicular cE, meeting the circumference in E; make AF alſo perpendicular to BA and equal to a fourth proportional to Cc, cE, and ST; and, from F to the center O, draw OF; take $OM = OF$, and through M draw LN parallel to Cc, and the thing is done.

Demonſtration. It is evident, (Euc. 6. 2.) that $AM \times BM$ $(= OM^2 - OA^2 = OF^2 - OA^2) = AF^2$: And, by ſimilar triangles, $\begin{array}{l} ML:BM::Cc:Bc \\ MN:AM::Cc:Ac \end{array}$; therefore $ML \times MN : AM \times BM\ (AF^2) :: Cc^2 : Bc \times Ac\ (cE^2)$. But (by conſtruction) $Cc^2 : cE^2 :: ST^2 : AF^2$; whence, by equality, $ML \times MN : AF^2 :: ST^2 : AF^2$, and conſequently $ML \times MN = ST^2$. *Q.E.D.*

Conſtruction of Caſe 2. (No. 2 and 3.) Draw Bb parallel to Cc, meeting PAP in b; upon Bc let a ſemicircle be deſcribed; and in cE, perpendicular to Bc, take cG a mean proportional between Bb and Cc, and take cK a fourth proportional to cG, Bc, and ST; draw KD parallel to BA, cutting the circumference of the circle in D, D; alſo draw DL perpendicular to BA, and then draw LMN parallel to Cc.

Demonſtration. By ſimilar triangles, $\begin{array}{l} LM:BL::cC:Bc \\ MN:cL::Bb:Bc \end{array}$; therefore $LM \times MN : BL \times cL\ (LD^2) :: cC \times Bb\ (cG^2) : Bc^2$. But (by conſtr.) $cG^2 : Bc^2 :: ST^2 : cK^2\ (LD^2)$; whence, by equality, $LM \times MN : LD^2 :: ST^2 : LD^2$; and conſequently $LM \times MN = ST^2$. *Q.E.D.*

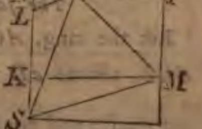
The ſixth poſition of LN has the very ſame conſtruction with the firſt; and the fourth and fifth are determined like the ſecond and third: But theſe four, when ST exceeds a certain limit, are impoſſible.——Beſides the two caſes above, there are other particular ones; ſuch as, when all the three lines meet in a point, or when two of them are parallel, &c. But theſe, being much eaſier than the above, need not, I preſume, be inſiſted on.

Mr *Moſs* ſent a conſtruction of this problem, very little different from the above.

⁂ *In the fig. for OE read* cE.

IX. QUESTION 384 *anſwered by Mr.* Rich. Gibbons.

Conſtruction. By the firſt bearing and diſtance lay down the right-angled triangle SMK, ſo that M may repreſent the iſland, and S the ſhip, which, at the ſecond obſervation, muſt be in the eaſt and weſt line KM: Produce SK to R, ſo that $KR = KM$; and draw MR, in which the ſhip muſt be at the laſt obſervation. Moreover, as the given diſtances are in the ratio of 3 to 4, make $KN = \frac{4}{3}$ of SK; and draw NO parallel to KM, meeting RM in O, which is the ſhip's true place. From



this conſtruction (taking $SL = 28$) the diſtance LO, run by the current in ſeven hours, is found to be 18·25, and its direction (NLO) $= 64^\circ\ 58'$.

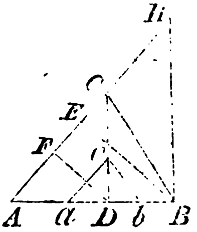
The ſame anſwered by Mr. S. Bamfield.

Let S be the place of the ſhip at the firſt obſervation, SN the meridian, M the iſland, and MK a perpendicular to SN, &c. Then (per trig.) $MK (= NP) = 36·95$, and $SK = 15·31$; which, per log is but 12; ſay therefore $12 : 15·31 :: 16 : 20·413 = KN = MP = PO$ (becauſe the angle $POM = 45^\circ$). Hence $SN = 35·723$, LN $(= 35·723 - 28) = 7·723$, and $ON (= NP - OP) = 16·54$. Hence, per trig. $SO = 39·36$ = the true diſtance ſailed, $NLO = 64^\circ\ 59'$ = the current's direction, and $LO = 18·258$ = the diſtance run by the current in ſeven hours, being at the rate of 2·608 miles per hour. *W. W. R.*

In the ſame manner it was anſwered by Meſſ. *Aſh*, *Brown*, *Brownbridge*, *Enefer*, *Kingſton*, *Milbourn*, *Moſs*, *Peart*, *Robinſon*, *Thearle*, and *Widd*.

X. QUESTION 385 *anſwered by Mr.* Harland Widd, *of* Whitby.

Let c = tang. $70^\circ = ACB$, $y = AB$, and $x = AD$; then, by the queſtion, $\sqrt{xy} = CD$; and (per trig.) $\sqrt{xy} : 1 :: x : \frac{x}{\sqrt{xy}}$ = tangent ACD; alſo $\sqrt{xy} : 1 :: y - x : \frac{y-x}{\sqrt{xy}}$ = tang. BCD: Whence, by the known theor. for the tangent of the ſum of two angles, we have $\frac{yy}{x\sqrt{xy}} = c$; and therefore $y = x\sqrt[3]{cc} = rx$. Again, (per Euclid 47. 1.) $\sqrt{xx + rxx} + \sqrt{xx - rxx + rrxx}$ $(= AC + BC) = 100$; whence $x = \frac{100}{\sqrt{1+r} + \sqrt{1-r+rr}} = 29·24$, and y $(= rx)$ $= 57·36 = AB$.



The ſame anſwered by Sylvius.

Let the tang. ACB $(70^\circ) = t$; and in acb, ſimilar to ACB, aſſume $cD = 1$, and $aD = x$; then will $ab = \frac{1}{x}$ (per queſt.) and conſequently $Db = \frac{1}{x} - x = \frac{1-xx}{x}$: But

ab

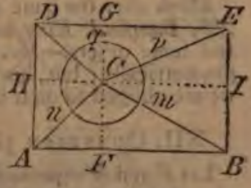
$\frac{ab}{1-aD\times Db}=t$ (= tang. acb), that is, $\frac{1}{x^3}=t$; whence $x=0{\cdot}713984=$ tang. $35^\circ\ 31'\ 34''=acD$: From which the angle $bcD=34^\circ\ 28'\ 20''$; and it will be $ca+cb$ (the ſum of the ſecants of theſe two angles) : $AC+BC :: ac : AC = 50{\cdot}3215$; whence the reſt are eaſily found.

The ſame anſwered by Mr. Henry Watſon.

Upon the ſide AC, of the required triangle ABC, conceive the perpendiculars BE and DF let fall; then, per ſimilar triangles, $\begin{matrix} AC : AD :: AB : AE \\ AC : CD :: CD : CF \end{matrix}$. From whence, as the rectangles under the means of both proportions are equal (per queſt.) we have $AE=CF$, and conſequently $CE=AF$. But $AF\ (CE) : DF :: AD : CD$, and $DF : BE :: AD : AB$: Therefore, by compoſition, $CE : BE :: AD^2 : AB\times CD :: AD^3 : AD\times AB\times CD\ (=CD^3)$; whence $\frac{BE}{CE}=\frac{CD^3}{AD^3}$, and conſequently $\frac{1}{3}$ log. tang. ECB ($\frac{1}{3}$ log. $\frac{BE}{CE}=$ log. $\frac{CD}{AD}$) = log. tang. of $A=54^\circ\ 28'\ 25''$. Now, if in AC produced, there be taken $CH=BC$, and B, H be joined, in the triangle ABH will be given the ſide AH and all the angles; whence $AB=57{\cdot}36$, $BC=49{\cdot}67$, and $AC=50{\cdot}32$.

Meſſrs. *Aſh*, *Bamfield*, *Bevil*, *Brown*, *Cottam*, *Gibbons*, *Johnſon*, *Metcalf*, *Moſs*, *Peart*, *Todd*, and *Widd* alſo anſwered the ſame, in a conciſe and elegant manner.

XI. QUESTION 386 *anſwered by Mr.* William Cottam, *at his Grace the Duke of Norfolk's.*



Let Bm, Ep, Dq, and An, (= 60, 52, 28, and 40) be denoted by m, p, q, and n, reſpectively; and let the radius of the fountain = x: Then (per figure) $\overline{m+x}|^2-\overline{p+x}|^2\ (=BI^2-EI^2=AH^2-DH^2)=\overline{n+x}|^2-\overline{q+x}|^2$; whence $x=\frac{1}{2}\times\frac{m^2+q^2-p^2-n^2}{p+n-m-q}=10$. Now let m, p, q, n, (= 70, 62, 38, and 50) = BC, EC, DC, and CA, reſpectively, and let $A=4840=$ the area of an acre in yards; alſo let x, now, $=GC$; and then we have $\overline{\sqrt{pp-xx}+\sqrt{qq-xx}}\times\overline{x+\sqrt{nn-qq+xx}}=A$.

$= A$; which, ſolved, gives $x = 15'11145$: Hence $DE = AB = 94'99631$, and $AD = BE = 50'94936$. *W.W.R.*

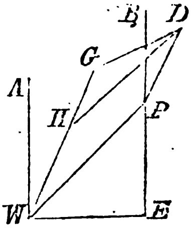
The ſame anſwered by Mr. Hugh Brown.

It is manifeſt that the radius of the pond muſt be 10; becauſe $BC^2 + DC^2 = EC^2 + AC^2$; conſequently $BC = 70\ (= a)$, $EC = 62\ (= b)$, $AC = 50\ (= c)$, $DC = 38\ (= d)$. If through C, the center of the pond, HI and FG be drawn parallel to the ſides of the rectangle, and there be put $AB = x$, and $AD = y$; then will $AF = \frac{x}{2} - \frac{aa-cc}{2x}$, and $AH = \frac{y}{2} + \frac{cc-dd}{2y}$; from whence, and the queſtion, we ſhall (becauſe $AF^2 + AH^2 = AC^2$) have the two following equations, $\frac{xx}{4} + \frac{yy}{4} + \frac{\overline{aa-cc}|^2}{4xx} + \frac{\overline{cc-dd}|^2}{4yy} = \frac{aa}{2} + \frac{dd}{2}$, and $xy = 4840 = A$. Let, now, the firſt equation be multiplied by 4, and $\frac{A}{x}$ ſubſtituted therein for y; ſo ſhall $xx + \frac{A^2}{xx} + \frac{\overline{aa-cc}|^2}{xx} + \frac{\overline{cc-dd}|^2}{A^2} \times xx = 2aa + 2dd$; and conſequently $\overline{1 + \frac{\overline{cc-dd}|^2}{A^2}} \times x^4 - 2 \times \overline{aa+dd} \times xx = -A^2 - \overline{aa-cc}|^2$: Put $1 + \frac{\overline{cc-dd}|^2}{A^2} = g$, $aa + dd = b$, $A^2 + \overline{aa-cc}|^2 = k$, and the equation will ſtand thus, $gx^4 - 2bx^2 = -k$; whence $x = \sqrt{\frac{b + \sqrt{bb - gk}}{g}} = 94'9961$, and $y = \frac{A}{x} = 50'9494$.

Much after the ſame manner it was anſwered by Meſſrs *Aſh*, *Bamfield*, *Kingſton*, *Milbourn*, *Moſs*, *Nichols*, *Widd*, and ſome others.—Mr. *O'Cavanah* conſtructs this problem geometrically; but, his demonſtration being ſomewhat long, we are obliged to omit the whole, for want of room.

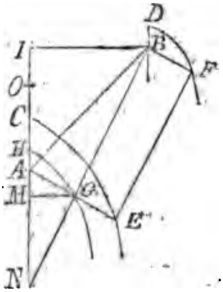
XII. QUESTION 387 *anſwered by Mr.* W. Bevil.

Let P and W repreſent the places of the two pillars, whoſe given heights (40 and 60) let be denoted by a and b, reſpectively; then, ſuppoſing $v =$ the co-tangent of the ſun's altitude, the lengths of their reſpective ſhades, DP and WG, will be av and bv: Draw DH parallel to PW, and WE perpendicular to the two meridian lines WA and BPE, putting $c = DH$ $(= PW) = 200$, $f = b - a$, $s =$ ſine $45° = EWP = AWP$, $z =$ ſine of AWG (= the ſun's azimuth from the ſouth), and $u =$ its coſine; then GH $(= bv - av) = fv$, and $sz + su =$ coſine of $GWP = GHD$: Therefore $HD^2 + HG^2 - GH \times HD \times 2$ coſ. $\angle H = GD^2 = c^2 + f^2v^2 - 2csfvz - 2scfvu$, a minimum: Put $p = \frac{f}{2cs}$, $q = \frac{c}{2fs}$, then $q + pv^2 - vz - vu$, a minimum; let e and $d =$ the ſine and coſine of the latitude, $n =$ the ſine of the ſun's declination, and x and y the ſine and coſine of his altitude, then we have $q + \frac{py^2}{x^2} - \frac{y}{x}\sqrt{1 - \overline{\frac{ex-n}{dy}}\,^2} - \frac{y}{x} \times \frac{ex-n}{dy}$; which, by reduction, (putting $-dp + dq - e = r$, $ee + dd = 1$, and $dd - nn = h$) becomes $\frac{dp + rx^2 + nx - x\sqrt{h - x^2 + 2enx}}{dx^2}$. This, thrown into fluxions and reduced, gives $\overline{2dp + nx} \times \sqrt{h - x^2 + 2enx} = hx + enx^2$; whence $x = .39359048 =$ ſine of 23° 10′ 41″, the ſun's altitude; and from thence the time of the day is found 5 h. 27 m. 3 ſec. in the afternoon.



The ſame anſwered by Mr. T. Moſs.

Let A be the place of the ſouthermoſt pillar, and B that of the northermoſt; and ſuppoſe AE and BF to be any contempoary poſitions of the two ſhadows, taken as parallel: Then, if BG be made parallel to FE, it will be equal to FE, and GE likewiſe $= BF$. But AE is to BF as the height of the pillar A, is to that of the pillar B; whence AG muſt be to AE in the conſtant ratio of the difference of the ſaid heights to the altitude of the pillar at A: So that the path HG of the point G will be exactly ſimilar to that of the point E, and is moreover the very ſame as would be deſcribed by



the ſhadow of an object at *A*, whoſe height is the exceſs of the height of the pillar at *A* above the pillar at *B*: Which path being an hyperbola, ſuppoſe *O* to be its center, and upon *AOC* produced let fall the perpendiculars *GM* and *BI*, producing *BG* to meet *IA* in *N*. Then, *AB* being = 200, and the angle $IAB = IBA$, we have $AI\ (= BI) = 141{\cdot}41$: Moreover, from the ſun's meridian altitude, the length of the meridional ſhadow *AH* (ſuppoſing the height of the object to be 20) is given = 10·656: And (by art. 467 of Simpſon's Fluxions) it will be as, the rectangle of the ſines of the ſun's altitude at noon and depreſſion at midnight, is to the rectangle of the radius and the ſine of twice the ſun's declination, ſo is (20) the height of the object projecting the ſhadow, to (64·072) the tranſverſe axis of the deſcribed hyperbola: Alſo, as the ſquare root of the former of the ſaid rectangles, is to the coſine of the ſun's declination, ſo is the height of the object, to (38·402) the ſemi-conjugate axis of the hyperbola. Put now $a\ (= 32{\cdot}036) = OH$, $b = 38{\cdot}401$, $c\ (= 98{\cdot}728) = OI$, and $d\ (= 141{\cdot}42) = BI$; and let $OM = az$. Then, by the property of the hyperbola, $MG = \frac{b}{a}\sqrt{aazz - aa} = b\sqrt{zz-1}$, and $MN = \frac{bb}{aa} \times az = \frac{bbz}{a}$ (ſince it is evident that *BG*, to be the ſhorteſt poſſible, muſt fall upon the curve at right angles): Hence, becauſe of the ſim. triangles, we have $\frac{MN}{MG} = \frac{NI}{BI}$; that is, $\frac{bz}{a\sqrt{zz-1}} = \frac{\overline{c + a + \frac{bb}{a}} \times z}{d}$, or $\frac{z}{\sqrt{zz-1}} = \frac{ac}{bd} + \overline{\frac{aa}{bd} + \frac{b}{d}} \times z$; which, in numb. becomes $\frac{z}{\sqrt{zz-1}} = 0{\cdot}58237 + 0{\cdot}460532z$; whence $z = 1{\cdot}56204$; $OM\ (= az) = 50{\cdot}04$; $AM = 7{\cdot}348$; MG ($= b \times$ tang. whoſe ſecant is z) $= 46{\cdot}083$; and the angle *MAG* (= the ſun's azimuth from the north) = 80° 56′ 27″; from which the required time is found to be 5 h. 26 m. 58 ſec. after noon.

The ſolution by Mr. *T. Peart* is both conciſe and elegant, as, indeed, is every thing ſent us by this author.

XIII. QUESTION 388 *anſwered by Mr.* E. Rollinſon.

In order to give a general ſolution to this problem, it will be proper to premiſe the following

Lemma. If $\frac{\dot{p}}{1-mpp} + \frac{\dot{q}}{1-mqq} + \frac{\dot{r}}{1-mrr} + \frac{\dot{s}}{1-mss}$, &

&c. $= \frac{\dot{x}}{1 - mxx}$; *wherein* m *is conſtant, and* p, q, r, s, *&c. variable; then, if the ſum of all the quantities* p, q, r, s, *&c. be denoted by A, the ſum of all their rectangles by* B, *the ſum of all their ſolids by* C, *&c. I ſay, that* $x = \frac{A + mC + m^2E + m^3G + m^4I, \&c.}{1 + mB + m^2D + m^3F + m^4H, \&c.}$. For, by taking the fluent, we have hyp. log. $\frac{1 + m^{\frac{1}{2}}p}{1 - m^{\frac{1}{2}}p}$ + hyp. log. $\frac{1 + m^{\frac{1}{2}}q}{1 - m^{\frac{1}{2}}q}$ +, &c. = hyp. log. $\frac{1 + m^{\frac{1}{2}}x}{1 - m^{\frac{1}{2}}x}$; and conſequently $\frac{1 + m^{\frac{1}{2}}p}{1 - m^{\frac{1}{2}}p} \times \frac{1 + m^{\frac{1}{2}}q}{1 - m^{\frac{1}{2}}q}$, &c. $= \frac{1 + m^{\frac{1}{2}}x}{1 - m^{\frac{1}{2}}x}$. Put this value of $\frac{1 + m^{\frac{1}{2}}x}{1 - m^{\frac{1}{2}}x} = \mathcal{Q}$; whence x will be found $= \frac{\mathcal{Q} - 1}{m^{\frac{1}{2}} \times \overline{\mathcal{Q} + 1}} =$

$$\frac{\overline{1 + m^{\frac{1}{2}}p} \,.\, \overline{1 + m^{\frac{1}{2}}q} \,.\, \overline{1 + m^{\frac{1}{2}}r}, \&c. - \overline{1 - m^{\frac{1}{2}}p} \,.\, \overline{1 - m^{\frac{1}{2}}q} \,.\, \overline{1 - m^{\frac{1}{2}}r}, \&c.}{m^{\frac{1}{2}} \,.\, \overline{1 + m^{\frac{1}{2}}p} \,.\, \overline{1 + m^{\frac{1}{2}}q} \,.\, \overline{1 + m^{\frac{1}{2}}r}, \&c. + m^{\frac{1}{2}} \,.\, \overline{1 - m^{\frac{1}{2}}p} \,.\, \overline{1 - m^{\frac{1}{2}}q} \,.\, \overline{1 - m^{\frac{1}{2}}r}, \&c.}$$

(by ſubſtituting the value of $\mathcal{Q}$): But, by multiplication, $\overline{1 + m^{\frac{1}{2}}p} \,.\, \overline{1 + m^{\frac{1}{2}}q} \,.\, \overline{1 + m^{\frac{1}{2}}r}, \&c. = 1 + m^{\frac{1}{2}} \times \overline{p + q + r}, \&c. + m \times \overline{pq + pr}, \&c. = 1 + m^{\frac{1}{2}}A + mB + m^{\frac{3}{2}}C$, &c. &c. Hence our equation becomes $x = \frac{A + mC + m^2E + m^3G, \&c.}{1 + mB + m^2D + m^3F, \&c.}$ *Q.E.D.*

If $m = -1$, the given equation will become $\frac{\dot{p}}{1 + pp} + \frac{\dot{p}}{1 + qq} + \frac{\dot{q}}{1 + rr}$, &c. $= \frac{\dot{x}}{1 + xx}$; and the value of $x = \frac{A - C + E - G, \&c.}{1 - B + D - F, \&c.}$

Now, to apply this to the queſtion propoſed, let the arcs P, $\mathcal{Q}$, R, &c. and their tangents p, q, r, &c. be conſidered as in a flowing ſtate; and let x be the required tangent of $P + \mathcal{Q} + R$, &c. Then, it being known that $\dot{P} = \frac{\dot{p}}{1 + pp}$, $\dot{\mathcal{Q}} =$

$\dot{Q} = \frac{\dot{q}}{1+qq}$, &c. we thence have $\frac{\dot{p}}{1+pp} + \frac{\dot{q}}{1+qq} + \frac{\dot{r}}{1+rr}$, &c. $= \frac{\dot{x}}{1+xx}$; and conſequently $x = \frac{A - C + E - G + I,\ \&c.}{1 - B + D - F + H,\ \&c.}$, by the preceding corollary; A being the ſum of all the tangents p, q, r, s, &c. B the ſum of all their rectangles, C the ſum of all their ſolids, &c. &c. *W. W. R.*

Corollary. If all the arcs P, Q, R, &c. are equal, and their number be denoted by n; then will $A = np$, $B = n . \frac{n-1}{2} p^2$, $C = n . \frac{n-1}{2} . \frac{n-2}{3} p^3$, &c. and therefore $x =$

$$\frac{np - n . \frac{n-1}{2} . \frac{n-2}{3} p^3 + n . \frac{n-1}{2} . \frac{n-2}{3} . \frac{n-3}{4} . \frac{n-4}{5} p^5,\ \&c.}{1 - n . \frac{n-1}{2} p^2 + n . \frac{n-1}{2} . \frac{n-2}{3} . \frac{n-3}{4} p^4,\ \&c.}$$

The ſame anſwered by Mr. W. Bevil.

It is proved, by the writers on trigonometry, that *the tangent of the ſum of two arcs* (*the radius being unity*) *is equal to the ſum of the tangents of thoſe arcs divided by the exceſs of the ſquare of the radius above their rectangle or product.*

Hence the tang. of $P + Q$ will be $\frac{p+q}{1-pq}$; and, if $P+Q$ be conſidered as one arc, then the tangent of $\overline{P+Q}+R$ will, by the ſame rule, be $= \frac{\frac{p+q}{1-pq} + r}{1 - \frac{pr+qr}{1-pq}} = \frac{p+q+r-pqr}{1-pq-pr-qr}$:

After the ſame manner the tangent of $P+Q+R+S$ is found to be $= \frac{p+q+r+s-pqr-pqs-prs-qrs}{1-pq-pr-ps-qr-qs-rs+pqrs}$: And thus, by carrying on the proceſs a ſtep or two farther, the law of continuation will appear manifeſt; being ſuch, that, if the ſum of all the given tangents be denoted by A, the ſum of all their rectangles by B, the ſum of all their ſolids by C, &c. then will the tangent of the ſum of all the arcs be $\frac{A - C + E - G,\ \&c.}{1 - B + D - F,\ \&c.}$.

In this laſt manner it was anſwered likewiſe by Mr. *Hugh Brown*, Mr. *G. Burgeſs*, Mr. *T. Moſs*, Mr. *Harland Widd*, and ſome others.

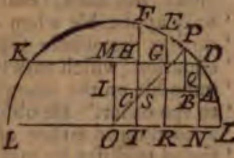
XIV. QUES-

XIV. QUESTION 389 *anſwered by the propoſer,* *Mr.* E. Rollinſon.

Let R and r be the ſemi-diameters of the orbits of the earth and moon, P and p the periodic times in thoſe orbits, S and s the ſun's mean apparent ſemi-diameter and moon's mean horizontal parallax, and N and n any two numbers in the required ratio of the denſities of the ſun and earth, reſpectively. Then, the real ſemi-diameters of the ſun and earth being in the ratio of RS to rs, their maſſes will be as $R^3S^3 \times N : r^3s^3 \times n$; and conſequently their forces, at the diſtances R and r, as $\frac{R^3S^3N}{R^2} : \frac{r^3s^3n}{r^2}$, or as $RS^3N : rs^3n$. But theſe (by the laws of central forces) are alſo as $\frac{R}{PP} : \frac{r}{pp}$; therefore, by dividing the antecedents of theſe equal ratios by RS^3, and the conſequents by rs^3, we have as $N : n :: \frac{1}{P^2S^3} : \frac{1}{p^2s^3} :: 1 : \frac{P^2}{p^2} \times \frac{S^3}{s^3}$; which, in numbers, (taking $P = 365$ d. 5 h. 49 m. $p = 27$ d. 7 h. 43 m. $S = 16$ m. $5\frac{1}{4}$s. and $s = 57$ m. $17\frac{1}{2}$s.) will come out as 1 to 3·957, for the ratio of the denſity of the ſun to that of the earth. *W.W.R.*

XV. QUESTION 390 *anſwered by Mr.* J. Morland.

Conſtruction. In any right line AI ſet off SA, SB, and SC, equal to the three given longitudes of the moon from the ſun; and make AD, BE, and CF perpendicular to AI, ſo as to expreſs the given latitudes correſponding: Then, through the three points D, E, and F, let the circumference of a circle be deſcribed; and from O the center thereof, through S, draw the radius OP, and upon AI let fall the perpendicular PQ: So ſhall SP be the diſtance of the two centers, at the time of the greateſt obſcuration, and SQ the required difference of longitudes at that time. For, ſince the circumference of the circle thus deſcribed, coincides with the real curve (whatever it is) in three points (D, E, F) which are but at a ſmall diſtance from one another, it muſt neceſſarily have nearly the ſame degree of curvature, and therefore likewiſe coincide with it in the intermediate ſpaces, very near. To derive the numerical ſolution from this conſtruction, let the chord DK be parallel to AI, and let OIM be perpendicular



dicular thereto; alſo let *FHC* and *EGB* be produced to meet the diameter *LL*, at right angles, in *T* and *R*.

Put *DG* (*AB*) $= a$, *DH* (*AC*) $= b$, *EG* (*BE* $-$ *AD*) $= c$, *FH* (*CF* $-$ *AD*) $= d$, *OM* $= x$, and *MK* (*MD*) $= y$: Then, by the property of the circle, $GD \times GK = GE \times \overline{2GR + GE}$, and $HD \times HK = HF \times \overline{2HT + HF}$; that is, $a \times \overline{2y - a} = c \times \overline{2x + c}$, and $b \times \overline{2y - b} = d \times \overline{2x + d}$: Whence x is found $= \dfrac{a \times \overline{bb + dd} - b \times \overline{aa + cc}}{2bc - 2ad}$, and $y = \dfrac{c \times \overline{bb + dd} - d \times \overline{aa + aa}}{2bc - 2ad}$: From which values, thoſe of *OK* (*OP*), *OI*, *OS*, *PS*, *SQ*, and *AQ* will all become known.—As to the time anſwering to this (or any other) given value of *AQ*, it is beſt determined from the common method of interpolating by differences: According to which, the two given intervals correſponding to *AB* and *BC* being denoted by p and q, the required interval, between the poſition *Q* and the firſt poſition *A*, will be repreſented by $\dfrac{\overline{pbb - qaa} \times AQ + \overline{qa - pb} \times AQ^2}{ab \times \overline{b - a}}$

(See M. de Caille's Aſtronomy, p. 60.)

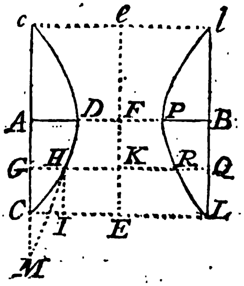
The propoſer reſolves this problem by means of a parabolic curve deſcribed through three given points: And obſerves, 'That, if the equation $yy = g + hx + kx^2$ were to be aſſumed for the general relation of the latitude (y) to the difference of longitude (x), the reſult would come out more neat and ſimple than from any curve of the parabolic kind:' But adds, 'that this laſt method is not general, being only applicable when the moon has a conſiderable latitude during the whole time of the eclipſe; ſince the aſſumed equation (which anſwers to an ellipſis or hyperbola) becomes impoſſile on the moon's paſſing from one ſide of the ecliptic to the other.' He obſerves farther, 'That the concluſions, according to either of the above methods, will ſeldom be found to differ by more than one minute in time from thoſe ariſing from the common way of computation.' Which laſt he therefore thinks may be uſed as ſufficiently near, till the theory of the moon's motion is known to a greater degree of exactneſs.

Mr. *Harland Wild* ſent a ſolution to this problem.

The PRIZE QUESTION *anſwered by the Propoſer.*

Suppoſing *Cc* and *Ll* to be the lengths, and *AD* and *BP* the ſemi-breadths of two adjacent piers (or ſtarlings), let *Ee*, parallel to *Cc*, biſect *AB* at right angles, in *F*; and let *IH*, parallel to *Ee*, be the direction of a particle of water im-

pinging



pinging upon the ſurface *CD* in *H*; alſo let *HM* be a tangent at *H*, interſecting *AC* produced in *M*.

Call *AC*, *a*; *AD*, *b*; *CE*, *c*; *CG*, *x*; and *GH*, *y*; and let the tangents of the angles *ECD* and *ADC* (to the radius 1) be denoted by *p* and *q* reſpectively. The celerity wherewith the ſtream paſſes any ſection *HR* being inverſely as the breadth, the velocity of the particle acting at *H* will therefore be as $\frac{1}{c-y}$, and its force (by mechanics) as $\frac{1}{c-y} \times \frac{GH^2}{MH^2} = \frac{1}{c-y} \times \frac{\dot y\dot y}{\dot x\dot x+\dot y\dot y}$; which, drawn into $\dot y$, gives $\frac{1}{c-y} \times \frac{\dot y^3}{\dot x\dot x+\dot y\dot y}$ for the fluxion of the reſiſtance upon *CH*. But (by art. 408 of Simpſon's Fluxions) it appears that, if *S* be aſſumed to denote any quantity expreſſed in term of *y* and given coefficients, the fluent of $S \times \frac{\overline{\dot x\dot x+\dot y\dot y}|^{n}}{\dot y^{2n-1}}$ (correſponding to any given value of *x*) will be a max. or min. when the relation of *x* and *y* is ſuch that the value of $\frac{S\dot x \times \overline{\dot x\dot x+\dot y\dot y}|^{n-1}}{\dot y^{2n-1}}$ is every-where the ſame. Therefore, by transforming our fluxion, $\frac{1}{c-y} \times \frac{\dot y^3}{\dot x\dot x+\dot y\dot y}$, to $\frac{1}{c-y} \times \frac{\overline{\dot x\dot x+\dot y\dot y}|^{-1}}{\dot y^{-3}}$, and then comparing it with that above, we ſhall get $\frac{1}{c-y} \times \frac{\dot x\dot y^3}{\overline{\dot x\dot x+\dot y\dot y}|^2} = d$, (a conſtant quantity). In order to the reſolution of this equation, put $\dot x = v\dot y$ (v being the tang. of the angle *MHG*) then, by ſubſtitution, &c. $\frac{1}{c-y} \times \frac{v}{\overline{vv+1}|^2} = d$; and conſequently $d \times \overline{c-y} = \frac{v}{\overline{1+vv}|^2}$: Which, in fluxions, gives $-d\dot y = \frac{\dot v - 3v^2\dot v}{\overline{1+vv}|^3}$, and $dv\dot y = d\dot x) = \frac{3v^3\dot v - v\dot v}{\overline{1+vv}|^3}$; whereof the corrected fluent will be $dx = \frac{1+3pp}{2\cdot\overline{1+pp}|^{2}} - \frac{1+3vv}{2\cdot\overline{1+vv}|^{2}}$.

To find d, take $y=0$; in which circumſtance, v being $=p$, the equation $\frac{1}{c-y} \times \frac{v}{\overline{vv+1}|^{2}} = d$ becomes $\frac{p}{c\cdot\overline{1+pp}|^{2}} = d$; which value being ſubſtituted for d, our two equations, after proper reduction, will become $y = c - \frac{cv\times\overline{1+pp}|^{2}}{p\times\overline{1+vv}|^{2}}$, and $x = c\times\frac{1+3pp}{2p} - c\times\frac{\overline{1+3vv}\times\overline{1+pp}|^{2}}{2p\times\overline{1+vv}|^{2}}$.

Theſe equations give the general relation of x, y, and v; but to apply them to any particular caſe propoſed, ſomething further remains to be done, ſince the value of p (the tangent of the angle ECH) is not given, but muſt be found from the known values of CE, CA, and AD. In order to this, ſuppoſe H to coincide with D; then, x becoming $=a$, $y=b$, $v=q$, if theſe values be ſubſtituted in the aforeſaid equations, we ſhall, after due reduction, have $\frac{\overline{1+pp}|^{2}}{\overline{1+qq}|^{2}} = \frac{c-b}{c} \times \frac{p}{q}$, and $a = c\times\frac{1+3pp}{2p} - \overline{c-b}\times\frac{1+3qq}{2q}$.

Put $r = \frac{2a}{c} + \frac{c-b}{c} \times \frac{1+3qq}{q}$, and then $p = \frac{\sqrt{rr-12}+r}{6}$; from which equations the values of p and q may be found. Thus, for example, if a be ſuppoſed $=12$, $b=5$, and $c=8$; then p will come out $=2$, and $q=3$: So that the angles ECH and ADC are here 63° 26′ and 71° 34′, reſpectively.

Corollary. If c be ſuppoſed exceeding great, or, which comes to the ſame, if every particle of the fluid impinges with the ſame velocity, then $\frac{y}{c}$ will vaniſh, and the equation $\frac{y}{c} = 1 - \frac{v\times\overline{1+pp}|^{2}}{p\times\overline{1+vv}|^{2}}$ will become $\frac{v\times\overline{1+pp}|^{2}}{p\times\overline{1+vv}|^{2}} = 1$, and conſequently $v=p$; therefore the angle CHI being every-where the ſame, CD will, in this caſe, become a right line. From whence it appears, that the leſs DF is in reſpect of CE, the greater muſt be the curvature of the ſurface upon which the water acts.

This was the only true ſolution received.

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